Non-performing loans due to inefficient capital reallocation

Philip Schnattinger *

Please click here for the latest version

August 12, 2021

Abstract

Keywords: *Business cycles, Search frictions, Investment, Capital allocation, Non-performing loans*

A high share of non-performing bank loans in total loans (sNPL) has been shown to negatively affect aggregate investment and economic growth. While these empirical facts have been well established, neither the mechanisms causing the great cross-country heterogeneity in the sNPL, nor the channels through which they affect the real economy are well understood. A commonly invoked channel is that the sNPL leads to reduced credit supply. This paper first shows that focusing solely on this channel would provide an incomplete picture. Reduced credit supply implies higher rates of return to capital in economies with a higher non-performing loan burden as profitable projects are not met with sufficient credit supply. This can neither be observed in country cross-sections nor in time series data. The more important channel through which the sNPL affects the economy seems to stem from the credit demand side with the sNPL providing a mirror image of real capital misallocation. The paper then proposes a structural model with search frictions in used capital markets that links non-performing loans and sluggish capital reallocation to explain the observed sNPL and investment dynamics. The structural model shows that long and persistent sNPL increases in response to a negative shock, are either a symptom of a low option value of foreclosure for banks due to inefficient used capital markets or a symptom of forbearance incentives for banks due to balance sheet weaknesses and regulatory requirements. Both frictions are captured parsimoniously in the model, and since they imply different impulse responses for capital prices their individual impact on sNPL is estimated from capital price data. Both inefficiencies lead to more misallocated capital and reduce the marginal product of fresh capital, thereby impacting credit creation. A higher sNPL following a negative shock, such as the Covid-19 pandemic, will lead to more prolonged output and investment below equilibrium in countries with less efficient used capital markets. The tractable model can provide an explanation for observed non-performing loans levels and macroeconomic outcomes and may serve as a ready framework for analysing macroeconomic policies preventing the build-up and working towards resolving non-performing loans.

*DPhil candidate at the University of Oxford. Contact: philip.schnattinger@economics.ox.ac.uk
I would like to thank Francesco Zanetti, Petr Seláček, Masashige Hamano, Inessa Benchora, and seminar participants at the University of Oxford and participants at the ICMAIF 2021 for insightful comments and suggestions. The work is preliminary and not ready to be cited yet. Any mistakes or errors are my own.

"... At the end of 2016, the stock of gross NPLs in the EU banking sector was around € 1 trillion. This number, however, does not take into account the fact that that collateralised lending plays an important role in Europe. For example, including collateral and provisioning, the coverage of NPLs is, on average, 82% in the euro area ...
... The outstanding stock of NPLs is a consequence of cyclical and structural factors. First, the severe recession resulting from the global financial crisis led to a deterioration of the quality of banks' loan books... At the same time, structural weaknesses still persist. These include inadequate internal governance structures in banks, ineffective and costly debt recovery procedures in some Member States and misaligned incentives that prevent a quick resolution of NPLs."[1]

1 Introduction

The share of non-performing bank loans in total loans (sNPL) in a country has received a lot of attention by policymakers ever since the global financial crisis in 2007 led to a persistent rise of the share in some countries. The recession induced by the Covid-19 pandemic is expected to lead to a similar rise (Kasinger et al., 2021). A high share of badly performing loans in total loans and subdued aggregate output go hand in hand. For good reasons, both have been suspected to be the cause of the other. The sNPL may increase when negative supply or demand shocks cause the economy to contract, causing previously profitable firms and solvent households to default on payments. On the other hand, a high sNPL may lead to more restrictive lending and investment keeping the economy subdued. This paper builds on existing and predominantly empirical literature on the connection between NPLs and macroeconomic performance to develop a structural macroeconomic model capturing NPL statics and dynamics. Based on the model it presents new insights explaining the highly different dynamic reactions of NPL shares to macroeconomic shocks for different countries. The model can parsimoniously capture and describe the feedback loops between NPL shares and economic busts. The model allows for an evaluation of the importance of forbearance frictions and used capital market efficiencies. Judging which of these fictions are more prevalent in an economy, providing a basis for formulating and prioritising the most effective policies to resolve NPLs across Europe. The paper finds inefficient used capital markets to be a more likely driver of the sNPL, meaning that NPLs may be understood as a symptom of an economy's inability to efficiently reallocate capital from unproductive to productive use.

Even though non-performing loans have recently taken a center stage in the policy discussion in many countries, we do not know what the sNPL tells us about the state or dynamic response of a country's economy to macroeconomic shocks. Figure 1 shows that much like unemployment the sNPL is commonly cyclical, and remains persistently elevated following a recession. The right graph in figure 1 suggests that the speed of recovery from the great recession by means of investment correlated with the ability of a

[1] Speech by Mario Draghi, at the time President of the ECB and Chair of the European Systemic Risk Board, at the second annual conference of the ESRB, Frankfurt am Main, 21 September 2017

country to keep NPL stocks low following the negative shock. Recently, a growing empirical and policy-oriented literature has emphasized this negative correlation of NPL stocks on consumption, investment, and more generally on the macroeconomic performance of a country. A non-exhaustive list of empirical and policy papers in this area include (Louzis et al., 2012), (Klein, 2013), (Beck et al., 2013), (Jassaud and Kang, 2015), and (Ari et al., 2020). (Balgova et al., 2016)s argue using an event study approach that reducing the sNPL leads to an increase in real growth and investment. Even though it is difficult to determine the direction of causation between macroeconomic variables and sNPL, Institutions have become concerned with levels of NPLs and started a lively discussion about approaches to reduce and prevent these loans from arising. Policy proposals focus on macro-prudential policies, asset management strategies, as well as faster default processes and capital reallocation in the form of liquidating collateral.

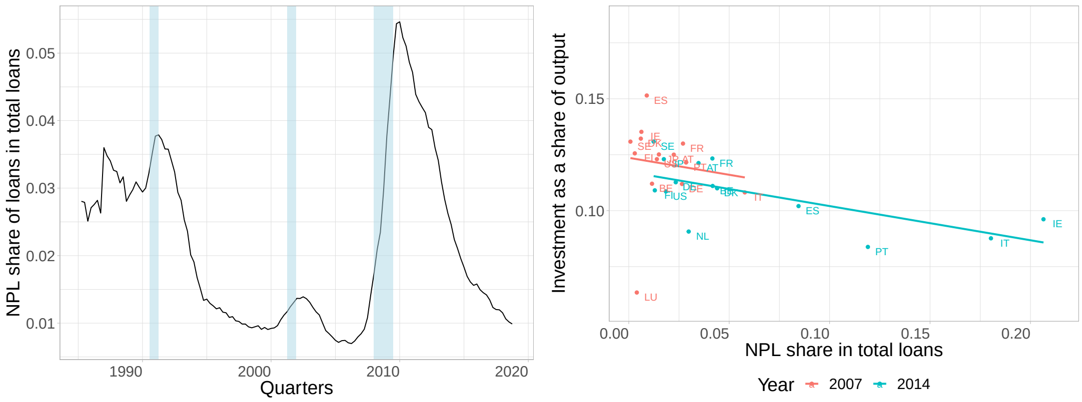

Figure 1: Left: NPLs as a share of total gross loans in the US from the FRED database. Right: Change in investment share of GDP and NPLs for OECD countries between 2007 and 2014.Investment calculated from the KLEMS database, NPL ratios from World bank and IMF data.

This policy discussion has so far largely focused on data and empirical models, while structural and business cycle models studying non-performing loan dynamics remain scarce. The general narrative for interpreting the empirical observations on sNPL has been that NPLs on a bank's balance sheet lead to lower bank profits and a regulatory need for higher cash reserves to compensate loan losses. Banks with a higher stock of NPLs on their balance sheet will then have less balance sheet space to lend out capital and thereby reduce credit supply.[2]. While the general intuition is compelling, it is difficult to find clear evidence that the lending behaviour of individual banks is differing due to their non-performing loan stocks ((Bredl, 2018) and (Accornero et al., 2017)). The cross-country comparison in this paper shows that this theorised reduction in credit supply cannot be the only channel through which the sNPL affects the macroeconomy as it would imply higher real rates of return for capital in the long run. The paradox of a policy discussion paired with a lack of structural models has been pointed out by a senior policymaker of the bank of Italy, which is due to the high stock of NPLs in Italy a major stakeholders in this discussion: *"To my knowledge, there is no clear theory suggesting that high volumes of NPLs*

[2]See, for instance, https://www.bankingsupervision.europa.eu/banking/priorities/npl/html/index.en.html

impair the credit allocation mechanism."[3].

The empirical part of the paper finds the sNPL to be strongly counter-cyclical. The sNPL correlates negatively with investment activity, aggregate returns on capital, capital prices, and capital reallocation. A VAR model with long-run restrictions (Blanchard and Quah, 1988) shows that positive supply shocks will lead to a reduction in the sNPL, while a rise in the sNPL has an ambiguous short-run effect on output. Meanwhile, short-run restricted VARs suggest that, controlling for output, an increase in the SNPL will lead to reduced investment, reduced capital returns, and delayed capital reallocation. On the other hand, positive investment shocks have an ambiguous effect on the SNPL, while a capital return shock reduces them. A country cross-section using aggregated firm micro and sectoral data shows that a higher SNPL is linked with lower aggregate returns on capital, lower investment, and a higher prevalence of non-profitable firms. Capital reallocation slows down as the SNPL rises.

The paper then presents a structural business cycle model where banks act as intermediaries for household lending to firms to match observed dynamic data and identify cross-country differences in SNPL outcomes as either the result of skewed bank forbearance incentives or inefficiently working markets for used capital. In the model, loans are provided with underlying collateral to firms for them to produce. The search and matching framework applied to bank-firm credit relationships combined with frameworks developed for capital reallocation ((Cao and Shi, 2017), (Ottonello, 2017), (Gopinath et al., 2017), (Lanteri, 2018), (Eisfeldt and Shi, 2018), and (Cui and Radde, 2020)) is shown to be particularly useful for modelling the decision-making of banks on whether to foreclose or forbear a loan. It can parsimoniously capture heterogeneity in the quality of a loan and heterogeneity in capital allocation while allowing for a tractable model that can explain non-performing loans, investment, and capital allocation dynamics. Loans may become non-performing in the model, upon which a bank must decide whether to foreclose the loan and reallocate the foreclosed collateral or to forbear the loan incurring real cost and hoping that the loan will become profitable in the future. This dynamic decision-making problem is modelled by assuming search frictions in used capital markets. This is a way to capture the fact that used capital has high asset specificity ((Bertola and Caballero, 1994) and (Kermani and Ma, 2020)) meaning it may hold a heterogeneous value for heterogeneous firms involving information and market search problems.

