Price Points and Price Dynamics*

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Abstract

This paper proposes a macroeconomic model with positive trend inflation that involves an important role for price points as well as sticky information. We argue that, in particular, a variant of our model that allows for a general distribution of price points is more successful in explaining several stylized facts of individual price setting than a benchmark model that is based on Calvo price-setting. More specifically, it makes empirically reasonable predictions with regard to the duration of price spells, the sizes of price increases and decreases, the shape of the hazard function, the fraction of price changes that are price increases, and the relationship between price changes and inflation. Moreover, our model implies plausible aggregate effects of monetary policy in contrast with a model with a prominent role for price points but no information rigidities.

Keywords: price stickiness; price point; sticky information

JEL: E31, E37

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1 Introduction

Understanding the nature of microeconomic price rigidities is central to monetary economics. The leading paradigm in monetary economics, the new Keynesian model, starts from the premise that the extended durations of price spells found in the data point to the existence of price-adjustment costs, which are often modeled in a shorthand manner by time-dependent pricing. These costs and the resulting price rigidities are considered to be instrumental for understanding why monetary policy has real effects.[1] This paper explores an alternative mechanism for explaining spells of constant prices. In particular, it develops a monetary model with two features. First, it follows Knotek (2016) and includes a prominent role for price points in firms' price-setting decisions. Second, it incorporates information rigidities in a way that was first proposed by Mankiw and Reis (2002).[2] We compare our model with a standard new Keynesian model based on Calvo (1983) price-setting and show that it is arguably more successful in replicating a number of stylized facts about the dynamics of individual prices. Moreover, it also produces plausible responses of economic aggregates to monetary shocks.

As discussed in more detail in Section 2, there is strong evidence in favor of the relevance of price points, as some prices are chosen much more frequently compared to other prices (see Kashyap (1995), Blinder et al. (1998), Dhyne et al. (2006), Levy et al. (2011) and Chen et al. (2017)). In this paper, we therefore build on an idea by Knotek (2016), who explores the possibility that price points rather than menu costs can explain why prices typically remain fixed for two to three quarters, which has been documented by Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008).[3]

[1] See Woodford (2003) for a textbook treatment.

[2] We discuss papers that compare Mankiw and Reis' sticky-information model with the more traditional sticky-price approach in Section 8.

[3] To see that price points have the potential for explaining this pattern, consider a simple thought experiment. Suppose that the desired price of a firm, i.e. the price that would maximize profits if the firm could set its price to any real number costlessly in each period, would increase continuously at a

In particular, Knotek (2016) considers a model where firms incur menu costs and additional costs when they choose prices that are not price points. According to his estimation, menu costs are effectively irrelevant as a source of price rigidity. He shows that, as a consequence, monetary shocks have almost no effects on real variables. As the latter implication is at odds with the general consensus that monetary policy has short-term real effects, we propose a model that, in addition to price points, also incorporates information frictions. As a result, our model can explain spells of constant prices, while at the same time being compatible with the empirical regularity that monetary policy can affect output and other real variables in the short run. In contrast with Knotek (2016), who focuses on the potential of price points to explain the empirically observed durations of price spells, we also explore whether our model is compatible with a host of other stylized facts of price dynamics.

More specifically, we examine a simple general-equilibrium model with positive trend inflation that is populated by individual price setters, which are subject to idiosyncratic productivity shocks and aggregate monetary disturbances.[4] Our workhorse model is an extension of a textbook macroeconomic model (see Woodford (2003)) with the following two modifications. First, we abstract from menu costs and impose a price-point restriction (PPR), i.e. a requirement that firms may only select prices from a discrete set of price points. Second, because a model with the PPR but without additional costs of adjusting prices has the counterfactual implication that monetary policy is completely ineffective, we incorporate sticky information as in Mankiw and Reis (2002) into our model.

We evaluate this model's ability to explain several stylized facts of microeconomic price adjustment, which were documented by Klenow and Kryvtsov (2008) (henceforth: KK)

constant rate equal to the average rate of inflation. Suppose next that the firm would be constrained to choose prices only from a discrete set of price points, say prices that end with the digit 9. In this case, it appears plausible that, while the desired price is moving upwards gradually, the posted price would move in a step-wise fashion and remain constant for considerable time intervals.

[4]A positive rate of trend inflation allows us to study the differences in magnitudes of price increases and decreases.

and Nakamura and Steinsson (2008) (henceforth: NS), and others.[5] Moreover, we compare our main model to a variant without the PPR but price stickiness à la Calvo (1983).[6] In particular, we derive the following findings.

Our benchmark model with the PPR and sticky information (henceforth: PP), as well as the sticky-price model with Calvo pricing (henceforth: SP), are consistent with the following stylized facts, which are reported in KK: First, prices stay constant on average for 2-3 quarters while, second, duration spells are significantly variable. Third, the magnitude of relative price changes is 11% on average. Fourth, the intensive margin dominates the variance of inflation.

The PP model outperforms the SP model along several dimensions: First, the magnitude of price decreases is larger than the magnitude of price increases (Burstein and Hellwig (2007), KK). Second, prices move back and forth between a few rigid values (Eichenbaum et al., 2011; Knotek, 2016).[7] Third, the frequency of price changes co-varies with inflation (KK).

The basic PP model has two major shortcomings. In particular, the hazard curve of price adjustments has a maximum at around 6 to 7 quarters, although the data suggest that hazard curves are roughly flat (KK, NS). Moreover, a plot of the magnitude of relative price changes as a function of the age of the price reveals a minimum at around 6 to 7 quarters, which is at odds with the empirical finding that this curve should be approximately flat as well. Both problems can be traced back to the assumption made in our basic PP model that all price points are located on an equally spaced grid. As a consequence, we introduce an extended PP model with a more general

[5] For a comprehensive review of the literature on individual prices see Klenow and Malin (2010) and Nakamura and Steinsson (2013).

[6] In order to isolate the effects of sticky information on price dynamics we analyze a third variant that includes the PPR but no information stickiness in a separate appendix, which is available upon request.

[7] Several papers (Eichenbaum et al., 2011; Matějka, 2015, 2016; Alvarez et al., 2016) provide a rationale for this observation by modeling firms as following a price plan consisting of a finite number of points.

distribution of price points in Section 7 and demonstrate that this extension alleviates both shortcomings of the basic PP model.

The remainder of our paper is organized as follows. In Section 2, we review the empirical evidence on price points. Section 3 presents our model. Analytical results for log-linearized versions of our model are derived in Section 4. Our simulation strategy is described in Section 5 and our main findings about price dynamics are presented in Section 6. We discuss the extension to our basic PP model that allows for a richer distribution of price points in Section 7. Section 8 studies the impact of monetary shocks on aggregate variables. In Section 9, we discuss the relationship between our PPR and menu costs. Section 10 concludes.

2 Evidence on Price Points

In the last two decades, a rich literature documenting the dynamics of individual prices has emerged. One of the striking regularities observed in the data is the presence of price points, i.e. prices with special endings, for instance the digits 5 or 9, which are used substantially more often than other prices.

Several cognitive and behavioral mechanisms have been proposed as a rationale for price points.[8] A major reason for firms to choose threshold prices like \$1.99 may be that consumers perceive the difference between \$1.99 and \$2.00 to be larger than, say, the difference between \$1.98 and \$1.99. Hence demand may drop disproportionately when firms raise their price from \$1.99 to \$2.00, which makes it comparably likely that they choose \$1.99. A related concept is that of convenient prices, i.e. prices chosen because they require few pieces of money or little change (see Knotek (2008)). This concept helps to explain why certain goods like newspapers are often sold at prices

[8] For surveys see Monroe (1973) and Hamadi and Strudthoff (2016).

such as \$1.00, \$1.50, or \$2.00. For the purpose of our paper, it is only important that firms prefer a certain class of prices to other prices; the exact mechanism why these prices are preferred is not relevant.

Early evidence on the role of price points for price rigidity stems from Kashyap (1995), who analyzes prices in retail catalogues, and Blinder et al. (1998), who conduct a survey on price stickiness among U.S. firms. More recently, Levy et al. (2011) use both scanner and online prices in the U.S. to document that prices with 9-endings occur more frequently than other prices, that they are less likely to change and that the magnitude of price changes is larger for these prices in comparison to prices with non-9-endings.[9]

Price points are important for a broad set of consumer prices as well. Figure 1, which shows the distribution of last digits of consumer prices in the United Kingdom, demonstrates that the distribution clearly differs from a uniform distribution, which one would plausibly expect if price points played no role. Interestingly, while "9" occurs comparably often as a last digit, the most frequently chosen last digit is "0." This could point to the relevance of convenient prices or the existence of large threshold prices like £49.00, where the last digit before the decimal point is "9."[10] A closer look at the data reveals that for some categories of products in the ONS database, "9" is the most frequent last digit, whereas "0" occurs most often for other product categories.

It may also be instructive to examine the most frequently chosen prices in the ONS database. As can be seen from Table 1, most of the fifteen most frequently used prices end with "99," "00," or "50." It may also be noteworthy that the three largest prices

[9]Price points have been found to be empirically relevant in other countries too. Dhyne et al. (2006) summarize the evidence from the Eurosystem's Inflation Persistence Network and document that across various European countries price points matter for the frequency of price changes. Recently, Chen et al. (2017) provide additional evidence for the relevance of price points. They show that after the Israeli parliament restricted prices to have a zero ending in January 2014, 90-ending prices became the new price points.

[10]Even prices that do not obviously qualify as price points, like a price of \$1.34, may be the result of price adjustments in discrete steps. For example, a price of \$1.34 could be the result of a 10% discount on an original price of \$1.49.

