# The Crude Oil Futures Curve, the U.S. Term Structure and Global Macroeconomic Shocks

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# Disclaimer

The views expressed in this paper are the authors' own and do not necessarily represent those of the Bank of Canada.

# Motivation: What Macroeconomic Forces Drive Oil-Futures Risk Premia?

- Increasing consensus that oil price and term structure set in globally integrated market.
  - Oil: Barsky and Kilian (2002); Hamilton (2009); and Kilian (2009).
  - Bonds: Perignon, Smith, and Villa (2007); Bekaert and Wang (2009); Dahlquist and Hasseltoft (2011); and Bauer and Diez de los Rios (2012).
- Firms with no physical interest in oil now trade more in futures market (e.g., Büyüksahin and Harris 2011; Bassam, Kilian, and Fattouh 2012; and Hamilton and Wu 2012).
- As trading of longer-horizon oil futures contracts increases, shifts in term structure should matter more for pricing oil futures contracts.

# Motivation: How Do We Model These Macroeconomic Forces?

- Price the oil futures curve by computing the term structure of convenience yields using affine term-structure model.
- Convenience yield captures value of having physical access to crude oil.

$$\delta_{t,n} = y_{t,n} - \frac{(f_{t,n} - s_t)}{n}$$

  - Assumed to be concave in inventories.
- Model integrates oil futures curve into a term structure model.

# Outline

- Model
- Estimation
- Preliminary Results
- Conclusions

# Model

## Standard term structure model

- Our term structure model is standard (e.g., Cochrane and Piazzesi 2008; Josline, Priebsch, and Singleton 2010; Joslin, Le, and Singleton 2011; Wright 2012; Joslin, Singleton, and Zhu 2012).

- Linear relationship between yields $\mathbf{y}_t$ and pricing factors, $\mathbf{x}_t = (\mathbf{b}_t', \mathbf{m}_t')'$:
  - Bond market factors: $\mathbf{b}_t = \mathbf{P}_1' \mathbf{y}_t$
  - Unspanned macroeconomic factors: $\mathbf{m}_t$
- Measurement equation (cross-section):

$$\mathbf{y}_t = \boldsymbol{\alpha}_{\mathbf{y}}(\boldsymbol{\theta}^Q) + \boldsymbol{\beta}_{\mathbf{y}}(\boldsymbol{\theta}^Q)\mathbf{b}_t + \varepsilon_t$$

- Transition equation (time-series):

$$\mathbf{x}_{t+1} = \boldsymbol{\mu}(\boldsymbol{\theta}^Q, \boldsymbol{\lambda}) + \boldsymbol{\Phi}(\boldsymbol{\theta}^Q, \boldsymbol{\lambda})\mathbf{x}_t + \mathbf{v}_{t+1}$$

  with $\mathbf{v}_{t+1} \sim iid\ N(0, \boldsymbol{\Sigma})$
  - $\boldsymbol{\theta}^Q$ : parameters driving the risk-neutral dynamics
  - $\boldsymbol{\lambda}$: *restricted* parameters driving the prices of risk

- Short rate is affine function of bond market factors:

$$y_{t,1} = \psi_0 + \boldsymbol{\psi}' \mathbf{b}_t$$

- Prices of risk affine in state variables $\mathbf{x}_t$ :

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\lambda} \mathbf{x}_t$$

- Stochastic discount factor (e.g., Ang and Piazzesi 2003; and Cochrane and Piazzesi 2008):

$$\xi_{t+1} = \exp\left(-y_{t,1} - \frac{1}{2}\boldsymbol{\lambda}_t' \boldsymbol{\Sigma}^{-1} \boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t' \boldsymbol{\Sigma}^{-1} \mathbf{v}_{t+1}\right)$$

# Model

## Pricing the bonds

- Use $\xi_{t+1}$ to price zero-coupon dollar bonds:

$$P_{t,n} = E_t \left[ \xi_{t+1} P_{t+1,n-1} \right]$$

- Continuously compounded yield at any maturity affine function of bond factors:

$$y_{t,n} = \alpha_{y,n} + \boldsymbol{\beta}'_{y,n} \mathbf{b}_t ,$$

- Yields are function of factors directly and macroeconomic variables through prices of risk $\boldsymbol{\lambda}_t$.