The bank's foreclosure decision will depend on the value of forbearance incentives and the efficiency of markets for the collateral. Forbearance incentives are regulatory or other frictions that result in banks incurring real economic losses when foreclosing a loan. Higher forbearance incentives and lower efficiency of used capital markets will both cause higher non-performing loan levels in equilibrium. However, they imply different dynam-

[3]Paolo Angelini, at the time Deputy Director General for Financial Supervision and Regulation, Bank of Italy; VOX EU CEPR, 12April 2018

ics regarding the price of capital and new investment activity. This allows using the response of countries to output shocks to judge the extent to which strong forbearance incentives or inefficient used capital markets drive sNPL persistence. The paper finds that in most European countries struggling with a high sNPL following the global financial crisis the inefficiency of used capital markets is at fault. This result correlates well with the resolving insolvency scores from the World bank's doing business indicators which should be a combination of asset specificity and used capital market efficiency.

Models with search frictions in credit markets have recently become more popular. The model presented in this paper build on many of the insights of models from search frictions in labour markets. It is kept simple in a random search fashion with intra-period heterogeneity similar to (Mortensen and Pissarides, 1994). However, the model setup is chosen in such a way as to allow for persistent heterogeneity dynamic directed search block recursive equilibrium extensions of the type developed in (Menzio and Shi, 2010a) and (Menzio and Shi, 2010b). Papers introducing search in credit markets in a similar manner to this paper like (Beaubrun-Diant and Tripier, 2015), (Boualam, 2018) or (Cui and Radde, 2020) have mostly set up the models with firm search and bank free-entry, leading to a challenge in determining firm stocks or assuming less-intuitive fixed stocks of entrepreneurs which may gather financing. The paper also speaks to the recently re-emerging zombie firm literature (Caballero et al., 2008) and (Acharya et al., 2020), which is unsurprising as corporate NPLs are likely to stem from zombie firms. While not focusing explicitly on either the financial crisis or the financial accelerator literature (Bernanke, Gertler, and Gilchrist, 1999), this model can also be straightforwardly integrated in such models, and may be used for studying the effect of unconventional central bank policies in models of the (Gertler and Karadi, 2011) type.

The next section of this paper presents the business cycle properties of non-performing loans and shows that the sNPL has an effect on the macro-economy beyond the pure credit supply channel. The third section presents the model focusing on bank foreclosure decisions and the consequences on the sNPL, capital reallocation, and capital productivity is presented. The model section first describes a simple partial equilibrium model, which serves to provide intuition for the main mechanism for the dynamic stochastic general equilibrium of the main model. Following this, the main model is presented featuring endogenous foreclosure decisions, search frictions in used capital markets and heterogeneity in the profitability of loans, and forbearance incentives. It is shown that the efficiency of used capital markets and forbearance incentives play a crucial role in determining the outside value of foreclosing capital versus forbearing capital. The model is then calibrated to show that it can explain the correlations of the business cycle with sNPL dynamics. Furthermore, some preliminary data correlations are provided highlighting the plausibility of the theoretical result that the efficiency of used capital markets determines NPL stocks and dynamics, that variation in this efficiency can explain the observed cross-country divergence following the global financial crisis and great recession. Finally, the fourth concludes.

2 Business cycle properties of NPL shares in bank balance sheets

This section first presents the business cycle properties of NPLs using aggregated US data for the period 1985 - 2018. US data on NPLs and capital reallocation is available for a longer time period than for most other economies. NPL shares in total loans are shown to be counter-cyclical. Long-run restrictions suggest that output growth leads to a decline in NPL, while the opposite impact is ambiguous. Controlling for Real GDP short-run restrictions suggest a rise NPLs will drive down investment, increase delinquency rates, reduce property prices, and capital returns.

The correlations in table 1 present aggregate business cycle properties of the sNPL. Series are downloaded from the federal reserve of St. Louis' database unless otherwise specified. The sNPL series describes the share of non performing loans in total loans. Return on capital is calculated as the value-added accruing to capital over the capital stock calculated via a perpetual inventory method from capital formation and consumption. Investment is the GDP share of gross fixed capital formation, while property prices are captured by the house price index. Reallocation is calculated similarly to (Eisfeldt and Rampini, 2006) as the sum of firm acquisitions of existing property plant or equipment over total firm assets. Firm data is downloaded from WRDS Compustat database for the relevant period. The reallocation series which is only available at annual frequencies is linearly interpolated. All series are Hodrick-Prescott filtered at quarterly frequencies to highlight cyclical properties and remove trends.

The table shows that the sNPL is counter-cyclical. The correlations further suggest that low returns on capital lead NPLs, while NPLs lead reductions in investment, reduced reallocation, and reduced property prices. Delinquency rates increase as NPLs increase. The underlying series for this table and the IRFs from short-run restrictions in the next subsection can be found in Appendix B.

2.1 Shock identification and impulse responses

We are interested in the impact real output has on non-performing loans and vice versa. Given the long-run property of NPL shares in the US presented in figure 1 to return to an equilibrium, and their similarity to unemployment it is reasonable to identify the effects of NPLs on output and vice versa by assuming variation in them has only a temporary effect on output. Thus one can impose long-run restrictions of the type suggested by (Blanchard and Quah, 1988) to separate demand and supply shocks with NPLs capturing demand. While the available thirty years of data are not enough to provide clearer confidence intervals for either of the series responding to shocks from the other figure 2 suggests that it is

Variable	sNPL (-2)	sNPL (-1)	sNPL	sNPL (+1)	sNPL (+2)
Real GDP	-0.64	-0.67	-0.64	-0.56	-0.43
Return on capital	-0.34	-0.49	-0.60	-0.68	-0.73
Investment	-0.67	-0.75	-0.77	-0.74	-0.66
Reallocation	-0.51	-0.47	-0.41	-0.32	-0.20
Delinquency rates	0.75	0.81	0.81	0.77	0.68
Property prices	-0.79	-0.75	-0.69	-0.60	-0.49

Table 1: Business cycle properties of sNPL. sNPL, Real GDP, Delinquency rates, and property prices are downloaded from Fred. Aggregate capital returns are calculated based on BEA data. Capital reallocation is calculated following Eisfeldt and Rampini (JME, 2006). All series are calculated as deviations from a quarterly Hodrick-Prescott trend. Property prices stand in as capital prices. Sources: FRED, BEA, WRDS

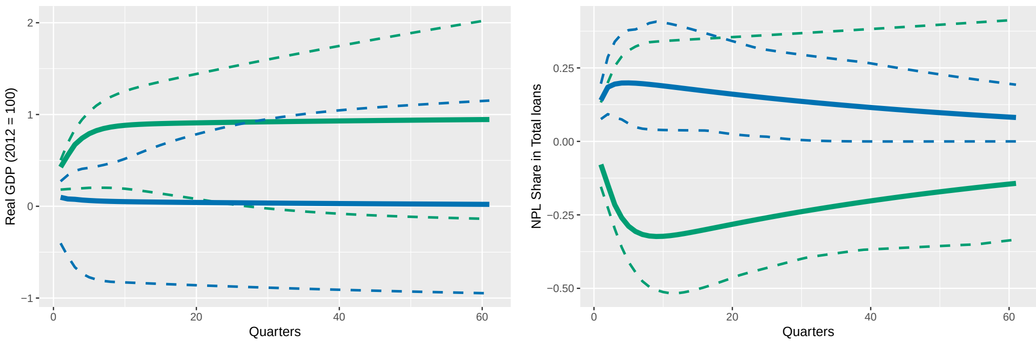

Figure 2: Green captures supply shocks (from real GDP) and blue demand shocks (from NPL shares in total loans). Dashed lines show bootstrapped confidence intervals.

very likely a rise in real GDP will reduce NPL shares.

It is thus clear that controlling for real GDP is necessary to identify correct impulse responses for a change in the NPL share with relation to other variables that provide indications about capital reallocation, such as return on capital, reallocation flows, delinquency rates, or property prices. Using the suggested ordering from the correlations and real GDP ordered before non-performing loan rates short-run restrictions are imposed. The number of lags (L) is chosen using the Hannan–Quinn information criterion. ϵ is a vector of identified shocks. Importantly, the main reason for the imposition of short-run restrictions here is not to identify causal relations, but to explore the dynamic behaviour of NPL shares in relation to other variables. Causality in this paper is implied by the mechanisms in the structural model. The VAR models for the relevant variables are found in equation 1 and 2. Equation 1 is used for variables for which the correlation table implies that they lead NPL shares, while equation 2 is used for variables that lag them.

$$\begin{bmatrix} Var\ leading\ NPL \\ Real\ GDP \\ NPL \end{bmatrix} = A(L) \begin{bmatrix} Var\ leading\ NPL(L) \\ Real\ GDP(L) \\ NPL(L) \end{bmatrix} + B\epsilon \tag{1}$$

Figure 3 suggests NPLs declining as capital returns increase, while the effect in the opposite direction is also negative meaning that a higher NPL share causes lower returns to capital, while controlling for the impact of output. Meanwhile higher levels of NPLs lead to lower investment as the graphs in 4 clearly show, but higher investment has an ambiguous impact on NPL stocks. Capital reallocation flows are affected negatively as NPLs rise as shown in figure 5. Delinquency rates and NPLs both affect each other positively as shown in figure 6. This means that capital reallocation flows decline with higher NPL stocks even though foreclosures by banks rise. This suggests the classic congestion mechanism inherent in markets with search frictions. Finally, figure 7 shows that high NPL rates will negatively affect the outside value of the underlying collateral capital as proxied for by house prices. Given the negative correlation of NPLs with capital reallocation, this is an expected fact highlighted by the capital reallocation literature. Search frictions in used capital markets are useful in explaining the strong correlation of the capital price with capital reallocation.