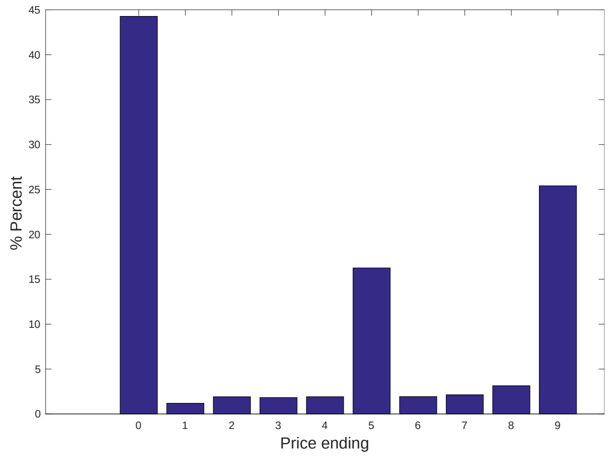

Figure 1: Distribution of final digits for all consumer prices in the UK from February 1996 to December 2016, all prices weighted with the weights used for the construction of the CPI. Source: Office of National Statistics (ONS), own calculations

in this list, £10, £20 and £25, appear to be rather special. Moreover, indirect evidence suggesting the relevance of price points stems from the observation that prices jump discontinuously between a few fixed values (NS, Eichenbaum et al. (2011), Knotek (2016)). This pattern cannot be reconciled with simple menu-cost models easily.

3 Model

Having discussed the empirical evidence in favor of price points, we propose two variants of a text book macroeconomic model (see Woodford (2003)) with positive trend inflation and idiosyncratic productivity shocks as in Gertler and Leahy (2008).[11] In our main model, which we label PP, we consider firms that can choose prices only from a discrete set of price points, i.e. subject to a price-point restriction (PPR). In addition, firms' pricing behavior is subject to information rigidities à la Mankiw and Reis (2002). As a benchmark, we examine a second version of our model. This version

[11]Idiosyncratic productivity shocks are necessary to generate the large price changes observed in the data (see Golosov and Lucas (2007)).

rank	price	relative freq.
1	2.00	1.11%
2	1.00	0.93%
3	2.50	0.85%
4	3.00	0.81%
5	1.50	0.81%
6	7.99	0.73%
7	10.00	0.71%
8	0.99	0.69%
9	1.99	0.67%
10	6.99	0.67%
11	3.50	0.62%
12	1.80	0.62%
13	2.20	0.61%
14	25.00	0.61%
15	20.00	0.60%

Table 1: Most frequent consumer prices in the UK from February 1996 to December 2016, all prices weighted. Source: ONS, own calculations

incorporates neither price stickiness nor a PPR but involves sticky prices as in Calvo (1983). We label the sticky-price model SP in the following.

Time is discrete and denoted by $t = 0, 1, 2, ...$ The model is populated by households and monopolistically competitive firms. We provide details about each of these groups in turn.

3.1 Households

There is a continuum of households that own identical shares of all firms and receive firms' profits as dividends. The households' instantaneous utility function in period t is

$$u(C_t, M_t/P_t, N_t) = \ln\left[C_t\left(\frac{M_t}{P_t}\right)^{\nu}\right] - \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{1}$$

where ν and φ are positive parameters, M_t represents nominal money holdings, P_t is the aggregate price level, N_t stands for the household's supply of labor and C_t is a consumption basket. C_t is given by a Dixit-Stiglitz aggregator function

$$C_t = \left[\int_0^1 (C_{j,t})^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $C_{j,t}$ denotes the quantity of good $j \in [0,1]$ consumed in period t and ε ($\varepsilon > 1$) stands for the elasticity of substitution between the differentiated goods.

Utility in future periods is discounted by the factor $\beta \in (0,1)$. In each period t, the real flow budget constraint is

$$\int_0^1 \frac{Q_{j,t}}{P_t} C_{j,t} dj + \frac{M_t - M_{t-1}}{P_t} + \frac{\frac{1}{R_t^n} B_t - B_{t-1}}{P_t} = \frac{W_t}{P_t} N_t + T_t, \tag{3}$$

where $Q_{j,t}$ denotes the price of one unit of good j, B_t bond holdings, R_t^n the nominal interest rate, W_t the nominal wage, and T_t a real transfer, which includes the profits of firms and the government's seigniorage revenues. Bonds are in zero net supply. The aggregate price level P_t is given by

$$P_t = \left[\int_0^1 (Q_{j,t})^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}. \tag{4}$$

The growth rate of the nominal money stock, which is denoted by g_t^m, follows an exogenously given stationary stochastic process. We allow for a positive unconditional mean of g_t^m, which enables us to model a positive inflation trend.

3.2 Firms

The economy is populated by a continuum of monopolistically competitive goods producers, indexed by $j \in [0,1]$. Each firm j produces the individual good j and sells it directly to consumers. The production function is of the form

$$Y_{j,t} = X_{j,t} N_{j,t}^{\gamma}, \tag{5}$$

with $\gamma \in (0,1]$ and where $X_{j,t}$ is an idiosyncratic productivity level and $N_{j,t}$ is the labor input of firm j at time t. Thus we allow for decreasing returns to scale when $\gamma < 1$, which could also be interpreted as the production function being of the Cobb-Douglas type but with fixed capital.

In every period, each firm j is hit by a productivity disturbance with probability $1-\alpha$. When this happens, the firm survives with probability τ. For surviving firms, productivity changes according to $X_{j,t} = X_{j,t-1}e^{\xi_{j,t}}$, where $\xi_{j,t}$ is an i.i.d. firm-specific shock that is uniformly distributed over the support $\left[-\frac{\chi}{2}, +\frac{\chi}{2}\right]$. If a firm does not survive, which happens with probability $1-\tau$, conditional on a shock, it is immediately replaced by a new firm with productivity one, i.e. $X_{j,t} = 1$. This can be interpreted as product substitutions. We note that the main purpose of the assumption that firms may die with probability $1-\tau$ is to guarantee a stationary distribution of productivities across firms as in Gertler and Leahy (2008). For our calibration, it will be convenient to introduce $\theta := (1-\alpha)(1-\tau)$ as the probability that a given firm exits in a given period.

In our main model (PP), firms receive information about the aggregate monetary disturbance and the idiosyncratic productivity shock in periods in which they are hit by an idiosyncratic shock. In all other periods, they must act on outdated information.[12] While firms can adjust the prices of their outputs in every period, they must choose these prices subject to a price-point restriction (PPR), as will be explained in the following.

We assume that each firm j chooses the price for a quantity U_j of the good, where we use $\tilde{Q}_{j,t}$ to denote this price. U_j is constant over time and exogenous for each

[12]This assumption about when firms receive information updates is closely related to the modeling strategy in Gertler and Leahy (2008), who assume that firms face costs of information acquisition that are too large for firms to search for information in the absence of idiosyncratic shocks but small enough such that firms always acquire information when they are hit by a shock. Klenow and Willis (2007) also consider a model where firms update their information about idiosyncratic shocks more frequently than their information about aggregate disturbances.

firm j. Thus the price of quantity U_j, $\tilde{Q}_{j,t}$, and the price per unit of the good, $Q_{j,t}$, are related via $\tilde{Q}_{j,t} = U_j Q_{j,t}$. We can think of the U_j's as different package sizes of the differentiated products.

The PPR implies that each firm j can only choose a log price $\tilde{q}_{j,t} := \ln\left(\tilde{Q}_{j,t}\right)$ that lies in the set of price points $\Delta \cdot \mathbb{Z}$, where $\mathbb{Z}$ is the set of positive and negative integers and Δ is the exogenously given relative distance between price points, which is identical for all firms. Moreover, we assume that the natural logarithms of the firms' U_j's, which are denoted by u_j's, are uniformly arranged on the interval $[0, \Delta[$. Let $q_{j,t}$ be the natural logarithm of the per-unit price $Q_{j,t}$. As $q_{j,t} = \tilde{q}_{j,t} - u_j$ and $\tilde{q}_{j,t} \in \Delta \cdot \mathbb{Z}$, the log per-unit price $q_{j,t}$ can only be chosen such that $q_{j,t} \in \Delta \cdot \mathbb{Z} - u_j$. Firm j's profits in period t are given by the difference between revenues and total labor costs,

$$\Pi_{j,t} = \frac{Q_{j,t}}{P_t} Y_{j,t} - \frac{W_t}{P_t} N_{j,t}. \tag{6}$$

Finally, a few comments on our assumptions regarding price points are in order. First, we would like to stress that we take the relevance of price points as given and introduce them into our model as an exogenous constraint on firms' price setting. Second, we note that our assumption about the relative distance between price points being constant is in line with the observation that, for \$0.89, the next price point would be \$0.99 but for a price point of \$8.99, the next price point would plausibly be \$9.99.[13] Hence, the assumption of constant relative differences between price points is plausible to be a reasonable first approximation. This assumption will be modified in Section 7. Third, the assumptions that prices refer to fixed quantities U_j of goods and that the U_j's are uniformly arranged have the plausible consequence that the fraction of firms choosing a price point below the price they would charge in the absence of a PPR and the fraction

[13] This argument is also supported by the evidence presented in Levy et al. (2011). They find that for small prices, prices with 9s in the penny and dime digits are particularly persistent. For more expensive products, they observe more persistence of prices with 9s in the \$1, \$10, and \$100 digits.

of firms choosing a higher price than they would select without a PPR are constant over time.[14]

In our benchmark case with sticky prices (SP), we abstract from information rigidities. Firms do not face a PPR but can only adjust their prices when they are hit by an idiosyncratic productivity shock, which happens with exogenous probability $1 - \alpha$. Thus firms in the SP face price stickiness as in Calvo (1983).