# Model

- Apply identical arguments to *synthetic zero-coupon oil bond* with discount rate equal to convenience yield:

$$O_{t,n} = \exp(-\delta_{t,n}) = \frac{F_{t,n} P_{t,n}}{S_t}$$

- 1-month convenience yield is affine in a convenience yield factor:

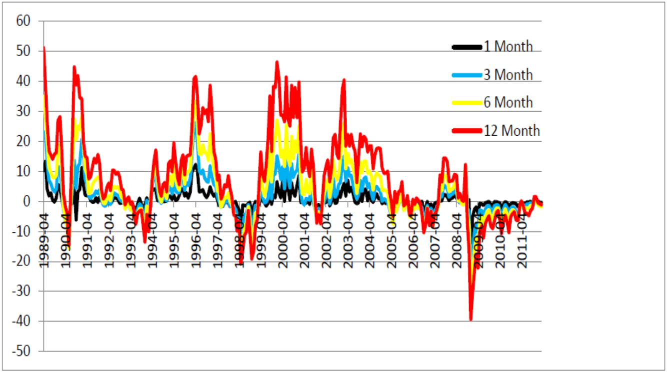
$$\delta_{t,1} = \phi_0 + \boldsymbol{\phi}' \mathbf{c}_t$$

- Synthetic zero-coupon oil bond can be priced using oil SDF $\xi^{oil}_{t+1}$:

$$O_{t,n} = E_t \left[ \xi^{oil}_{t+1} O_{t+1,n-1} \right]$$

# Model

**Figure 3**
**The Term Structure of the Oil Market Convenience Yields**
(The chart shows the 1, 3, 6 and 12 month convenience yields calculated from the no arbitrage condition shown in the text. The units are in per cent per annum.)



- Assume no arbitrage between oil and U.S. Treasury markets:

$$\Delta s_{t+1} = \log \xi^{oil}_{t+1} - \log \xi_{t+1}$$

- Given $\Delta s_{t+1}$and $\log \xi_{t+1}$, yields $\log \xi^{oil}_{t+1}$ :

$$\xi^{oil}_{t+1} = \exp\left[-\delta_{t,1} - \frac{1}{2}\boldsymbol{\lambda}^{oil\prime}_t \boldsymbol{\Sigma}^{-1} \boldsymbol{\lambda}^{oil}_t - \boldsymbol{\lambda}^{oil\prime}_t \boldsymbol{\Sigma}^{-1} \mathbf{v}_{t+1}\right]$$

- Continuously compounded convenience yield is:

$$\delta_{t,n} = \alpha_{t,n} + \boldsymbol{\beta}_{\mathbf{t,n}} \mathbf{c}_t$$

- The log price of the oil futures contract can be written as:

$$
\begin{aligned}
f_{t,n} &= s_t + ny_{t,n} - n\delta_{t,n} \\
&= s_t - (A_{y,n} + B'_{y,n}\mathbf{b}_t) + (A_{\delta,n} + B'_{\delta,n}\mathbf{c}_t)
\end{aligned}
$$

# Model

## Choice of macroeconomic state variables

- $\mathbf{m}_t = (\Delta prod_t, rea_t, \Delta s_t, \pi_t, s_t - p_t)'$
  - Percent change in global crude oil production
  - Global real activity index (Kilian 2009)
  - Change in log nominal price of WTI
  - U.S. CPI inflation
  - Real price of oil is an error-correction term
- Affine term structure models are VAR(1), ours is VECM(1).

# Estimation

Estimating the model proceeds in 3 steps.

1. Estimate risk-neutral parameters using OLS (Diez de los Rios 2012).
2. Estimate prices of risk after imposing restrictions : $\widehat{\boldsymbol{\lambda}}$.
3. Physical VAR: $\widehat{\boldsymbol{\mu}} = \boldsymbol{\mu}(\widehat{\boldsymbol{\theta}}^{Q}, \widehat{\boldsymbol{\lambda}})$ and $\widehat{\boldsymbol{\Phi}} = \boldsymbol{\Phi}\ (\widehat{\boldsymbol{\theta}}^{Q}, \widehat{\boldsymbol{\lambda}})$.