$$\begin{bmatrix} Real\ GDP \\ NPL \\ Var\ lagging\ NPL \end{bmatrix} = A(L) \begin{bmatrix} Real\ GDP(L) \\ NPL(L) \\ Var\ lagging\ NPL(L) \end{bmatrix} + B\epsilon \tag{2}$$

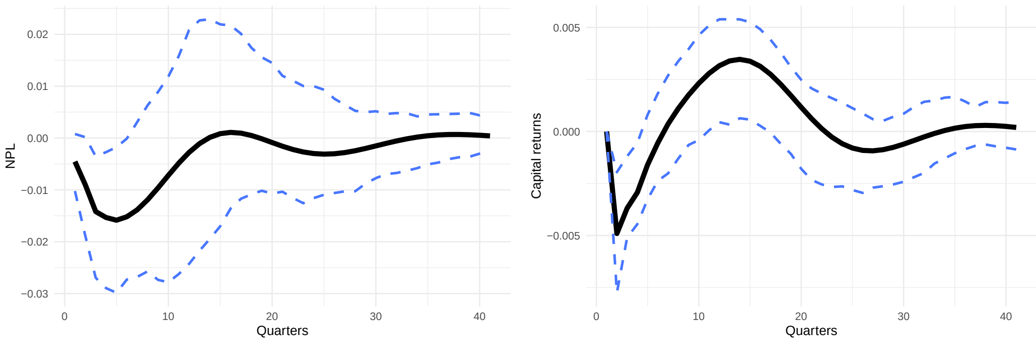

Figure 3: Impact of capital returns on NPLs (left) and vice versa (right)

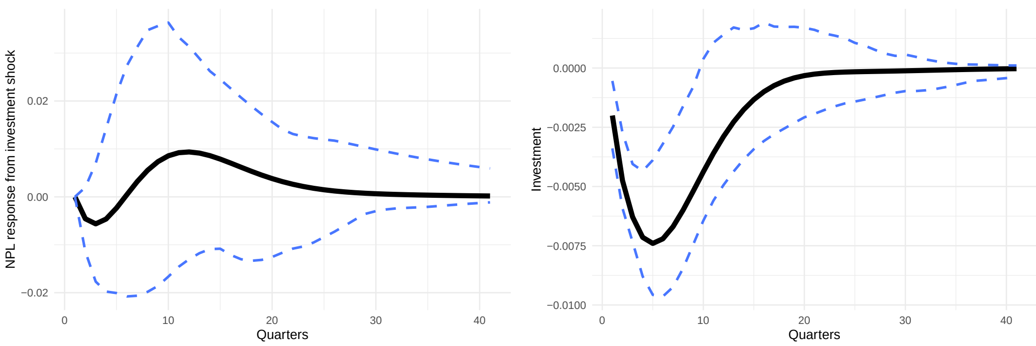

Figure 4: Impact of Investment on NPLs (left) and vice versa (right)

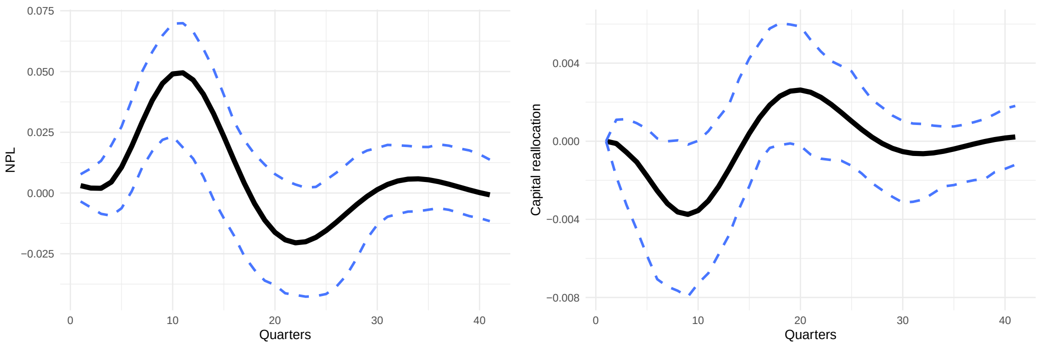

Figure 5: Impact of capital reallocation on NPLs (left) and vice versa (right)

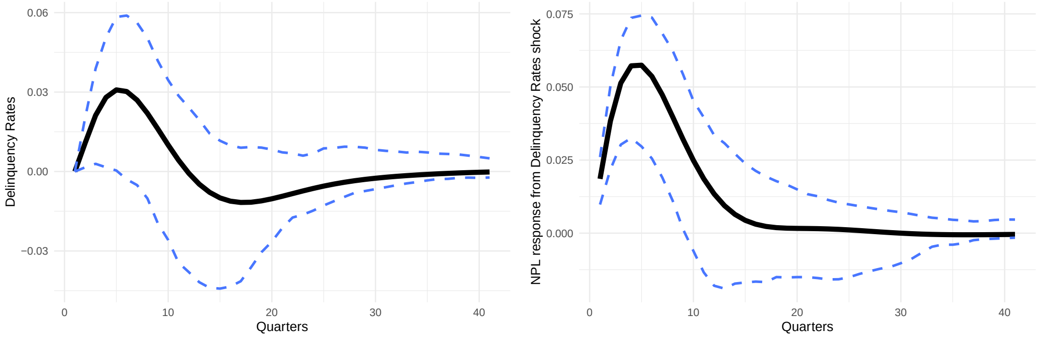

Figure 6: Impact of delinquency rates on NPLs (left) and vice versa (right)

2.2 Cross country comparison showing the capital misallocation channel captured by NPLs matters

A lemma from the assumption that high NPL stocks only reduce credit supply via the bank profit channel is that real returns of capital should increase. The reason for this is that in a frictionless credit market the price of capital should equal marginal returns to capital $r_k = MPK = r_l$. If the lending rate increases due to a reduction in credit supply then $r_k = MPK = r_l + u^{NPL}$. Thus reduced credit supply should lead to increased marginal real returns of capital due to unrealised opportunities.

However, figure 8 shows that real capital returns seem at best negatively correlated with NPL shares in total loans in OECD countries for the data available. In the figure, the mean NPL share between 1995 - 2017 is plotted versus the mean capital return over the same period. This result is robust to using other measures for the correlations such as NPL shares at the start of the dataset (start_npl), NPL share growth (d_npl), capital returns corrected by value-added growth (r_g_y), and their respective growth rates. The results of these simple regressions are in table 2 and all correlation plots can be found in Appendix B. The main takeaway from this section is that the credit supply channel cannot be the only way through which NPL shares in total loans correlate with other macroeconomic outcomes in general and capital productivity in specific. The structural model in the next section will argue that the results can be explained by NPL also indicating a reduction in marginal returns to capital in the economy via a misallocation

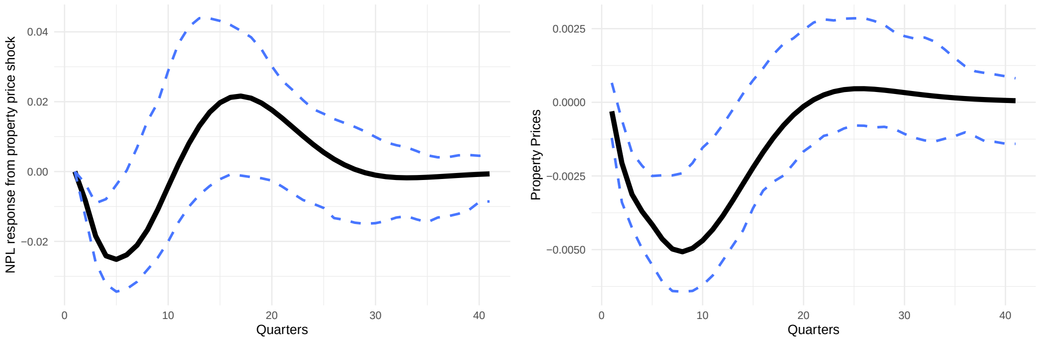

Figure 7: Impact of property prices on NPLs (left) and vice versa (right)

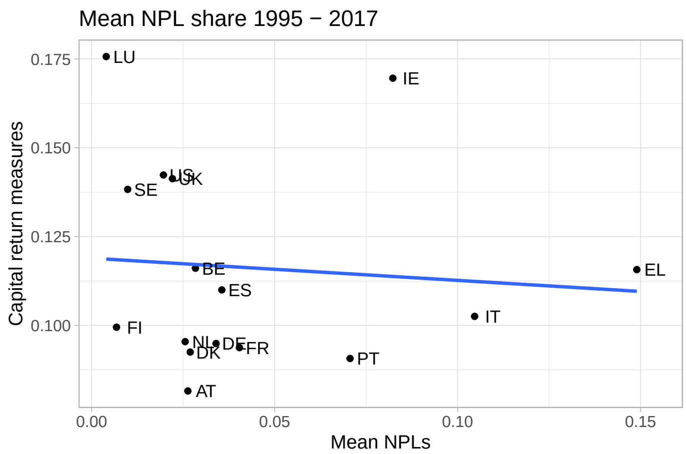

Figure 8: Capital return measures are from KLEMS data, while NPL measures are from merged world bank and IMF data.

channel $MP_{Misallocation}$. This can explain the equation before in very reduced form as $r_k + MP_{Misallocation} = r_l + u^{NPL}$. Thus a higher sNPL doesn't lead to higher capital returns, because higher NPL shares indicate that a larger part of the capital stock gets stuck in unproductive relationships and cannot escape these relationships due to market frictions.