4 Solution

4.1 Common equations for PP and SP

In the following, we consider log-linearized versions of the PP and the SP model. In both scenarios, the equations describing the optimal behavior of households, which are stated in Appendix A, have well-known log-linear approximations around the steady state

$$w_t - p_t = \ln\left(\frac{\overline{W}}{P}\right) + \varphi \hat{N}_t + \hat{Y}_t, \tag{7}$$

$$\hat{Y}_t = -\left(\hat{R}^n_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \mathbb{E}_t\left[\hat{Y}_{t+1}\right], \tag{8}$$

$$m_t - p_t = \ln\left(\frac{\overline{M}}{P}\right) + \hat{Y}_t - \frac{1}{\bar{R}^n - 1}\hat{R}^n_t, \tag{9}$$

$$\hat{Y}_{j,t} = -\varepsilon\left[q_{j,t} - p_t + \ln\left(\frac{\overline{Q}}{P}\right)\right] + \hat{Y}_t, \tag{10}$$

where here and henceforth small letters denote log levels, variables with a bar denote steady-state levels, and variables with a "hat" stand for relative deviations from the steady state.[15]

[14] Otherwise, under positive inflation there would be discontinuous jumps in the price level in periods where a large fraction of firms adjusted their price upwards to the next price point.

[15] We have used $\hat{Y}_t = \hat{C}_t$ for the derivation of (7)-(10).

4.2 Equations specific to the PP model

In the model with price points and information frictions, a firm hit by an idiosyncratic shock i periods ago sets the following price for one unit of its good:

$$\begin{aligned} q_{j,t}^{PP} &= \mathcal{T}_j \left\{ \mathbb{E}_{t-i} \left[q_{j,t}^{*\,PP} \right] \right\} \\ &= \mathcal{T}_j \left\{ \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left(-\frac{1}{\gamma} x_{j,t} + \mathbb{E}_{t-i} \left[\widehat{ulc}_t \right] \right) + \mathbb{E}_{t-i} \left[p_t \right] \right\}, \end{aligned} \tag{11}$$

where $\mathcal{T}_j : \mathbb{R} \rightarrow \mathbb{R}$ is an operator which maps the hypothetical optimal price of producer j in the absence of the PPR to the closest corresponding price point $q_{j,t}^{PP} \in \Delta \cdot \mathbb{Z} - u_j$. $\widehat{ulc}_t$ denotes the relative deviation of aggregate unit labor costs from their steady-state value. For details of the derivation see Appendix B.1. It may be worth stressing that $q_{j,t}^{PP}$ is the price for one unit of the consumption good. The price actually chosen by the firm for a package of log size u_j is $\tilde{q}_{j,t}^{PP} = q_{j,t}^{PP} + u_j$.

For the scenario with information frictions, our model results in a sticky-information Phillips curve à la Mankiw and Reis (2002):

$$\begin{aligned} \hat{\pi}_t =& \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \frac{\left(1-\alpha^{PP}\right)}{\alpha^{PP}} \widehat{ulc}_t \\ &+ (1-\alpha^{PP}) \sum_{i=0}^{\infty} \left(\alpha^{PP}\right)^i \ \mathbb{E}_{t-1-i} \left[\hat{\pi}_t + \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left(\widehat{ulc}_t - \widehat{ulc}_{t-1} \right) \right], \end{aligned} \tag{12}$$

where we have added the superscript PP to the parameter α in the PP model. The derivation of (12) can be found in Appendix B.2.

4.3 Equations specific to the SP model

In the SP model, the price setting equation is given by[16]

$$q_{j,t}^{SP} = \frac{\gamma}{\gamma+\varepsilon(1-\gamma)}\left(\hat{\psi}_t - \hat{\phi}_t\right) - \frac{1}{\gamma+\varepsilon(1-\gamma)}x_{j,t} + p_t + \ln\left(\frac{\overline{Q}}{P}\right), \tag{13}$$

where we have introduced the superscript SP for the SP model. The auxiliary variables $\hat{\psi}_t$ and $\hat{\phi}_t$ are given by

$$\hat{\psi}_t = \left(1-\alpha^{SP}\beta\bar{\pi}^{\frac{\varepsilon}{\gamma}}\right)\left[\widehat{ulc}_t - \hat{s}_t\right] + \alpha^{SP}\beta\bar{\pi}^{\frac{\varepsilon}{\gamma}}\left[\mathbb{E}_t\hat{\psi}_{t+1} + \frac{\varepsilon}{\gamma}\mathbb{E}_t\hat{\pi}_{t+1}\right], \tag{14}$$

$$\hat{\phi}_t = \alpha^{SP}\beta\bar{\pi}^{\varepsilon-1}\left[\mathbb{E}_t\hat{\phi}_{t+1} + (\varepsilon-1)\mathbb{E}_t\hat{\pi}_{t+1}\right]. \tag{15}$$

The deviation $\hat{s}_t$ of price dispersion from its steady-state value is given by

$$\hat{s}_t = \frac{\varepsilon(\bar{\pi}^{\frac{\varepsilon}{\gamma}} - \bar{\pi}^{\varepsilon-1})}{\gamma(1-\alpha^{SP}\bar{\pi}^{\varepsilon-1})}\alpha^{SP}\hat{\pi}_t + \alpha^{SP}\bar{\pi}^{\frac{\varepsilon}{\gamma}}\hat{s}_{t-1}. \tag{16}$$

The associated Phillips curve with trend inflation is then

$$\begin{aligned}
\hat{\pi}_t = {} & \frac{\gamma}{\gamma+\varepsilon(1-\gamma)}\frac{(1-\alpha^{SP}\bar{\pi}^{\varepsilon-1})(1-\alpha^{SP}\beta\bar{\pi}^{\frac{\varepsilon}{\gamma}})}{\alpha^{SP}\bar{\pi}^{\varepsilon-1}}\left[\widehat{ulc}_t - \hat{s}_t\right] \\
& + \beta\left[1+\varepsilon\frac{1-\alpha^{SP}\bar{\pi}^{\varepsilon-1}}{\gamma+\varepsilon(1-\gamma)}\left(\bar{\pi}^{\frac{\gamma+\varepsilon(1-\gamma)}{\gamma}} - 1\right)\right]\mathbb{E}_t\widehat{\pi}_{t+1} \\
& + \frac{\gamma}{\gamma+\varepsilon(1-\gamma)}\alpha^{SP}\beta\left(\bar{\pi}^{\frac{\varepsilon}{\gamma}} - \bar{\pi}^{\varepsilon-1}\right)\frac{1-\alpha^{SP}\bar{\pi}^{\varepsilon-1}}{\alpha^{SP}\bar{\pi}^{\varepsilon-1}}\mathbb{E}_t\widehat{\psi}_{t+1},
\end{aligned} \tag{17}$$

where $\hat{\psi}_t$ is given by equation (14).

[16] The derivation is standard. For completeness, it is given in a separate appendix.

5 Simulation Strategy

The main objective of this paper is to simulate the individual price dynamics implied by the PP model and the SP model in order to assess how well these models can explain the empirical findings about price-setting documented by KK, NS, and others. In this section, we explain our simulation strategy and the calibration of our model.

We compute the individual price dynamics for the PP and the SP model variants using the price-setting equations (11) and (13), respectively. More specifically, we simulate the prices set by 100.000 firms for the time period 1988Q1 - 2004Q4, which is the period considered by KK and NS. While the idiosyncratic shocks are generated by a random number generator in this simulation exercise, we use realized values for the current and past CPI and unit labor costs. As firms' optimal prices also depend on current and lagged expectations of unit labor costs and the price level, we follow Sbordone (2002) and Dupor et al. (2010) and estimate a vector autoregressive model to generate the corresponding forecasts. We use data from 1983Q1-2004Q4 for our VAR model, which enables us to calculate lagged expectations of current economic variables for the entire period 1988Q1-2004Q4.[17] The forecasting model includes CPI quarter-on-quarter inflation rates and unit labor costs.[18,19] The time unit is a quarter, as data on unit labor costs is not available at shorter time intervals.

We would like to comment on a difference between the simulations for the PP model and the ones for the SP model. As in our simulations firms utilize information about aggregate unit labor costs but do not observe a direct measure of costs on the individual firm level, they rely on a relationship between individual and aggregate costs that involves the measure of price dispersion $\hat{s}_t$.[20] In the PP model, $\hat{s}_t = 0$ is satisfied in

[17] This is necessary for the PP model.

[18] We follow Galí and Gertler (1999) and measure log real unit labor cost as the logarithm of the ratio of nominal compensation per hour to nominal output per hour in the non-farm business sector.

[19] We use a VAR model of order two, which is suggested by the standard information criteria.

[20] The details of the derivations are laid out in a separate appendix, which is available upon request.

every period t for a linear approximation around the steady state, which entails that firms do not have to calculate $\hat{s}_t$ when computing their own costs from aggregate unit labor costs. By contrast, $\hat{s}_t = 0$ does not hold in the SP model under the assumption of a positive inflation rate in the steady state. Therefore, firms use (16) to compute $\hat{s}_t$ in our simulations. In this sense, our simulations involve an additional condition for the SP model compared to the PP model. We have confirmed that all our results about individual price dynamics in the SP model are virtually unaffected if we make the assumption that firms (erroneously) use $\hat{s}_t = 0$ in the SP model as well.