- When using linear combinations of yields (factors) as state variables, model-implied values must be identical to those used in estimation (Cochrane and Piazzesi 2005).
- This identifies the risk neutral parameters (Joslin, Singleton, and Zhu 2012).

$$\mathbf{x}_{t+1} = \boldsymbol{\mu}^Q + \boldsymbol{\Phi}^Q \mathbf{x}_t + \mathbf{v}^Q_{t+1}$$

with $\mathbf{v}^Q_t \sim iid\ N(0, \boldsymbol{\Sigma})$.

- Risk neutral dynamics differ from the physical dynamics:

$$\begin{aligned} \boldsymbol{\mu}^Q &= \boldsymbol{\mu} - \boldsymbol{\lambda}_0 \\ \boldsymbol{\Phi}^Q &= \boldsymbol{\Phi} - \boldsymbol{\lambda} \end{aligned}$$

- Given risk neutral parameter estimates, estimate prices of risk.
- For example, estimate price of U.S. dollar bond risk:

$$b_{t+1} - \left( \widehat{\boldsymbol{\theta}}_1^Q + \widehat{\boldsymbol{\Phi}}_{11}^Q \mathbf{b}_t \right) = \lambda_{10} + \lambda_{11} b_t + \lambda_{12} c_t + \lambda_{13} m_t + \lambda_{14} \Delta s_t + v_{1,t+1}$$

where $\widehat{\boldsymbol{\theta}}_1^Q$ and $\widehat{\boldsymbol{\Phi}}_{11}^Q$ are estimates of risk-neutral parameters.

- Necessary to impose restrictions on prices of risk.
  - Risk-neutral distribution can provide information about time series dynamics of yields (Cochrane and Piazzesi 2008).
  - Physical dynamics would be same as risk-neutral dynamics (Joselin, Singleton, and Zhu 2012).

- Restrictions:
  - $\mathbf{b}_t'$: Bond slope, global real activity, inflation, and real price of oil.
  - $\mathbf{c}_t'$: Bond level, lagged conv. yield, % change global oil production, global real activity, and real price of oil.
  - $\Delta s_t$: Bond level, lagged conv. yield, % change global oil production, global real activity, and real price of oil.

# Estimation

## Step 3: Recover Physical Distribution

- Assemble risk-neutral parameters obtained in step 1 with prices of risk from step 2:

$$
\begin{aligned}
\widehat{\boldsymbol{\mu}}^{Q} + \widehat{\boldsymbol{\lambda}}_0 &= \widehat{\boldsymbol{\mu}} \\
\widehat{\boldsymbol{\Phi}}^{Q} + \widehat{\boldsymbol{\lambda}} &= \widehat{\boldsymbol{\Phi}}
\end{aligned}
$$

- Thus we can decompose oil-futures curve in expectational component and time-varying risk premia (ignoring Jensen's inequality term):

$$E_t\left[s_{t+1} - s_t\right] = \left(y_{t,1} - \delta_{t,1}\right) + orp_t$$

- Sample period: 1989.4-2012.3.
- U.S. Treasury yields: 1-month to 10 years.
- Nominal WTI spot price and the prices of WTI futures contracts to 12 months.