Table 2: Regressions of capital returns on NPL variables

	Dependent variable:											
	r_k	r_g_y	d_r_k	dr_g_y	r_k	r_g_y	d_r_k	dr_g_y	r_k	r_g_y	d_r_k	dr_g_y
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
m_npl	-0.062	-0.053	-0.038	-0.027								
	(0.194)	(0.183)	(0.057)	(0.056)								
start_npl					-0.150	-0.128	-0.093*	-0.084				
					(0.189)	(0.178)	(0.051)	(0.051)				
d_npl									0.336	0.348	0.133	0.322
									(1.843)	(1.732)	(0.543)	(0.527)
Constant	0.119***	0.116***	-0.0001	0.0003	0.121***	0.118***	0.001	0.002	0.116***	0.113***	-0.002	-0.001
	(0.011)	(0.011)	(0.003)	(0.003)	(0.010)	(0.009)	(0.003)	(0.003)	(0.008)	(0.008)	(0.002)	(0.002)
Observations	16	16	16	16	16	16	16	16	16	16	16	16
R^2	0.007	0.006	0.032	0.016	0.043	0.035	0.189	0.160	0.002	0.003	0.004	0.026

Note: *p<0.1; **p<0.05; ***p<0.01

This result is confirmed by aggregated micro-data from the 7th Vintage CompNet dataset. Figure 9 shows that a larger percentage change in the sNPL between 2007 and

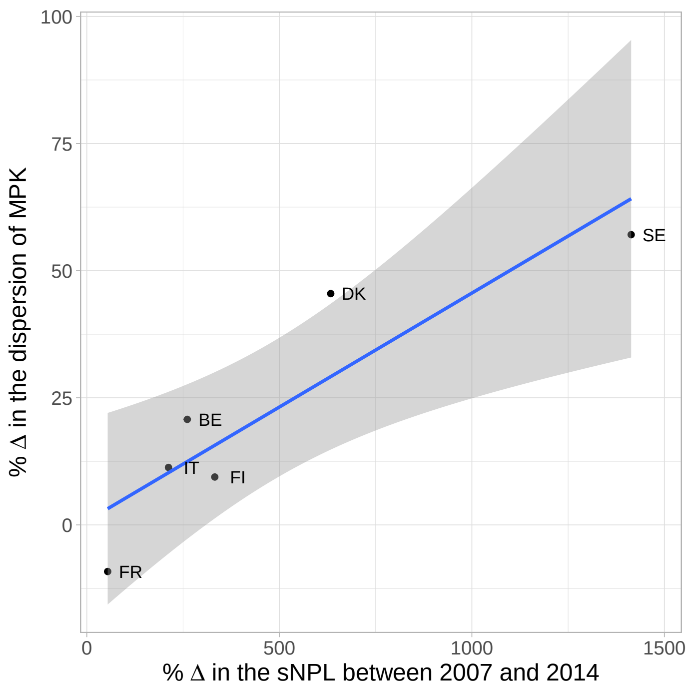

Figure 9: As sNPL changes estimated MPK dispersion from C-D estimation by sector increases proportionally. sNPL measures are from merged world bank and IMF data.

2014 went hand in hand with a proportional increase in the dispersion of the marginal product of capital of firms in a country. In this case, the marginal product was estimated via OLS between sectors, but results are similar for an estimation of marginal products via the method proposed by Wooldridge.

Finally, an important figure for judging the importance of forbearance incentives versus frictions in used capital markets is figure 10. It shows the change in the price of capital as non-performing loans increase. As the sNPL increases capital is becoming cheaper. This is unlikely to be the case if forbearance incentives rise strongly in recessions and this was to drive the sNPL. As the structural model shows rising forbearance frictions lead to more tight used capital markets, which would lead to an expected increase in the real price of capital.

3 Model

The model assumes that credit markets for used capital experience matching frictions summarising similar heterogeneities as labour markets, and that the aggregate product of capital experiences marginally diminishing returns, whether the capital is put to use or not. Modelling used capital with higher matching frictions than fresh capital is meant to capture the heterogeneities entrepreneurs who want to employ used capital productively face when reusing specified capital, for example, a specific factory. If entrepreneurs were to receive fresh capital they could build the factory to any specification. An entrepreneur seeking to employ a reused factory may find an excellent an inexpensive fit, but may also spend a long time on search not finding a suitable production location. Capital experi-

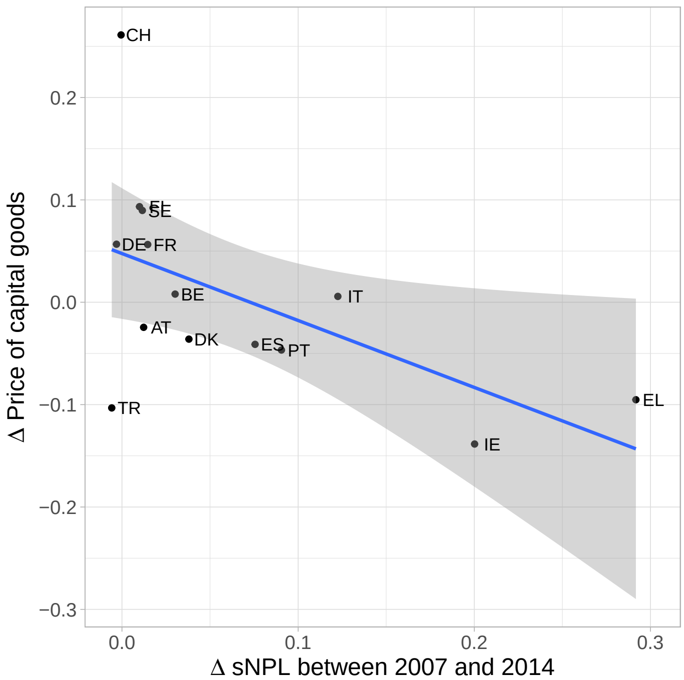

Figure 10: As sNPL increase capital prices fall. sNPL measures are from merged world bank and IMF data. The real price of capital is from Eurostat.

encing marginally decreasing returns is a common assumption and can be micro-founded by unused real estate or factories occupying the most productive locations, or occupying other non-modelled resources such as labour performing maintenance tasks or engaging in production with foreclosed capital.

The model is placed into a general equilibrium framework to assess the impact of frictions in used capital markets on the dynamics and long-run outcomes of reallocation, non-performing loans, and capital returns. Figure 11 sketches an overview of the workings of this general equilibrium model. Households provide fresh resources to create capital in the form of deposits to banks. Households own firms, which search for profitable opportunities to borrow this capital from banks, which then becomes a new loan with underlying collateral. Firms, which can be also viewed as entrepreneurs, make proposals for using the capital to banks, which are at the heart of the capital market. Banks provide capital in form of loans to firms. Once firms become unproductive and can no longer pay a share of profits in the form of interest to banks in return for the financing the banks have to decide whether to foreclose the loan and seize the collateral capital. If the bank decides on doing this it will seek to re-lend the foreclosed capital to entrepreneurs. Banks pass any profits or losses generated with household deposits on to households. The capital markets with search frictions in this model are closed on the loan demand side via the free entry by firms, while they are closed on the loan supply side via the deposit provision from households, which is derived from the intertemporal Euler equation.

The first subsection in this section aims to illustrate the key mechanism, namely how the efficiency of used capital markets affects the bank forbearance or foreclosure decision, in a partial equilibrium model. The second subsection then presents the full dynamic stochastic general equilibrium model with endogenous foreclosure.

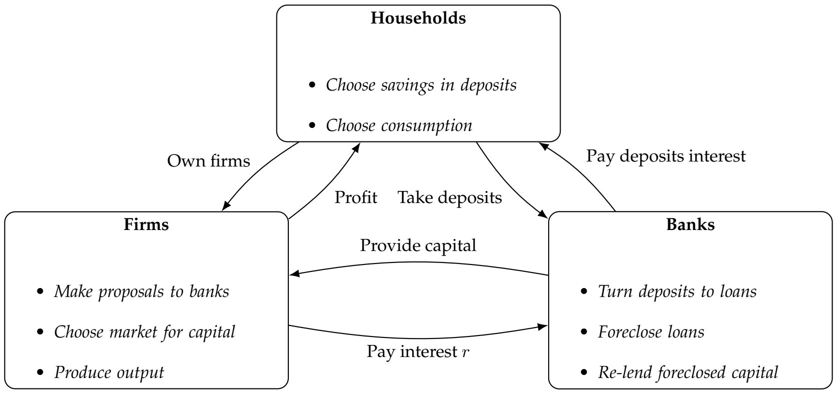

Figure 11: Model relations

3.1 Partial equilibrium foreclosure decision

The purpose of this simple model is to show that the loan foreclosure decision will be determined by the ability of the bank to reuse foreclosed capital productively. For illustration, very simple assumptions are taken leading to a closed-form solution. These are subsequently relaxed in the full model following this section.

Assume a model that only runs for one period. Banks come into the period with all capital lent to firms. For simplicity, banks learn about the productivity of lent capital units, decide whether to foreclose the loan, and have the option in case of foreclosure to re-lend the capital within one period.. Banks will aim to maximise profits by making an optimal foreclosure decision.

A firm matched with a bank will pay the bank a stochastic interest rate realisation $r(\epsilon)$ depending on the firms marginal productivity. $r(\epsilon)$ is assumed to be the realisation of $\bar{r}-\epsilon$, where $\log(\epsilon)$ is assumed to be normally distributed. Thus the set of possible interest rate realisations for legacy loans is $(-\infty, \bar{r})$.

Let p be the probability for the bank of finding a new firm if it decides to foreclose a current loan. p is assumed to be a function of match efficiency μ in used capital market and the tightness of the market θ. Tightness is the number of entrepreneurs offering a business plan for the capital unit g over the number of capital units searching to be matched s. $p(\mu, \theta)$ is a function with the properties $\frac{\partial p}{\partial \mu} \geq 0$ and $\frac{\partial p}{\partial \theta} \geq 0$. Let $V_u = [p(\mu,\theta)\bar{r} + (1 - p(\mu,\theta))b]$. Thus the reward of finding a new loan is to be matched at the productivity frontier $\bar{r}$. The bank will decide to foreclose a legacy loan when $V_m = r(\epsilon) < V_u$. The cutoff value $\tilde{\epsilon}$ at which a bank decides to foreclose a unit of capital is then given by 3. p is specified as the result of a Cobb-Douglas matching function with $p = \mu\theta^{0.5}$. Finally, there

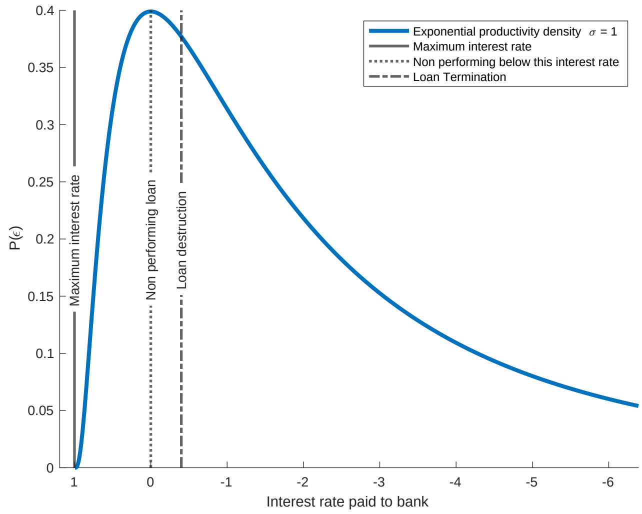

Figure 12: Simple illustration of a mass of NPLs in the distribution of existing loans

may be forbearance incentives for banks τ which would increase the benefit from keeping a beginning of period loan.