For our calibration, we proceed as follows. First, we rely on external information to calibrate β, φ, ε, γ, θ, and Δ.[21] Since we use quarterly data, we choose a discount factor of $\beta = 0.99$. In line with Gertler and Leahy (2008), we utilize $\varphi = 1$ for the inverse of the Frisch elasticity of the labor supply, and $\varepsilon = 11$, which implies a steady-state markup of 10% over marginal costs. We set γ to the labor income share of approximately 62% (Elsby et al., 2013). For the exit probability θ, we select $\theta = 0.087$, which is in line with the evidence from KK and NS that the monthly rate of forced item substitutions is around 3%. To calibrate Δ, which is only relevant for the PP model, we refer to the following observations. First, Levy et al. (2011) find that the price points in cents contained in the data set from Dominick's supermarkets end with the digit 9. Moreover, we note that the modal price in this data set is \$1.99 , which implies that the two closest price points are \$1.89 and \$2.09. As the relative differences of these prices from \$1.99 are approximately 5%, we set $\Delta = 0.05$.[22]

Second, parameters α and χ are calibrated to values that differ across the two scenarios.[23] To determine the respective values of α, we implement the procedure that was

[21]The utility parameter ν does not affect our simulations.

[22]Our results are qualitatively robust to reasonable changes in this parameter value. The results are available upon request. A scenario with multiple values of Δ is considered in Section 7.

[23]As $\theta = (1-\alpha)(1-\tau)$ is independent of the scenario, it is clear that τ differs across both scenarios.

introduced by Sbordone (2002) and was also used by Dupor et al. (2010), more recently. For this purpose, we use equations (12) and (17) to compute the model-implied inflation in the PP and the SP model as functions of α, where current and lagged real unit labor costs are set to the values in the data and the VAR model is used to generate forecasts of inflation and real unit labor costs.[24] We then select α by minimizing the sum of quadratic deviations of realized inflation rates from inflation rates predicted by the respective Phillips curve, i.e. (12) for the PP model and (17) for the SP model, for the time horizon 1988Q1-2004Q4. This procedure results in $\alpha^{PP} = 0.73$ and $\alpha^{SP} = 0.67$ for the two scenarios.

It remains to calibrate χ, the width of the support for the idiosyncratic shocks, where we also allow for different values for the two scenarios. We apply the simulated method of moments to equations (11) and (13) and determine these parameters by targeting the mean magnitude of price changes, which is 11.3% in US CPI data (see Table III in KK). This procedure results in $\chi^{PP} = 2.20$ and $\chi^{SP} = 1.84$. Table 2 summarizes our calibration.

6 Individual Prices

We now turn to our simulation results regarding individual price dynamics. These simulations show that our PP model can explain several pieces of the evidence on individual price dynamics at least as well as the SP model. In Section 7, we will demonstrate that the PP model is even more successful in explaining the stylized facts of individual price dynamics when we allow for a more general distribution of price points.

[24] As is common in the literature, we restrict the number of lagged expectations included in the sticky-information Phillips curve. In particular, we consider only expectations that are formed up to 18 periods ago. For $\alpha^{PP} = 0.73$, the value implied by our calibration exercise, only 0.3% of all firms have not updated their information after 18 periods. We proceed analogously when computing $\hat{s}_t$ and $\hat{\psi}_t$ for the SP model.

data	
targeted period	1988Q1 - 2004Q4
mean price change (KK)	11.3%
VAR period	1983Q1 - 2004Q4
mean q-o-q inflation	0.76%
annualized mean inflation	3.08%
calibrated externally	
β	0.99
θ	0.087
Δ	0.05
ε	11
γ	0.62
φ	1
calibrated internally	
α^{PP}	0.73
α^{SP}	0.67
χ^{PP}	2.20
χ^{SP}	1.84

Table 2: Calibration summary

6.1 Duration of price spells and the magnitude of price changes

Duration of price spells How often do prices change? Empirical studies find that the mean duration of regular prices is roughly three quarters, depending on the sample period, the weighting of prices and the treatment of product substitutions. For example, KK estimate the frequencies of price adjustments for different categories of products and compute the mean of the implied durations as 2.9 quarters.[25,26] As shown

[25]See the implied durations for regular prices in their Table I. This value includes ends of price spells due to product substitutions.

[26]Gorodnichenko et al. (2018) find that, even in online markets, where physical price adjustment costs are negligible, prices remain fixed for comparably long periods.

in Table 3, even though this statistic has not been targeted, both models can match this evidence very well. The PP model implies a mean duration of 2.9 quarters exactly as in KK, and the SP model involves a slightly higher duration of 3.2 quarters. Despite the higher value of α in the PP model, the duration of price spells is shorter in the PP scenario since firms can change their prices not only when they are hit by an idiosyncratic shock but can adjust their prices freely even in periods where they receive no new information.

	KK	PP	SP
mean price duration in quarters	2.9	2.9	3.2
std. dev. of price dur.'s in q.'s	1.7	1.9	2.7
mean magn. of changes (targeted)	11.3%	11.3%	11.3%
median magn. of changes	9.7%	10.0%	10.2%
mean price increases	10.6%	9.9%	12.3%
mean price decreases	13.3%	14.3%	10.1%
share of price decreases	43.4%	31.4%	43.7%

Table 3: Simulation results

Variance of price durations KK document that the standard deviation of durations between price adjustments for a given item in the BLS data is around 1.7 quarters (see their Table V). Our simulations yield 1.9 for the PP case and 2.7 for the SP case. Due to the idiosyncratic productivity shocks that arrive with a fixed probability in every period, both models can generate variances of price spells that are broadly consistent with the empirical evidence, although the SP model generates price spells that are somewhat more volatile than in the data.

Magnitude of price changes The empirical evidence that prices change by much more than necessary to catch up with inflation has been emphasized since Bils and

Klenow (2004). As argued by Golosov and Lucas (2007), a model has to involve idiosyncratic shocks in order to be able to explain this pattern. As both model variants include idiosyncratic shocks, we are able to hit the calibration target of an average magnitude of price changes of 11.3% in both cases.[27] In both models and in the data, the median magnitude of relative price changes is smaller than the mean.

Magnitude of price changes for increases vs. decreases It is a puzzling asymmetry in empirical data that the magnitude of price decreases tends to be larger than the size of price increases. Burstein and Hellwig (2007) document this fact for the Dominick's database and KK provide evidence that for regular prices in the BLS data set, increases average 10.6% whereas decreases average 13.3%.

Table 3 shows that the PP model outperforms the SP model in this regard since, in contrast with the SP model, the PP model generates larger price decreases than increases. In particular, the PP model generates average increases of 10% and average decreases of 14% similarly to the data whereas the SP model implies average increases of 12% and average decreases of only 10%.

How can this relative success of the PP model be explained? Roughly speaking, most of the price decreases in the PP model are driven by idiosyncratic productivity shocks, which have a comparably large variance. By contrast, increases in prices also occur because of the positive trend in inflation. These increases are small, as firms adjust their price upwards by Δ in these cases, which is the smallest possible price change in the PP model. As a consequence, increases are smaller on average than decreases.

In the SP model, price increases are larger than decreases on average because every time a firm is allowed to adjust its price, the new price is determined by two main

[27] If we exclude the relatively large price changes that occur when products are replaced, this value drops to 9.7% in the PP and to 9.9% the SP model. This is in line with the finding reported in Footnote 9 in KK that price changes are 1 to 2 percentage points smaller if price changes at substitutions are excluded.

factors: the idiosyncratic productivity shock, which has zero expected mean, and the change in the price level since the price was adjusted last, which is positive under a positive trend inflation rate. It is clear that the resulting price change involves larger average increases than decreases.

Fraction of price changes that are price decreases KK find that 43.4% of all price changes are price decreases (see their Table VI). Both models considered in this paper are broadly in line with this findings. The SP model predicts a fraction of 44% and the PP model implies that 32% of all price changes are price decreases. While the value predicted by the PP model is lower than the one found in KK, it is still in line with the empirical findings in other papers. For example, for essentially the same data that is used in KK, NS report a value of roughly one third.[28]

Price points Data on individual prices indicate that prices move back and forth between a few rigid values (NS, Eichenbaum et al. (2011), Knotek (2016)). Obviously this fact is matched by the PP model by construction. Nevertheless it is still one of the most puzzling empirical observations and it is not trivial for macro models to be consistent with this pattern (see Kehoe and Midrigan (2015)).

6.2 Frequency and magnitude of price changes as functions of the age of the price

Hazard rates Simple state-dependent pricing models like menu-cost models typically predict an increasing hazard curve, i.e. an increasing probability of a price change

[28]These differences arise because KK and NS proceed slightly differently, e.g. when removing price changes due to sales. For more detailed information on the differences in findings between KK and NS, see NS.

as a function of the duration of a price spell. As shown by KK, NS and Klenow and Malin (2010), this implication is not supported by the data.[29,30]

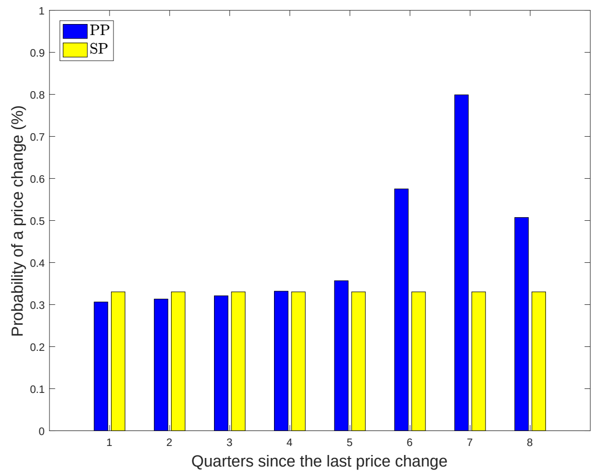

Figure 2: Hazard rates

Time-dependent pricing models based on Calvo pricing produce flat hazard curves by construction, which can be seen from Figure 2 in the case of our SP model. By contrast, while the hazard curve in the PP model is rather flat up to the fourth and fifth quarter, it features a significant peak around the seventh quarter. The reason for this outcome is straightforward. In the PP model there are two main reasons for price changes: First, idiosyncratic shocks may occur. These shocks alone would produce a flat hazard curve. Second, in the absence of idiosyncratic shocks, firms form expectations about changes in the aggregate price level, which require an adjustment of their prices from time to time due to the positive level of trend inflation. As the average quarter-on-quarter inflation rate is 0.76% for our sample and the relative difference between price points is $\Delta = 5\%$, one would expect that firms are comparably likely to adjust their prices after $5/0.76 \approx 6.6$ quarters, which is exactly what Figure 2 shows.