# Preliminary Results

## Fitted Yields

**Table 4**

**Model Implied Pricing Errors**

(RMSPE is the model's root mean squared pricing error, while MAD gives the mean absolute pricing error. The affine model of the oil futures curve and the U.S. term structure of interest rates was estimated over the April, 1989 to March, 2012 period.)

| | RMSPE | MAD |
|---|---|---|
| Dollar Bonds | 0.0305 | 0.0183 |
| Oil Bonds | 5.8519 | 3.681 |
| Oil Futures | 1.3072 | 0.0368 |

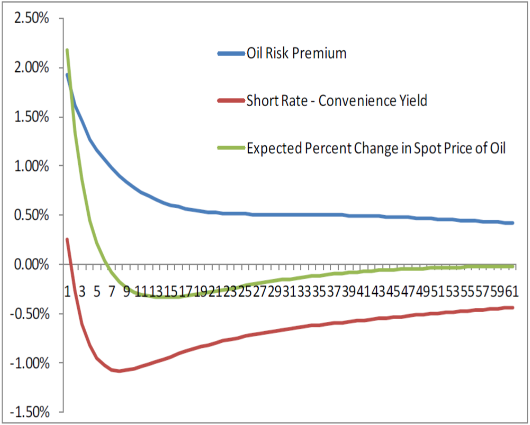
# Preliminary Results

- Restricted model estimated in levels based on recursive ordering.
  - Cointegration imposed between nominal price of oil and U.S. CPI (i.e., model is a VAR(2)).
- Oil-market variables ordered first: oil production, global real activity, the convenience yield factor, nominal price of oil, and U.S. CPI.
- U.S. term structure variables (curvature, slope, and level) ordered last to identify monetary policy shocks (e.g., Christiano, Eichenbaum, Evans 1999).

# Preliminary Results

## Impulse response to an unexpected 25 b.p. increase in short rate

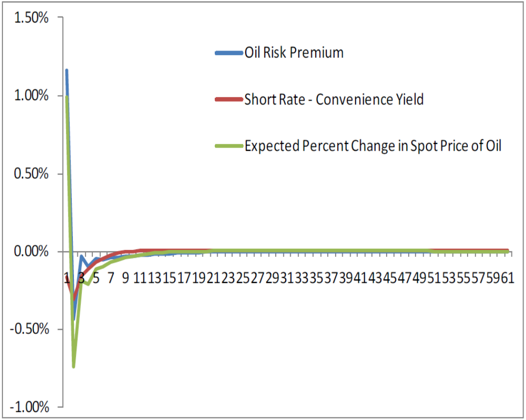
Figure 5.
Impulse Responses of Oil Futures Risk Premia and Expectations to a 25 b. p. Contractionary Monetary Policy Shock



# Preliminary Results

## Impulse response to an unexpected 1% decrease in global crude oil production

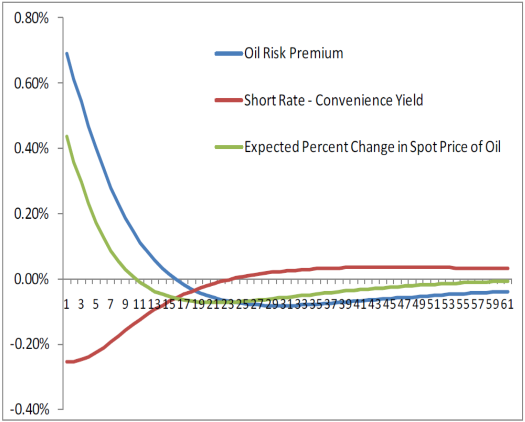
**Figure 6.**
**Impulse Responses of Oil Futures Risk Premia and Expectations to a 1% Decrease in Global Crude Oil Production**



# Preliminary Results

Impulse response to an unexpected 10% increase (approx. one-std. dev.) in global real activity

Figure 7.
Impulse Responses of Oil Futures Risk Premia and Expectations to a 10% Increase in the Global Real Activity Index



# Conclusions

- Develop model of the oil futures curve, term structure of interest rates, and global macroeconomic factors.
- Estimate response of oil-risk premia and expectations to unexpected monetary policy shocks, and oil supply and demand shocks.
- Preliminary estimates suggest different shocks have different effects on response of oil-risk premium.

# Conclusions

## Next steps

- Address econometric issues: Compute standard errors and correct for finite-sample bias in VAR estimates (Bauer, Rudebusch, and Wu 2012).
- Estimate model omitting period during which interest rates were at zero-lower bound.
- Compute decomposition of time-varying risk premium in crude oil market and expected oil-price changes, and their responses for longer horizons.