$$r(\tilde{\epsilon}) = \bar{r} - \tilde{\epsilon} = p(\mu,\theta)\bar{r} + (1 - p(\mu,\theta))b - \tau = p(\mu,\theta)[\bar{r} - b] + b - \tau \tag{3}$$

With an appropriate calibration of $b < 0$ to capture possible losses of capital due to depreciation it is then clear that values of $V_u < 0$ are possible depending on the productivity of used capital markets, the value of $\bar{r}$, and match probabilities $p(\mu,\theta)$. A reasonable assumption is to define an NPL cutoff where $r(\epsilon_{NPL}) = 0$, meaning those legacy loans that are maintained by banks even though $r(\epsilon) < 0$ are NPLs. The distribution presented in figure 12 with the calibration with $\bar{r} = 1$, $b = -1$, $\mu = 0.3$, $\theta = 1$ and the distributional parameters for $\log(\epsilon)$ set to mean 0 and standard deviation 1 shows that there can be a significant mass on NPLs in this model with matching frictions.

The reason NPLs arise in this model is that foreclosing a loan doesn't necessarily mean for the bank that it will be able to re-lend the underlying capital to a new firm and gain $\bar{r}$. The bank may make a loss b on the capital instead as it stays idle, depreciates, or possibly requires maintenance costs or management by the bank. The probability of this negative event happening depends on match probabilities p. Low probabilities due to low used capital efficiencies will mean that the bank is willing to accept more and more negative interest rates increasing the NPL share in total loans given in equation 4, where $\Phi()$ is the cumulative distribution function of a normal distribution.

$$NPLshare = \frac{\Phi(\log(\tilde{\epsilon})) - \Phi(\log(\epsilon_{NPL}))}{\Phi(\log(\tilde{\epsilon})) + p[1 - \Phi(\log(\tilde{\epsilon}))]} \tag{4}$$

$\Phi(\log(\tilde{\epsilon}))$ captures is the share of surviving legacy loans while $p[1 - \Phi(\log(\tilde{\epsilon}))]$ is the value of newly created loans. Mean expected interest rates r_m received by banks can be

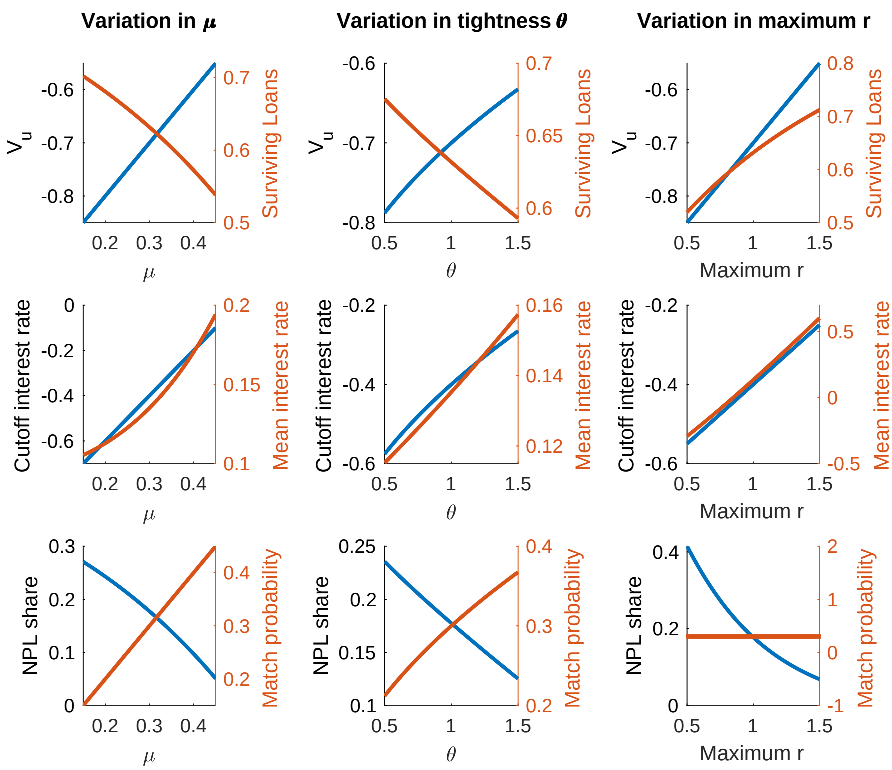

Figure 13: Comparison of the value of foreclosing a loan V_u, surviving legacy loans, the cutoff interest rate, the mean interest rate paid to banks, the share of NPLs in total loans and the match probability to variations in μ, θ, and $\bar{r}$

computed from the mills ratio as $r_m = \bar{r} - \exp(-\frac{\phi(\log(\tilde{\epsilon}))}{\Phi(\log(\tilde{\epsilon}))})$ and 50 % variation in parameters μ, θ, and $\bar{r}$ can be compared to provide an idea of the forces in the model. Here μ is an increase in used capital market efficiency, while θ is an increase in entrepreneurial activity, thus an increase in investment and $\bar{r}$ can be interpreted as a rise in total factor productivity lifting interest rates. The underlying outcomes of these variations on the partial equilibrium model are shown in figure 13. These variations show that a higher value of foreclosure only leads to more loans being called, hence more reallocation if the increase is due to a rise in match probabilities, which happens when θ or μ increase. While increases in θ, however, experience marginally diminishing returns increases in match productivity exponentially increase mean interest rates and decrease NPL shares due to increased reallocation. When $\bar{r}$ rises reallocation falls as shown by the rise in surviving loans. The only reason NPLs fall, in this case, is due to the rising denominator and the shift of the distribution, but not due to reallocation. This shows that match efficiency and reallocation go hand in hand. However, to study the business cycle properties of this mechanism it has to be included in the dynamic general equilibrium setting, which is done in the next section.

Figure 14 finally shows the comparison between an increase in the efficiency of used capital markets and a decrease in forbearance incentives. From the statics, it is clear that both would lead to a similarly shaped decrease in the sNPL as both affect the cutoff condition for $r(\tilde{\epsilon})$. However, while a higher forbearance incentive leads to loans being kept due

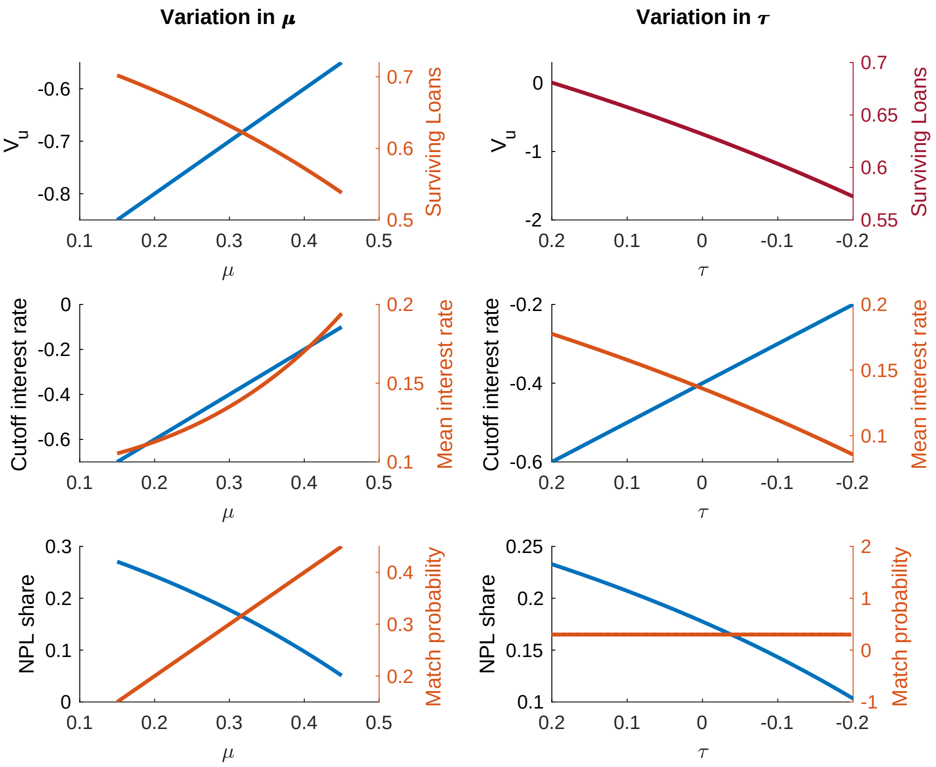

Figure 14: Comparison of the value of foreclosing a loan V_u, surviving legacy loans, the cutoff interest rate, the mean interest rate paid to banks, the share of NPLs in total loans and the match probability to variations in μ, τ.

to the cost of dissolving the loan, a lower efficiency of used capital markets drives down the probability of rematching and thereby V_u. This ultimately will lead to less tight capital markets as more capital is on the market searching to be matched with a lesser likelihood of success. The dynamic general equilibrium setting can use this distinction to identify the importance of forbearance incentives in comparison to the efficiency of used capital markets using the behaviour of the observed price of capital following a shock.

3.2 General equilibrium model with endogenous loan foreclosure

The decision on whether to foreclose a NPL, on which the partial equilibrium model of the previous section focused, can be integrated into a dynamic stochastic general equilibrium. The expected value of a capital unit underlying as collateral a loan for banks to entrepreneurs then creates an infinite value function, which depends on the ability of the entrepreneur to pay for the capital unit of the loan hence the entrepreneur's capital productivity. Other parts of the model are kept as simple as possible. Some kind of heterogeneity in the productivity of loans will be necessary to model non-performing loans. Non-performing loans are those where capital is below a certain productivity $z < \tilde{z}$, but above the bank's cutoff value z_s. Further, there is a certain level below which a bank chooses to call the non-performing loans $\tilde{z}$. From this time on the loan will not produce and the only purpose will be to rematch the capital with another entrepreneur. Taking reasonable definitions for when to consider a loan as non-performing it is straightforward to show that with matching frictions in used capital markets $\tilde{z} < z < \hat{z}$, and thus there exists a share of loans that are non-performing but not foreclosed at all times.