[29]Estimating the shape of hazard functions is empirically challenging because of substantial heterogeneity in the frequency of price adjustments across goods.

[30]NS highlight that the hazard function can take many different forms in models with idiosyncratic shocks.

The peak of the hazard curve can be seen as an artifact of our assumption that the relative distance between price points is identical in all cases, which is arguably not particularly realistic. Consider for example, the price point \$0.99. The next price point would be \$1.09, which would imply a relative difference of roughly 10%, compared to the difference of 5% between \$1.99 and \$2.09. We take up this idea in the subsequent Section 7 and show that the hazard curve becomes significantly flatter once we allow for a richer distribution of Δ's.

Size of price changes as a function of age Another important empirical fact is that the mean magnitude of relative price changes is approximately independent of the time since the last adjustment (KK, Klenow and Malin (2010)). This empirical finding is at odds with the typical prediction of time-dependent pricing models that prices are adjusted more strongly if they have not been adjusted for a longer time period.[31]

Interestingly, although our SP model falls into the class of time-dependent pricing models, it is quite successful in replicating the empirical finding under consideration, as can be seen from Figure 3. The success of the SP model is due to the fact that our calibration selects a comparably large variance of idiosyncratic shocks. Hence the size of price changes is mostly driven by the realization of the idiosyncratic shock and hardly influenced by the comparably modest changes in the price level that occurred since the price was last adjusted.

By contrast, the PP model implies a sizable trough at around 7 quarters. The intuition is straightforward. As we have explained in our previous discussions, positive trend inflation causes firms in the PP model to adjust prices upwards by Δ from time to time even in the absence of idiosyncratic shocks. Due to our assumption of a single

[31] This prediction can be understood by noting that the optimal price drifts away from the current price as time passes and therefore larger adjustments are necessary for prices that have not been adjusted for a long time.

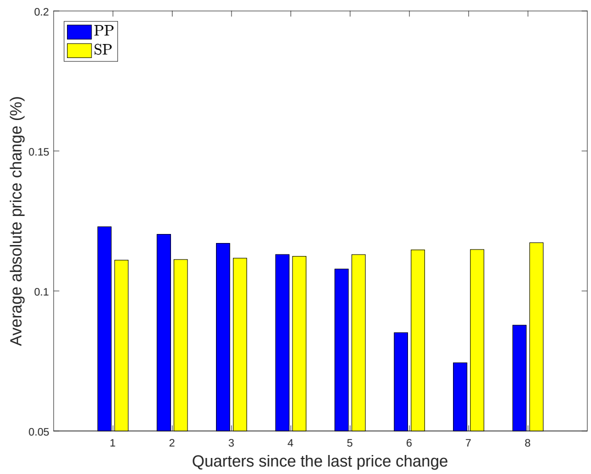

Figure 3: The absolute value of relative price changes conditional on the price's age

value of Δ, this occurs always after approximately 7 quarters.[32] It appears plausible that a more general specification of the set of price points would thus improve the performance of the PP model substantially. In fact, we will show in Section 7 that this conjecture is correct.

6.3 Price changes and inflation

Relationship of frequency and size of price adjustment with inflation KK find that the overall frequency of price changes co-moves with inflation. In particular, they find that, for monthly data, the correlation between the frequency of price adjustment and inflation equals 0.25. The positive correlation between the frequency of price adjustment and inflation is strong evidence in favor of state-dependent pricing and indeed the PP model can match this evidence qualitatively, as can be seen from Table 4. Similarly, estimating

$$fr_t = \beta_\pi \pi_t + b + \epsilon_t \tag{18}$$

[32]It is noteworthy that Figure 3 shows that, at the trough of the graph for the PP model, prices change by only slightly more than $\Delta = 5\%$ on average.

variable	source	mean (%)	std dev (%)	correlation with π_t	β_π
	KK	26.60	3.2	0.25	2.38
fr_t	PP	33.44	1.15	0.49	1.61
	SP	30.17	0.12	-0.05†	-0.02†
	KK	0.98	1.19	0.99	3.55
dp_t	PP	2.20	0.67	0.76	1.45
	SP	2.40	0.92	0.88	2.31

Table 4: Time-series moments

Notes: fr_t = the fraction of items with changing prices, dp_t = the average relative price change; values marked with a † are highly insignificant as both p-values are approximately 0.7.

with error terms ϵ_t and intercept b via OLS produces a significant positive coefficient $\beta_\pi = 1.61$, which is qualitatively in line with the value of 2.38 found in KK.[33,34] By contrast, the SP model assumes a fixed probability of price adjustment and thus implies no correlation of the frequency of price adjustment with inflation.

KK find a large positive correlation between inflation and the mean price change. Both models are compatible with this finding. Using the average of relative price changes, dp_t, as a dependent variable in (18) produces significant positive coefficients β_π in all three cases KK, PP, SP. To sum up, while only the PP model can correctly predict that the frequency of price adjustments co-moves with inflation, both models are in line with the data when it comes to the correlation between the average size of price changes and inflation.

Relationship of frequencies of price increases and price decreases with inflation NS and, more recently, Nakamura et al. (2016) have documented that the

[33] The coefficients are not directly comparably as KK use monthly data for which short-term fluctuations due to sales have been filtered out, whereas we consider quarterly data.

[34] Klenow and Malin (2010) also find that the frequency of price changes co-varies with inflation.

frequency of price increases changes substantially over time and co-varies with inflation. By comparison, the frequency of price decreases is more stable.

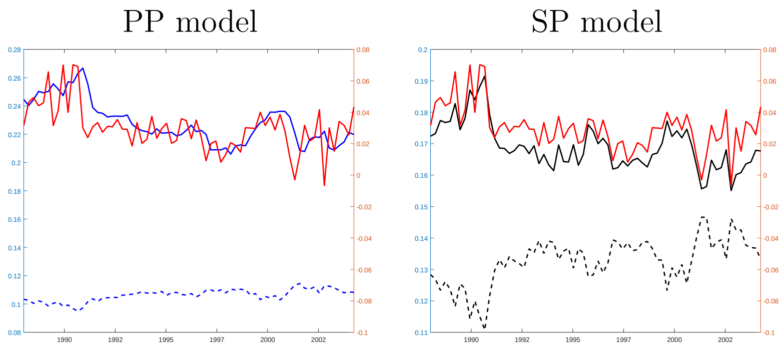

Figure 4: Frequency of price increases: blue/black solid line; frequency of price decreases: blue/black dashed line; inflation: red line.

Figure 6.3 shows that this pattern can be reproduced qualitatively by the PP model but not by the SP model. In the SP model, when an idiosyncratic shocks hits a firm, the price is adjusted in response to the size of the idiosyncratic shock and changes in the aggregate price level. As a consequence, a comparably large fraction of price changes are increases when inflation is high. As the frequency of price changes is fixed by assumption, a higher frequency of price increases automatically translates into a lower frequency of price decreases.

In the PP model, the current inflation rate mainly affects the frequency of the small positive adjustments that are caused by expected increases in the price level. In periods of high inflation, there are more of these increases and thus the frequency of price increases is higher.[35] By comparison, the frequency of price decreases is affected only to a smaller extent by higher inflation rates, as most price decreases are triggered by negative idiosyncratic shocks.

[35]Note that firms that have updated their information recently are more successful in predicting unusually high inflation rates compared to firms that have not received an idiosyncratic shock for a longer time period.

6.4 Intensive margin dominates the variance of inflation

A major purpose of studying price dynamics is to understand what drives fluctuations in inflation: Is it that the number of firms that change their prices varies or that firms change prices by different amounts? Put differently, are inflation dynamics driven by the extensive margin (EM) or the intensive margin (IM)? Figure 5 shows the time series predictions of the PP and the SP model for the average price adjustment and the frequency of price adjustment against the realized path of inflation. The extensive margin is relatively stable in both scenarios but its correlation with inflation is different across models: 0.49 in the PP and not significantly different from zero in the SP model. In both models and in the data, the intensive margin is more volatile and co-moves closely with inflation, which is in line with KK's empirical findings.

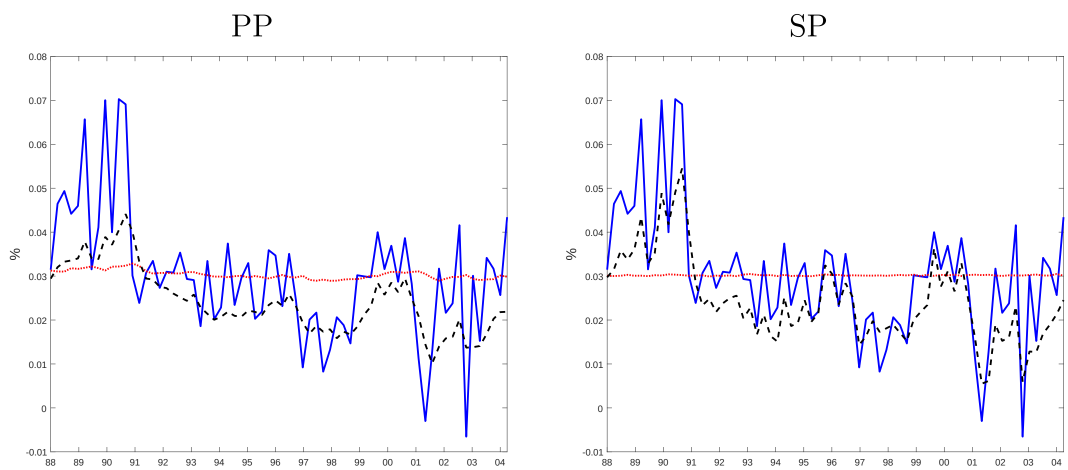

Figure 5: Extensive and intensive margins vs. inflation

Notes: The blue solid line is the realized path of annualized inflation. The red dotted line represents the extensive margin, i.e. the frequency of price adjustments (fr_t), divided by 10. The black dashed line stands for the intensive margin, i.e. the mean relative price change (dp_t).

To further explore this finding, KK use a decomposition of the inflation variance into terms capturing the intensive margin and terms capturing the extensive margin. More specifically, the decomposition is given by[36]

$$var(\pi_t) = \underbrace{var(dp_t)\overline{fr}^2}_{\text{IM term}} + \underbrace{var(fr_t)\overline{dp}^2 + 2\,\overline{fr}\;\overline{dp}\;cov(fr_t, dp_t) + O_t}_{\text{EM terms}}, \tag{19}$$

where dp_t is the average relative price change, fr_t is the fraction of prices in a given period that are adjusted and the values with a bar correspond to time averages. O_t are higher-order terms that are functions of fr_t.

The results from the variance decomposition are displayed in Table 5.[37] Obviously, the SP model assigns all fluctuations in the inflation rate to the intensive margin because the frequency of price adjustments is fixed by assumption. The PP model also attributes positive weight to the extensive margin.

	IM (in percent)	EM (in percent)
KK	91	9
PP	83	17
SP	100	0

Table 5: Variance decomposition: extensive margin vs. intensive margin

For completeness, we also consider another decomposition proposed by KK, which addresses the question whether fluctuations in inflation are the consequences of changes in price increases or price decreases. KK note that inflation can be written as

$$\pi_t = fr_t^+ dp_t^+ - fr_t^- dp_t^-, \tag{20}$$

where fr_t^+ and fr_t^- denote the fractions of price changes that are increases or decreases at time t, respectively, and dp_t^+ and dp_t^- denote the average magnitudes of increases

[36]See KK for a derivation.

[37]Note again that the numerical values are not directly comparable because KK use monthly data that have been filtered to eliminate sales, while our SP and PP simulations use quarterly data.

and decreases. With the help of (20), the variance of inflation can be expressed in the following way

$$
\begin{aligned}
var(\pi_t) = &\underbrace{var(fr_t^+ dp_t^+) - cov(fr_t^+ dp_t^+, fr_t^- dp_t^-)}_{\text{POS term}} \\
&+ \underbrace{var(fr_t^- dp_t^-) - cov(fr_t^+ dp_t^+, fr_t^- dp_t^-)}_{\text{NEG term}}.
\end{aligned} \tag{21}
$$

As shown in Table 6, both models imply reasonable values for the POS and NEG terms defined in (21). Around 60% of the variance of inflation can be traced back to changes in price increases, the remaining 40% are due to changes in price decreases.

	POS (in percent)	NEG (in percent)
KK	59	41
PP	62	38
SP	59	41

Table 6: Variance decomposition: price increases vs. price decreases

6.5 Summary

In Table 7, we provide an arguably subjective summary of our findings from the comparison of the PP and the SP model. Importantly, while the PP model is quite successful in explaining several stylized facts of price adjustment, it does not imply a flat hazard curve, i.e. Fact 3, and fails to reproduce Fact 5, which involves that the mean magnitude of price changes does not change with the age of the price. We have already mentioned that both of these problems may arise because we have considered a particularly simple distribution of price points until now. In the next section, we will therefore study a version of our PP model that allows for a more general distribution of price points.[38]

[38] Alvarez et al. (2016) and Nakamura et al. (2016) document that price dispersion is unresponsive to inflation at low rates of inflation. Thus one might also be interested in the implications of our two

	Facts	PP	SP
1.	regular prices stay constant on average for 2-3 quarters	yes	yes
2.	variable price durations	yes	yes
3.	flat hazard curves	no	yes
4.	large average magnitude of price changes (targeted in both cases)	yes	yes
5.	size of price changes does not change with price duration	no	yes
6.	magnitude of price decreases exceeds the size of increases	yes	no
7.	approximately 40% of regular price changes are decreases	yes	yes
8.	prices move back and forth between a few rigid values	yes	no
9.	frequency of price changes co-moves with inflation	yes	no
10.	the freq. of price increases co-varies strongly with inflation	yes	yes
11.	the frequency of price decreases changes little with inflation	yes	no
12.	intensive margin dominates the variance of inflation	yes	yes

Table 7: Sticky price economies vs. stylized facts of price adjustment

Notes: Sources of stylized facts: Facts 1-7, 9, 12 can be found in KK, fact 8 stems from Eichenbaum et al. (2011), facts 10 and 11 taken from Nakamura and Steinsson (2008) and Nakamura et al. (2016)

7 General Distribution of Price Points

In this section we relax our assumption that the log price points for all firms are distributed on evenly spaced grids with distances that are identical across firms. By contrast, we assume that there are n different values Δ_1, Δ_2, ..., Δ_n of relative differences between price points with n positive weights ρ_1, ρ_2, ... ρ_n satisfying $\sum_{k=1}^{n} \rho_k = 1$.

models in this regard. For the measure of price dispersion s_t, we have already highlighted that $\hat{s}_t = 0$ holds in the PP model for a log-linear approximation. By contrast, the SP model implies a value of $\hat{s}_t$ that is typically different from zero (see (16)).

The set of firms $[0,1]$ can be split into n subsets $[0,\rho_1)$, $[\rho_1,\rho_1+\rho_2)$, ... $[1-\rho_n,1]$. Firms in the kth interval can choose log prices $\tilde{q}_t^j$ only from the set $\Delta_k\mathbb{Z}$. The package sizes u_j are uniformly arranged on $[0,\Delta_k[$ for these firms. It is straightforward to see that in this variant of our model, a firm's optimal price continues to be given by (11), where u_j and Δ have to be replaced by the appropriate values. Moreover, inflation still follows the sticky-information Philips curve (12).

It remains to determine the values for the Δ_k's and the ρ_k's. For this purpose, we draw on the frequently used Dominick's Finer Foods database. First, we define price points as all posted prices that make up at least 10% of all observations in a window of ±10% around the price.[39] Second, we compute the relative difference when moving upward from one price point to the next one and weight the resulting differences with the relative frequencies of the observed price points. Figure 6 shows the different values of Δ_k's in the Dominick's database and the corresponding weights ρ_k. The weighted average $\sum_{k=1}^{n}\rho_k\Delta_k$ equals approximately 7%. We observe that it is unnecessary to recalibrate α for the general distribution of Δ's, as aggregate inflation dynamics are unaffected by the distribution of Δ's. By contrast, we need to adjust the value of χ. Following the same procedure as in Section 5, we obtain a value of $\chi = 1.96$ for the generalized PP model.

We are now in a position to simulate price dynamics for our generalized PP model. As can be seen from the left panel of Figure 7, the pronounced peak in the hazard curve observed in Figure 2 vanishes almost completely. This is due to the fact that in the basic PP model, most firms who have not encountered an idiosyncratic shock for some time adjust their prices after approximately $\Delta/(0.76\%)$ quarters, where 0.76% is the average quarterly inflation rate in our sample. With different values of Δ_k, these adjustments occur after different numbers of periods $\Delta_k/(0.76\%)$, which smooths

[39] The price points selected by this procedure represent 62% of all prices. This number is roughly in line with Levy et al. (2011) and Knotek (2016) who find that 9-ending prices account for about two thirds of price observations.