3.3 Aggregate production function

Similar to (Ottonello, 2017), who distinguishes between several types of capital based on their "employment status", there are three types of capital stocks in the economy. These are matched capital in safe loans and in weak loans denoted by K_t^N and K_t^E and unmatched capital denoted by K_t^U. Matched capital is employed in firms in production. Matched capital is split into matched capital in safe loans K_t^N and capital in weaker loans where payment of interest rate is uncertain and possibly lower than the market rate and which may become non-performing K_t^E. Unmatched capital is owned by banks and only produces output, but still forms part of the capital stock. The aggregate capital stock in the economy is $K_t = K_t^N + K_t^L + K_t^U$. Firms produce output with capital units. Each capital unit can either be in a loan or be foreclosed and held by the bank. When a capital unit is in a fresh loan, which will not be foreclosed it produces output according to equation 5. When a capital unit is in a loan that may be foreclosed it produces output according to equation 6. When a capital unit is foreclosed it produces output according to equation 7.

$$\bar{y} = AK^{\alpha-1}\bar{z} \tag{5}$$

$$y_i = AK^{\alpha-1}z \tag{6}$$

$$y_i = AK^{\alpha-1}g \tag{7}$$

The total aggregate capital stock negatively affects returns to the capital unit, but the distribution of capital over states does not. Employed capital K_t^E is the sum of all employed capital units K_i^L over all states $K_t^E = \sum_{i=0}^{Z} z_i K_i^L$. Total output is given by equation 8

$$Y = \bar{y}K_t^N + \int_{z_s}^{\bar{z}} y(z)\, d(z) K^L + gK^U = AK^{\alpha-1}(\bar{z}K^N + \int_{z_s}^{\bar{z}} z\, d(z)K^L) + gK^U \tag{8}$$

Output may be used for consumption by the households, as well as investment into bank deposits to create more productive capital, or as a resource to set up a business plan proposal by the entrepreneur.

K_u is the capital banks have foreclosed. This capital remains idle and only produces with g, which may also be a negative output consuming resources. Banks seek to rematch foreclosed capital in secondary capital markets with productive entrepreneurs. The lifecycle of a physical capital unit underlying a bank loan is sketched in figure 15. Fresh capital can be added to the existing capital stock via investment but matched and foreclosed unmatched capital remain in the economy. Matched capital changes exogenously with probabilities π^N and π^L from being matched in a safe or weak loan and vice versa. Capital switches states from weak loans to unmatched capital owned by banks according to the agent decision-making. The only way for physical capital to exit the economy is via depreciation.

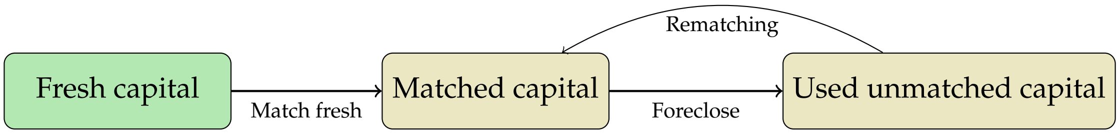

Figure 15: Life cycle of collateral underlying lent capital

3.4 Households

The economy is populated again by a unit mass of identical households. Each household has an initial deposit wealth D_0. However, not all deposits are automatically turned to capital. Thus deposits may stay idle and remain bank cash reserves X. This means real deposit values won't always depreciate in the same way in the same way as the underlying collateral values of capital.

$$\max_{C_t, I_t} E_t\left(\sum_{s=0}^{\infty} \frac{C_{t+s}^{1-\sigma}}{1-\sigma}\right) \tag{9}$$

The maximisation problem is subject to a conventional budget constraint where consumption C and investment I equal firm profits Π_t and interest rates paid on bank deposits $\rho_t D_t$.

$$C_t + I_t = \Pi_t + \rho_t D_t \tag{10}$$

Deposits evolve according to equation 11 with the current value of $\delta_{d,t-1}$ taken as given by the individual household and defined at a later time.

$$D_t = (1 - \delta_{d,t-1}) D_{t-1} + I_{t-1} \tag{11}$$

This means the inter-temporal Euler equation is a conventional function of depreciation, interest rates ρ, as well as present and future consumption.

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta_{d,t+1} + \rho_{t+1})] \tag{12}$$

3.5 Credit market

3.5.1 Financial intermediaries (Banks)

Banks turn household deposits into physical capital units when they match with entrepreneurs making a convincing business proposal that receives financing. The rate at which proposals arrive is $p(\theta_{x,t-1})$, where $\theta_{x,t} = \frac{g_{x,t}}{s_{x,t}}$ denotes market tightness. $s_{x,t}$ is the share of deposits that may be lent out, which may be affected by regulatory policy $s_{x,t} = \psi_x x_t$. Denote deposits that have not yet been turned into capital with X_t. Fresh deposits evolve according to equation 13.

$$X_t = (1 - \delta_x)[X_{t-1} - p(\theta_{x,t-1})S_{t-1}] + I_{t-1} \tag{13}$$

A bank's present discounted value from a safely matched unit of capital that was just created by lending out deposits is denoted by $V_{\bar{z}}^{B,L}$. The present discounted value of a capital unit in a weak lending relationship is $V_z^{B,L}$. In both cases z describes the idiosyncratic productivity of the capital being used. The present discounted value of a capital unit that is unused because the bank has foreclosed it is $V^{B,U}$. The present discounted value of a new and a weak lent unit are defined by equations 14 and 15. For simplicity of notation define the mean expected surplus-value of a weak loan match over a foreclosed loan next period as $\hat{V}_{\hat{z},t+1}^{B,L} = \int_{z_{z,t+1}}^{\bar{z}} (\hat{V}_{z,t+1}^{B,L} - V_{t+1}^{B,U} + \tau)\, d(z)$. τ captures a loan forbearance incentive for banks, who depending on the calibration may incur real cost when foreclosing a loan and reducing the size of their balance sheet.

$$V_{\bar{z},t}^{B,N} = r_{\bar{z},t} + (1 - \delta_k)E_t[\mu_{t+1}[\pi^N(V_{\bar{z},t+1}^{B,L} - V_{t+1}^{B,U}) + (1 - \pi^N)(\hat{V}_{\hat{z},t+1}^{B,L} - \tau) + V_{t+1}^{B,U}]] \tag{14}$$

$r_{\bar{z}}$ denotes the interest rate paid by firms in safe loans, which will depend on the productivity of the capital underlying the loan, while $\mu_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\sigma}$ is the stochastic discount factor. δ_k is the depreciation rate of capital. π^N is the probability that a safe loan will remain safe, while it will turn into a weaker loan with probability $(1 - \pi^N)$. In the case where $\pi^N = 0$ and $\pi^L = 1$ the model is then similar to the interpretation of new loans being created by businesses at the technology frontier as is assumed in (Mortensen and Pissarides, 1994) for new jobs.

$$V_{z,t}^{B,L} = r_{z,t} + (1 - \delta_k)E_t[\mu_{t+1}[(1 - \pi^L)(V_{\bar{z},t+1}^{B,L} - V_{t+1}^{B,U}) + \pi^L(\hat{V}_{\hat{z},t+1}^{B,L} - \tau) + V_{t+1}^{B,U}]] \tag{15}$$

r_z denotes the interest rate, which will depend on the productivity of the capital underlying the loan, while $\mu_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\sigma}$ is the stochastic discount factor. δ_k is the depreciation rate of capital. The present discounted value of a foreclosed unit of capital is in equation 16.

$$V_t^{B,U} = g + (1 - \delta_k)E_t[\mu_{t+1}(p(\theta_{u,t})[V_{\bar{z},t+1}^{B,L} - V_{t+1}^{B,U}] + V_{t+1}^{B,U})] \tag{16}$$

g is the benefit or cost banks receive on a foreclosed capital unit. This may also be a cost. $p(\theta_u)$ is the probability with which a bank will find a new entrepreneur willing to take on the foreclosed capital unit. This probability will depend on market tightness in secondary capital markets θ_u.

3.5.2 Firms

Firms submit proposals for funding to banks. Opening a proposal and presenting it to financial intermediaries comes at a cost κ_j. j denotes here the possibility of the firm searching either in markets of fresh or markets of used capital. It is assumed that the expected benefit of receiving an old or a new capital unit is brought to the same level by market forces. This leads to the following equivalencies for allocating search between fresh and used capital markets in equation 17.

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = \frac{\kappa_u}{q_u(\theta_{u,t})} = (1-\delta_k)E_t[\mu_{t+1}V^{E,N}_{\bar{z},t+1}]) \tag{17}$$

The value a successful match to the firm is found in equation 19. The firm will produce the period output produced $A_t z K_t^{\alpha-1}$ with the collateral capital provided and pay interest rate $r_{z,t}$ to the bank for it. The future present discounted value of the collateral capital provided is $(1-\delta_k)E_t(\mu_{t+1}[V^{E,L}_{z,t+1}])$, which accounts for temporal discounting and capital depreciation. Again, for simplicity of notation define the mean expected surplus value of a weak loan match to the entrepreneur next period as $\hat{V}^{E,L}_{\hat{z},t+1} = \int_{z_{z,t+1}}^{\bar{z}} V^{E,L}_{z,t+1}\, d(z)$

$$V^{E,N}_{\bar{z},t} = A_t K_t^{\alpha-1}\bar{z} - r_{\bar{z},t} + (1-\delta_k)E_t\left[\mu_{t+1}[\pi^N V^{E,N}_{\bar{z},t+1} + (1-\pi^N)\hat{V}^{E,L}_{\hat{z},t+1}]\right] \tag{18}$$

$$V^{E,L}_{z,t} = A_t K_t^{\alpha-1} z - r_{z,t} + (1-\delta_k)E_t\left[\mu_{t+1}[(1-\pi^L)V^{E,N}_{\bar{z},t+1} + \pi^L\hat{V}^{E,L}_{\hat{z},t+1}]\right] \tag{19}$$

3.5.3 Equilibrium interest rate

The equilibrium interest rate is determined via Nash bargaining between the bank and the firm. This delivers a simple solution, though more complicated bargaining solutions may be implemented as well. Let η be the bargaining power of the bank.

$$\eta V^{E,L}_{\bar{z},t} = (1-\eta)(V^{B,L}_{\bar{z},t} - V^{B,U}_t) \tag{20}$$

$$\eta V^{E,L}_{z,t} = (1-\eta)(V^{B,L}_{z,t} - V^{B,U}_t + \tau) \tag{21}$$

The Nash bargaining solution for safe loans is in equation 20, while the solution for weak loans is in equation . For weak loans there is a possibility of separation, so not separating forms part of the surplus for banks.

$$r_{\bar{z},t} = \eta[A_t K_t^{\alpha-1}\bar{z} + \kappa_u\theta_{u,t}] + (1-\eta)g \tag{22}$$

$$r_{z,t} = \eta[A_t K_t^{\alpha-1} z + \kappa\theta_{u,t}] + (1-\eta)[g - \tau(1-\beta\pi^L(1-\delta_k))] \tag{23}$$

3.5.4 Loan creation decision

Substituting equation 22 into equation 18 and combining it with the free entry condition for entrepreneurs into capital proposals in equation 17 yields the loan creation condition in equation 24.