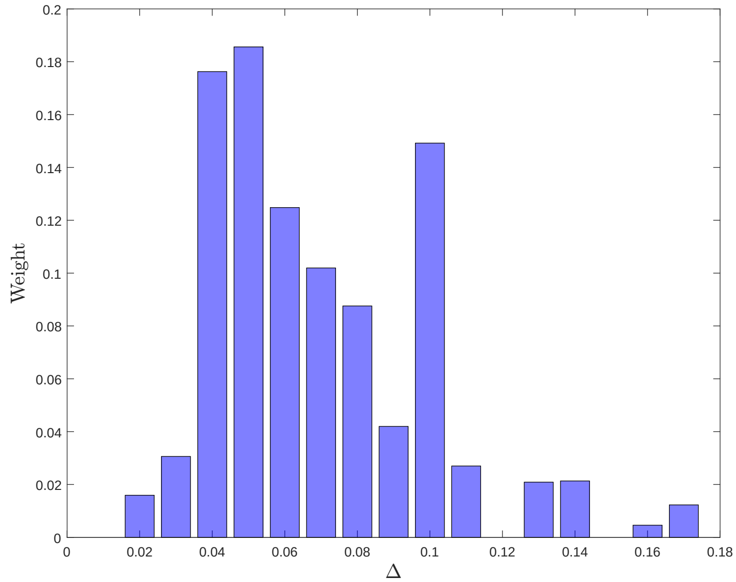

Figure 6: Distribution of Δ's constructed from the Dominick's Finer Foods Database

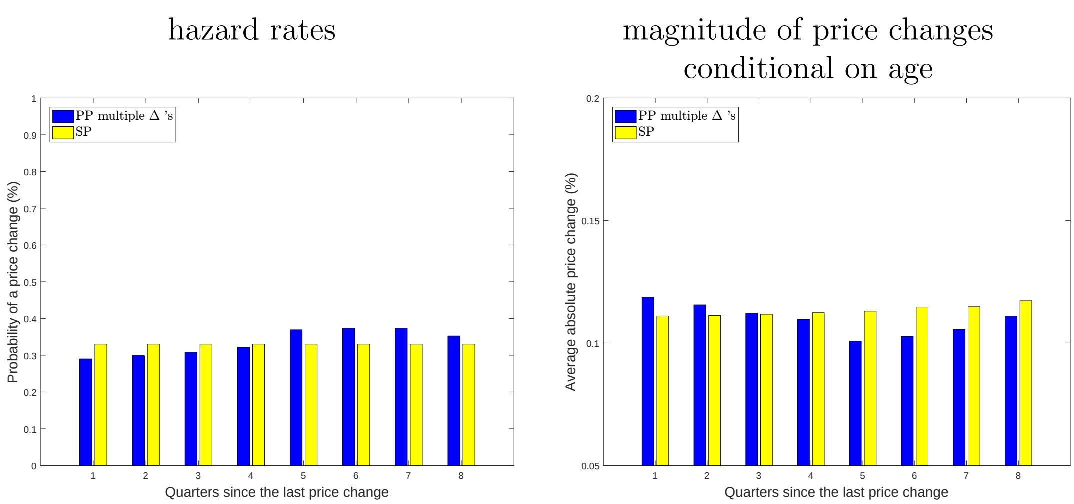

Figure 7: Comparison of the hazard rates and the average magnitude of price changes conditional on the price duration between the SP model and the PP model variant with multiple Δ's

out the peak considerably. In a similar vein, the trough for the graph displaying the magnitude of relative price changes as a function of age, which we observed in Figure 3, largely disappears. The right panel of Figure 7 shows that the generalized PP model becomes now more consistent with the approximately flat profile as documented by KK and others.

	KK	PP	generalized PP	SP
mean price duration in quarters	2.9	2.9	3.2	3.2
std. dev. of price dur.'s in q.'s	1.7	1.9	2.5	2.7
mean magn. of changes (targeted)	11.3%	11.3%	11.3%	11.3%
median magn. of price changes	9.7%	10.0%	10.0%	10.2%
mean price increases	10.6%	9.9%	10.2%	12.3%
mean price decreases	13.3%	14.3%	13.6%	10.1%
share of price decreases	43.4%	31.4%	32.7%	43.7%

Table 8: Simulation results including a generalized version of the PP model

We would like to mention that the more general distribution of Δ's does not qualitatively affect our results concerning the other stylized facts. In particular, Table 8 compares the main moments from the data with the simulation results for three different models: the PP model, the generalized PP model, and the SP model. It is obvious from the table that the generalized PP model's predictions are in several cases even slightly closer to the data (KK) than those of the PP model.

8 Impulse Responses

We now turn to the aggregate dynamics of the PP and the SP model that are triggered by monetary-policy shocks. Figure 8 plots the impulse responses to an unanticipated permanent negative shock to the money supply. In the PP case, i.e. the model variant with price points and information rigidities, the impulse responses are largely identical to those in the standard sticky-information model presented in Mankiw and Reis (2002) for the particular value of α^{PP} resulting from our calibration.[40] The impulse responses for the PP and the SP model are qualitatively similar except for the impulse response

[40] A minor difference is that Mankiw and Reis (2002) consider $m_t - p_t = \ln\left(\frac{M}{P}\right) + \hat{Y}_t$ instead of the money demand (9).

of inflation. This response is hump-shaped for the PP model, which Mankiw and Reis (2002) consider to be a major advantage of the sticky-information Phillips curve. We would like to stress again that a variant of our PP model without information stickiness would imply that monetary shocks have no effect on output. To sum up, the PP model does not only involve individual price dynamics that are broadly in line with the microeconomic evidence but entails responses of economic aggregates to monetary shocks that appear plausible as well.

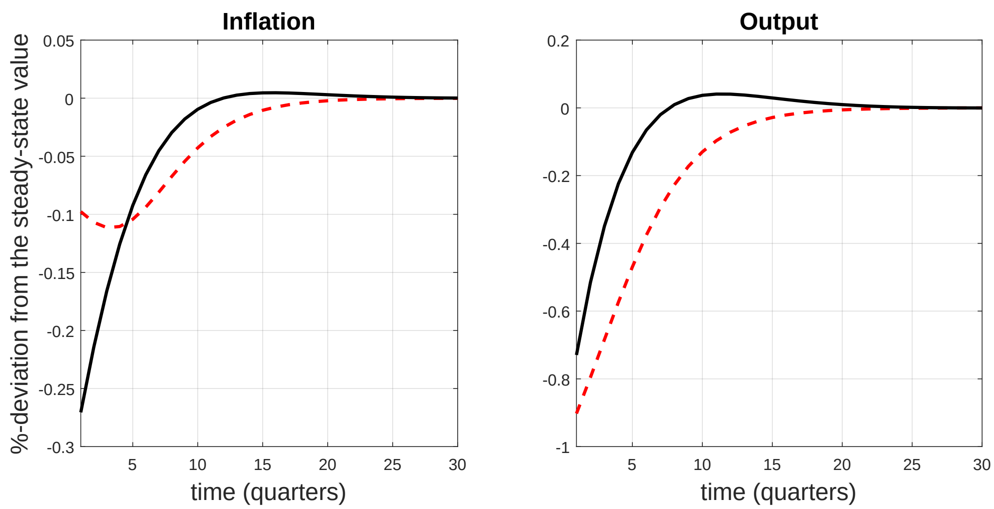

Figure 8: Impulse responses to a permanent negative money supply shock

Notes: PP: dashed red lines. SP: black solid lines.

There is a controversial debate in the literature about whether the sticky-information model or the sticky-price model are more in line with aggregate data. Some authors find that the sticky-price model is superior to the sticky-information model, provided that backward-looking agents are included (see e.g. Kiley (2007)). However, the introduction of backward-looking agents does not have strong microfoundations. Klenow and Willis (2007) find that firms' price decisions depend on outdated information about aggregate shocks, which supports the idea that sticky information is important for understanding inflation dynamics.[41] Another point of debate is the plausibility of the

[41] Dupor et al. (2010) show that a dual-stickiness model outperforms a hybrid sticky price model. Kaufmann and Lein (2013) find support in favor of a multi-sector sticky price model compared to a model with rational inattention.

models' policy implications in a liquidity trap. Kiley (2016) criticizes the sticky-price model in that regard and argues in favor of the sticky-information model, while Eggertsson and Garga (2017) claim that the sticky-information model may have similar policy implications as its sticky-price counterpart.

Interestingly, Coibion (2010) highlights that sticky-information models typically imply a delayed response of inflation to aggregate shocks, whereas sticky-price models predict a quick response. As empirically the response of inflation to monetary shocks occurs with a lag but the response to productivity shocks occurs fast, the sticky information model can explain the response to monetary shocks but predicts a too sluggish response to aggregate technology shocks.

In our model, there are aggregate monetary shocks but no aggregate technology shocks. If one wanted to include aggregate technology shocks into our model, an interesting avenue would be to take up an idea due to Coibion (2010, p. 100) and to assume that firms become more readily informed about aggregate productivity shocks compared to monetary shocks. Making such an assumption might entail a model with a fast response of inflation to productivity shocks but a delayed response to monetary disturbances.

9 Relationship to Menu-Cost Models

In this paper, we use a model with Calvo pricing as a benchmark for our comparisons but one might also ask how the PP model would fare against a variant with menu costs. While a rigorous analysis of such a model variant is beyond the scope of this paper, we offer a few thoughts about the relationship between our PPR and menu costs.

In some respects, menu costs and a PPR lead to similar predictions. For example, in the absence of idiosyncratic and aggregate shocks, both restrictions entail that all prices move upwards in a step-wise manner under positive trend inflation. Due to the selection effect, which involves that only the prices farthest away from their optimal values adjust, basic menu-cost models predict that changes in money growth rates have no effect on real variables (see Caplin and Spulber (1987)). This is closely related to the observation that our model with a PPR but without information frictions would imply that monetary policy has purely nominal effects. Moreover, some empirical findings like the larger sizes of price decreases compared to increases could be explained by menu cost models as well (see NS).

However, menu costs and our PPR do not always lead to identical predictions. Menu costs involve that relatively small price changes within a certain interval do not occur but that all price changes from a continuum outside this interval may occur.[42] By contrast, a PPR requires that all price changes come in discrete steps only. It is because of this difference that Knotek (2016) finds that price points are more relevant for understanding price dynamics than menu costs.

Finally, it is well known that jumps in the price level would induce many firms to adjust their prices simultaneously in a menu-cost model (see Caplin and Spulber (1987, p. 720)). At least temporarily, these coordinated price changes would substantially reduce any price dispersion that is not driven by differences in productivities or similar fundamental factors.[43] This arguably implausible effect does not occur under a PPR.

10 Conclusion

Kashyap (1995), Blinder et al. (1998) and Levy et al. (2011) have identified the empirical regularity that price points are relevant for firms' price setting decisions. Based on this observation, Knotek (2016) has shown that price points rather than menu costs

[42]For stochastic menu costs, the length of this interval may not be constant.

[43]In a similar vein, menu cost models would predict that the introduction of a new currency like the Euro in many European countries would lead to a temporary reduction in price dispersion.

may be responsible for extended price spells. However, a model where price points are the only source of price stickiness has the implication that monetary policy has no real effects, which contradicts the widespread consensus in monetary economics that central banks can influence real output in the short run.

As a consequence, this paper has proposed a model featuring a prominent role for price points as well information stickiness. Due to the presence of sticky information, monetary policy has real effects in our model. At the same time, our model can reproduce many stylized facts of price-setting, which cannot be easily reconciled with time-dependent pricing models such as those based on Calvo pricing. For example, our model is in line with the findings that the frequency of price adjustment is positively related to inflation and that the magnitude of price decreases exceeds the size of increases. By construction, it is also compatible with the observation that prices jump back and forth between a few rigid values.

Our framework could also be used to examine the impact of a change in trend inflation on price dynamics. The PP model would entail that price changes are more frequent and have a smaller mean magnitude if the trend rate of inflation is raised in a comparative statics exercise.[44] This follows from the observation that the comparably small price adjustments that are necessary from time to time to catch up with increases in the price level occur more frequently when inflation is higher. By contrast, the SP model would predict that the frequency of price changes is not affected by changes in trend inflation and that the magnitude of price changes increases with inflation. The evidence presented by Wulfsberg (2016) for Norway, namely that prices change more frequently and in smaller steps in periods of high inflation compared to periods of low inflation, appears to be more in line with the predictions of the PP model. Using our

[44] To be more precise, this is true for small and moderate inflation. For very large inflation rates, almost all prices are raised every period. In such a situation, the size of price changes would increase with inflation.

model to carefully examine how changes in trend inflation affect price dynamics would be an interesting avenue for future research.

A Households' Optimality Conditions

In this section, we state the first-order conditions that describe the optimal behavior of households. Minimizing costs for a given size of the consumption basket C_t yields the demand function

$$C_{j,t} = \left(\frac{Q_{j,t}}{P_t} \right)^{-\varepsilon} C_t, \tag{22}$$

where the aggregate price level P_t is given by

$$P_t = \left[\int_0^1 (Q_{j,t})^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}. \tag{23}$$

The household's utility maximization problem results in the following standard conditions:

$$\begin{aligned} \frac{W_t}{P_t} &= N_t^{\varphi} C_t, & (24) \\ \mathbb{E}_t \left[\beta \frac{C_t}{C_{t+1}} R_t^n \frac{P_t}{P_{t+1}} \right] &= 1, & (25) \\ \frac{M_t}{P_t} &= \nu C_t \frac{R_t^n}{R_t^n - 1}. & (26) \end{aligned}$$

□

B Firms' Optimal Pricing Decisions

B.1 Price setting in the PP model

In this appendix, we determine firms' optimal price-setting behaviors. Equation (5) and the demand function (22) can be used to write the profit function (6) as

$$\Pi_{j,t} = \left(\frac{Q_{j,t}}{P_t}\right)^{1-\varepsilon} Y_t - \frac{W_t}{P_t} X_{j,t}^{-\frac{1}{\gamma}} \left(\frac{Q_{j,t}}{P_t}\right)^{-\frac{\varepsilon}{\gamma}} Y_t^{\frac{1}{\gamma}}. \tag{27}$$

Producer j's first order condition is

$$(1-\varepsilon)\left(\frac{Q_{j,t}^*}{P_t}\right)^{-\varepsilon} Y_t + \frac{\varepsilon}{\gamma}\frac{W_t}{P_t} X_{j,t}^{-\frac{1}{\gamma}} \left(\frac{Q_{j,t}^*}{P_t}\right)^{-\frac{\varepsilon}{\gamma}-1} Y_t^{\frac{1}{\gamma}} = 0, \tag{28}$$

from which we obtain the optimal price $Q_{j,t}^*$ that would be chosen if there were no information frictions and if prices were not restricted to the set of price points:

$$\left(\frac{Q_{j,t}^*}{P_t}\right)^{\frac{\gamma+\varepsilon(1-\gamma)}{\gamma}} = \frac{\varepsilon}{(\varepsilon-1)\gamma}\frac{W_t}{P_t} X_{j,t}^{-\frac{1}{\gamma}} Y_t^{\frac{1-\gamma}{\gamma}} \tag{29}$$

A log-linear approximation of this condition yields the following hypothetical optimal log price in the absence of a PPR and information rigidities:

$$q_{j,t}^{*PP} = \frac{\gamma}{\gamma+\varepsilon(1-\gamma)}\left[-\frac{1}{\gamma}x_{j,t} + w_t - p_t - \ln\left(\frac{\overline{W}}{P}\right) + \frac{1-\gamma}{\gamma}\hat{Y}_t\right] + p_t \tag{30}$$

We observe that the expression $w_t - p_t - \ln\left(\frac{\overline{W}}{P}\right) + \frac{1-\gamma}{\gamma}\hat{Y}_t$ represents the deviation of aggregate unit labor costs from its steady-state value in this economy, $\widehat{ulc}_t = w_t - p_t - \ln\left(\frac{\overline{W}}{P}\right) + \hat{N}_t - \hat{Y}_t$, since the log-linearized aggregate production function is given by $\hat{N}_t = \frac{1}{\gamma}\hat{Y}_t$.[45] Therefore the hypothetical optimal log price in the absence of a PPR and

[45]We note that price dispersion $s_t = \int_0^1 \left(\frac{Q_{j,t}}{P_t}\right)^{-\varepsilon/\gamma}(X_{j,t})^{-1/\gamma}dj$ affects the relationship between employment and output, as $Y_t = N_t^\gamma/s_t$ (see Ascari and Sbordone (2014)). However, s_t reaches a minimum in the steady state of the PP model, which means that small perturbations have no first-

information frictions is given by

$$q_{j,t}^{*PP} = \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left[-\frac{1}{\gamma} x_{j,t} + \widehat{ulc}_t \right] + p_t. \tag{31}$$

In the following we analyze optimal price setting behavior under the assumption that firm j can only select price points. Consider a quadratic approximation of the profit function around its maximum. Then, given that firm j has last updated its information in period $t-i$, its profit-maximizing admissible log price $q_{j,t}$ is the element in the set $\Delta \cdot \mathbb{Z} - u_j$ that is closest to $\mathbb{E}_{t-i}[q_{j,t}^{*PP}]$.

Hence, a firm j that has received new information i periods ago selects

$$q_{j,t}^{PP} = \mathcal{T}_j \left\{ \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left(-\frac{1}{\gamma} x_{j,t} + \mathbb{E}_{t-i} \left[\widehat{ulc}_t \right] \right) + \mathbb{E}_{t-i} \left[p_t \right] \right\}. \tag{32}$$

Note that we have used the fact that $x_{j,t}$ does not change in periods where the firm is not subject to idiosyncratic shocks, i.e. $\mathbb{E}_{t-i} \left[x_{j,t} \right] = x_{j,t}$. $\square$

B.2 Sticky-information Phillips curve

In this appendix, we derive the Phillips curve for the PP model. Recall that firms update their information if and only if they are affected by an idiosyncratic shock, which happens with probability $1 - \alpha^{PP}$ in each period.

Let q_t be the average log price of firms that are hit by an idiosyncratic shock in period t, which implies an update of firms' information sets. Because the individual differences

order effect on s_t and thus $\hat{Y}_t = \gamma \hat{N}_t$ holds approximately. The same result does not hold under Calvo pricing when the steady-state inflation rate is positive.

of $q_{j,t}$ from $q^*_{j,t}$ wash out in the aggregation[46] and because the average value of $x_{j,t}$ is always zero, this price can be written as

$$q_t = \int_0^1 q_{j,t} dj = \int_0^1 q^*_{j,t} dj = \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \widehat{ulc}_t + p_t, \tag{33}$$

where we have utilized equation (31).

Moreover, we note that firms choose prices $\mathbb{E}_{t-i}[q_t]$ on average if they were hit by a shock i periods ago. Hence the log price level can be written as

$$\begin{aligned} p_t &= (1-\alpha^{PP}) \sum_{i=0}^{\infty} \left(\alpha^{PP}\right)^i \mathbb{E}_{t-i}\left[q_t\right] \\ &= (1-\alpha^{PP}) \sum_{i=0}^{\infty} \left(\alpha^{PP}\right)^i \mathbb{E}_{t-i}\left[\frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \widehat{ulc}_t + p_t\right], \end{aligned} \tag{34}$$

where we have used (33) to replace q_t. Equation (34) is equivalent to the expression obtained in Mankiw and Reis (2002, p. 1300). Therefore it can be used to formulate a sticky-information Phillips curve analogous to the one obtained by them:

$$\begin{aligned} \hat{\pi}_t =& \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \frac{1-\alpha^{PP}}{\alpha^{PP}} \widehat{ulc}_t \\ &+ (1-\alpha^{PP}) \sum_{i=0}^{\infty} \left(\alpha^{PP}\right)^i \mathbb{E}_{t-1-i}\left[\hat{\pi}_t + \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left(\widehat{ulc}_t - \widehat{ulc}_{t-1}\right)\right]. \end{aligned} \tag{35}$$

$\square$

[46] Note that $q_{j,t} = q^*_{j,t} + d_{j,t}$ where $d_{j,t}$ denotes the distance between the optimal price and the closest price point. We observe that $d_{j,t} \sim U[-\frac{\Delta}{2}, \frac{\Delta}{2}]$ given that $u_j \sim U[0, \Delta[$.

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