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = (1-\eta)(A_t K_t^{\alpha-1}\bar{z}-g) - \eta\kappa_u\theta_{u,t} + E_t\left[\pi^N \frac{\kappa_x}{q_x(\theta_{x,t+1})} + (1-\delta_k)(1-\pi^N)E_t(\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1})\right] \tag{24}$$

3.5.5 Foreclosure decision

A bank will only choose to foreclose capital when the benefit from the foreclosed capital exceeds the benefit from keeping the loan relation. The benefit from foreclosing a loan is $V_t^{B,U}$, which is the real value of the secondary specified capital in the match. This foreclosed capital can be sold in frictionless financial markets and purchased by other banks or kept by the bank itself. In either case, the bank will choose not to foreclose a loan as long as 25 holds.

$$V^{B,L}_{z,t} - V^{B,U}_t + \tau > 0 \tag{25}$$

The foreclosure condition in equation 26 is the result of combining equation 15, 16, and 23.

$$0 = [(1-\eta)(A_t K_t^{\alpha-1} z_{s,t} - g + \tau(1-\beta\pi^L(1-\delta_k))) - \eta\kappa\theta_{u,t} + E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1}\right]] \tag{26}$$

$$z_{s,t} = K_t^{1-\alpha}A_t^{-1}[g - \tau(1-\beta\pi^L(1-\delta_k))) + \frac{\eta}{1-\eta}\kappa_u\theta_{u,t} - \frac{1}{1-\eta}E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1}\right]] \tag{27}$$

This means that the productivity cutoff for weak loans will increase with the total capital stock, as loans become less productive due to declining marginal returns. The cutoff decreases with higher productivity levels, and decreases with lower levels of demand for newly built capital by entrepreneurs. Further forbearance incentives will lead to lower productivities being accepted by banks before they foreclose the entrepreneur.

3.5.6 Transition laws

The total real value of existing capital units should equal the total value of real deposits at all time.

$$D_t = X_t + K_t^N + K_t^L + K_t^U \tag{28}$$

The transition law for bank cash is found in equation 29. Cash is reduced by a small overhead that may be taken by the bank δ_x and the capital $p(\theta_{x,t-1})S_{t-1}$ lent out to firms. It is increased by deposits made by households I_{t-1}.

$$X_t = (1-\delta_x)[X_{t-1} - p(\theta_{x,t-1})S_{t-1}] + I_{t-1} \tag{29}$$

The transition law for collateral capital in safe loans K^N is given by equation 30 and states that present capital is the not depreciated $(1-\delta_k)$ part of past safely lent capital K^N_{t-1}, which has also not been foreclosed turned weak with probability π^N. Further loans that have turned from weak to strong loans again $(1-\pi^L)K^L_{t-1}$. Finally, the capital which has been successfully re-lent $p(\theta_{u,t-1})K^U_{t-1}$ plus the successfully added fresh capital built from cash $p(\theta_{x,t-1})S_{t-1}$ form part of K^N_t.

$$K^N_t = (1-\delta_k)[\pi^N K^N_{t-1} + p(\theta_{u,t-1})K^U_{t-1} + p(\theta_{x,t-1})S_{t-1} + (1-\pi^L)K^L_{t-1}] \tag{30}$$

The transition law for collateral capital in weak loans K^L is given by equation 31 and states that present capital is the not depreciated $(1-\delta_k)$ part of past lent capital K^L_{t-1}, which has also not been foreclosed $(1-H(z_{s,t}))$ and not turned safe again with $(1-\pi^L)$. To this capital, the loans which have turned from safe to weak in the last period are added $(1-\pi^N)K^N_{t-1}$.

$$K^L_t = (1-\delta_k)(1-H(z_{s,t}))[\pi^L K^L_{t-1} + (1-\pi^N)K^N_{t-1}] \tag{31}$$

Here $H(z)$ is the cumulative distribution function of the stochastic variable z.Equally the transition law for K^U in equation 32 is the not depreciated part $(1-\delta_k)$ of past unmatched capital plus foreclosed matched capital $H(z_{s,t})[K^L_{t-1} + p(\theta_{u,t-1})K^U_{t-1}]$ minus successfully re-matched foreclosed capital $p(\theta_{u,t-1})K^U_{t-1}$.

$$K^U_t = (1-\delta_k)[H(z_{s,t})(\pi^L K^L_{t-1} + (1-\pi^N)K^N_{t-1}) + (1-p(\theta_{u,t-1}))K^U_{t-1}] \tag{32}$$

Overall the transition law for deposits is then the addition of cash X, lent capital K^L, and foreclosed capital K^U.

$$X_t+K^N_t+K^L_t+K^U_t = (1-\delta_x)(X_{t-1}-p(\theta_{x,t-1})S_{t-1})+(1-\delta_k)[K^N_{t-1}+K^L_{t-1}+K^U_{t-1}+p(\theta_{x,t-1})S_{t-1}]+I_{t-1} \tag{33}$$

This transition allows for specifying the deposit depreciation $\delta_{d,t}$ in every period in equation 34.

$$\delta_{d,t} = \delta_k \frac{K^N_{t-1} + K^L_{t-1} + K^U_{t-1} + p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} + \delta_x \frac{X_{t-1} - p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} \tag{34}$$

This then leads to the aggregate deposit law of motion originally specified for the household.

$$D_t = (1-\delta_{d,t})D_{t-1} + I_{t-1} \tag{35}$$

3.5.7 Interest rates

As expected, the interest rate paid to the household ρ depends on the state of the lent capital in safe $r_{\bar{z},t}$ and weak loans $r_{\hat{z},t}$ and the proportion of lent $\frac{K_t^L}{D_t}$ and foreclosed capital $\frac{K_t^U}{D_t}$ on the active side of the bank balance sheet versus deposits, the passive side.

$$\rho_t = \frac{r_{\bar{z},t}K_t^N + r_{\hat{z},t}K_t^L + gK_t^U}{D_t} \tag{36}$$

3.6 NPL cutoff

It is necessary to define the NPL share endogenously. This share will be loans that are no longer profitable for the bank. In this case, NPLs are defined as the share of total weak loans K^L where the interest rate paid to banks doesn't cover the risk-less equilibrium rate paid to consumers.

$$r_{z,t} \leq \rho \tag{37}$$

We can thus find the real interest rate paid by firms to banks below which the loan turns non-performing by finding $\hat{z}$ as a function of equilibrium ρ.

$$r_{\hat{z},t} = \rho \tag{38}$$

3.7 Intra-period heterogeneity via the productivity distribution of loans

Each period loan productivity is drawn from a negative exponential distribution. The choice of a exponential distribution versus the log-normal distribution in the partial equilibrium model does not impact the model properties. The advantage of the exponential distribution over the log-normal is that it facilitates finding closed expressions for the equations above due to the closed-form expressions available for conditional expectations for this type of distribution. The idea of the negative exponential distribution is to assume that most loans are productive, but there is a subset in the tails that become costly to the banks due to being very unproductive. Thus there is a maximal value of z, which is $\bar{z}$. All safe loans are at productivity level $\bar{z}$. Weak loans may be at a productivity $z \in [\bar{z}, -\infty]$. This is the result of a realisation ζ of the exponential distribution that is subtracted from $\bar{z}$. The exponential distribution for ζ with the calibrated values is pictured in figure 16.

$$z = \bar{z} - \zeta \tag{39}$$

I general, ζ is exponentially distributed with probability density function $\gamma \exp(-\gamma\zeta)$.

The properties of the exponential distribution allow for a closed form computation of the mean value of not foreclosed weak loans $V_{z,t}^{\hat{E},L}$. Given a cutoff z_s, the cutoff of ζ will be $\zeta_s + \bar{z}$. This leads to a mean value of the exponential distribution, with γ as the distributional parameter of $-\zeta_s$, or transformed to z to a conditional expected value $z_s \in (-\infty, \bar{z}]$. We can transform a value of ζ to idiosyncratic productivity with equation

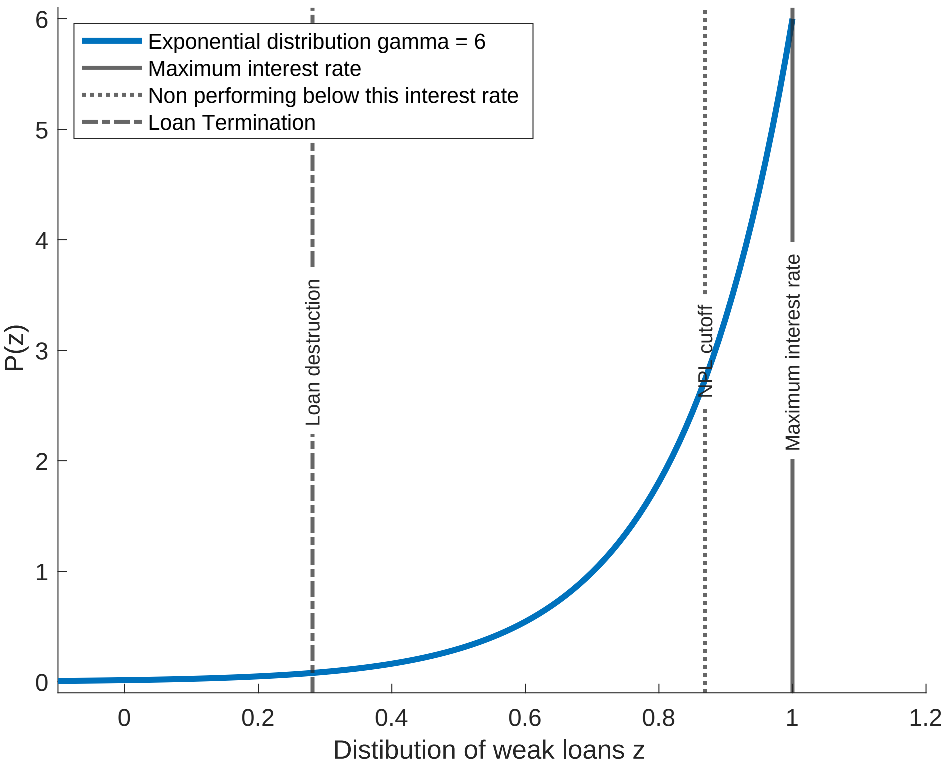

Figure 16: Pareto shape $\gamma = 2.4$

39. The probability of weak loans being foreclosed is then $H(z_s) = exp(-\gamma(\bar{z} - z_s))$. The mean productivity of a weak loan is then given by equation 40.

$$\hat{z} = \frac{\bar{z} - \frac{1-\gamma \exp(-\gamma[\bar{z}-z_{s,t}])(\bar{z}-z_{s,t})}{\gamma}}{1 - \exp(-\gamma[\bar{z} - z_{s,t}])} \quad (40)$$

3.8 Stochastic exogenous processes

There are potentially three stochastic processes in this economy. Equation 41 shows the auto-regressive process capturing typical deviations in aggregate productivity. Equation 42 consists of an auto-regressive process capturing shocks to the current forbearance incentive with $t\hat{a}u = \tau_t - \pi^L(1-\delta_k))E_t(\mu_{t+1}\tau_{t+1})$. $\sigma a, \tau$, which is suspected to be less or equal to 0 captures forbearance incentives possibly rising in line with an aggregate shock. Similarly, equation 43 captures possible changes to the efficiency of used capital markets. It is probable that $\sigma a, \mu \geq 0$, especially when shocks to aggregate productivity affect the functioning of capital transactions due to other turmoil in the economy. Further, the Cobb-Douglas matching function may not capture all changes in frictions in capital markets similar as in employment markets, where deep recessions have been shown to be accompanied by a significant decline in matching efficiency (Sedláček, 2014).

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \quad (41)$$

$$\hat{\tau}_t = (1-\rho_\tau)\tau + \rho_\tau \hat{\tau}_{t-1} + \sigma a, \tau \epsilon_t + \epsilon_\tau] \quad (42)$$

$$\mu_{u,t} = (1-\rho_\mu)\mu_u + \rho_\mu \mu_{u,t-1} + \sigma a, \mu \epsilon_t + \epsilon_\mu] \quad (43)$$

3.9 Equilibrium

The competitive equilibrium of the economy can be summarised by the following equations in the highlighted boxes 1, 2, 3, and 4. The matching function is specified in a Cobb Douglas form with the number of matches of new and foreclosed capital given as $M_x = \mu_x \psi X \theta_x^{1-\xi}$ and $M_u = \mu_u \psi K^U \theta_u^{1-\xi}$. $t\hat{a}u$ is defined as $t\hat{a}u = \tau(1-\beta\pi^L(1-\delta_k))$ capturing possibly dynamically changing forbearance incentives in the banking sector. The idea is that these incentives may increase when the economy is experiencing a negative shock. Similarly, the efficiency of used capital markets may decrease in economic crisis as market makers exit.

Dynamic competitive equilibrium: Exogenous processes

Stochastic aggregate process:

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t]$$

Stochastic forbearance cost process:

$$\hat{\tau}_t = (1-\rho_\tau)\tau + \rho_\tau \hat{\tau}_{t-1} + \sigma a, \tau \epsilon_t + \epsilon_\tau]$$

Stochastic used capital market efficiency process:

$$\mu_{u,t} = (1-\rho_\mu)\mu_u + \rho_\mu \mu_{u,t-1} + \sigma a, \mu \epsilon_t + \epsilon_\mu]$$

Highlighted Box 1: Competitive equilibrium: Exogenous processes

3.10 Comparative statics

The baseline calibration of the model parameters is set out in table 3. Variations of this baseline calibration are shown in this comparative statics section. The choice of the matching functions for fresh unspecified and used foreclosed capital is made such that fresh capital markets are in the baseline four times as efficient as specified capital markets with $\mu_u = 0.05$, while $m_x = 0.2$. The bargaining power of banks η is set to 0.3, meaning that the matching parameter ξ is set to 0.7 to fulfill the (Hosios, 1990) condition. κ_x and κ_u are both set to 0.01 keeping the cost for new proposals for capital from firms to banks at 1%. g is chosen at 0.07 for the real cost of foreclosed capital to be close to 0. The discount factor β is set to 0.99, while physical capital depreciation δ_k is set to 0.06, as is conventional. The share of capital in the economy α is set to 0.35 as is conventional. δ_x is set to 0 which can be considered a real cost of keeping the unmatched capital charged by the bank, but could also be set to $\delta_x > 0$ to obtain similar results. The maximum idiosyncratic productivity of a capital in a loan $\bar{z}$ is normalised to $1/$. The parameter of the exponential distribution is

Dynamic competitive equilibrium: Household and aggregate

Euler equation:

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta_{d,t+1} + \rho_{t+1})]$$

Resource constraint:

$$C_t + I_t = A_t K_t^{\alpha-1}(\bar{z}K_t^N + \hat{z}K_t^L) + gK_t^U - \kappa_x \theta_{x,t}\psi X_t - \kappa_u \theta_u K_{u,t}$$

Deposit depreciation:

$$\delta_{d,t} = \delta_k \frac{K_{t-1}^N + K_{t-1}^L + K_{t-1}^U + p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} + \delta_x \frac{X_{t-1} - p(\theta_{x,t-1})S_{t-1}}{D_{t-1}}$$

Interest rate paid to households:

$$\rho_t = \frac{r_{\bar{z},t}K_t^N + r_{\hat{z},t}K_t^L + gK_t^U}{D_t}$$

Aggregate capital and deposits:

$$K_t = K_t^N + K_t^L + K_t^U, \quad D_t = X_t + K_t^N + K_t^L + K_t^U$$

Highlighted Box 2: Competitive equilibrium: Household and aggregate constraints

then chosen to calibrate endogenous loan foreclosure. A higher value will mean a steeper distribution with more foreclosed loans in every period. In this case, a value of 6 means 3% of loans are foreclosed in every period. π^N is set top 0.8 meaning safe loans have a 20% chance of turning weak and, weak loans have a 20% chance of turning safe again due to $\pi^L = 0.8$. Finally, for foreclosure incentives $\tau = 0$ in the baseline calibration.

The persistence of the aggregate process ρ_a is set to 0.9. The other exogenous processes are assumed to be of similar persistence. In the baseline calibration $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ are set to 0 meaning that a shock to the aggregate process is not accompanied by increases in foreclosure costs or decreases in the efficiency of used capital markets. These parameters are then varied in the dynamic simulation to judge which one is more likely to capture the data.

The steady-state of the model is found with a non-linear solver solving the equations in Appendix A. Figures 17 - 20 shows the comparative statics when varying the efficiency of a match in used capital markets and forbearance incentives similar to the partial equilibrium section. Clearly, as the match efficiency in used capital markets increases or forbearance incentives decrease the sNPL decreases as figure 17 shows.

Dynamic competitive equilibrium: Credit markets

Loan creation condition:

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = E_t((1-\delta_k)\mu_{t+1}\left[(1-\eta)(A_{t+1}K_{t+1}^{\alpha-1}\bar{z}-g) - \eta\kappa_u\theta_{u,t+1} + \pi^N\frac{\kappa_x}{q_x(\theta_{x,t+1})} + (1-\pi^N)(1-\delta_k)\mu_{t+2}\hat{V}^{E,L}_{\hat{z},t+2}\right])$$

Loan foreclosure decision:

$$z_{s,t} = K_t^{1-\alpha}A_t^{-1}[g - \tau(1-\beta\pi^L(1-\delta_k)) + \frac{\eta}{1-\eta}\kappa_u\theta_{u,t} - \frac{1}{1-\eta}E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1}\right]]$$

Mean productivity of a weak loan and foreclosure share of weak loans:

$$\hat{z} = \frac{\bar{z}\gamma - 1 + \exp(-\gamma[\bar{z}-z_{s,t}])(\gamma[\bar{z}-z_{s,t}]+1)}{\gamma(1-\exp(-\gamma[\bar{z}-z_{s,t}]))}, \quad H(z_{s,t}) = \exp(-\gamma[\bar{z}-z_{s,t}])$$

Mean value of a weak loan:

$$\frac{\hat{V}^{E,L}_{\hat{z},t}}{(1-\exp(-\gamma(\bar{z}-z_{s,t})))} = [(1-\eta)(A_tK_t^{\alpha-1}\hat{z} - g + \tau(1-\beta\pi^L(1-\delta_k))) - \eta\kappa\theta_{u,t} + E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1}\right]]$$

Match probabilities:

$$p(\theta_{u,t}) = \mu_u\theta_u^{1-\xi}, \quad p(\theta_{x,t}) = \mu_x\theta_x^{1-\xi}, \quad q(\theta_{u,t}) = \mu_u\theta_u^{-\xi}, \quad q(\theta_{x,t}) = \mu_x\theta_x^{-\xi}$$

Interest rates to banks:

$$r_{\bar{z},t} = \eta[A_tK_t^{\alpha-1}\bar{z} + \kappa_u\theta_{u,t}] + (1-\eta)g, \quad r_{z,t} = \eta[A_tK_t^{\alpha-1}z + \kappa\theta_{u,t}] + (1-\eta)[g - \tau(1-\beta\pi^L(1-\delta_k))]$$

Highlighted Box 3: Competitive equilibrium: Credit market decision-making

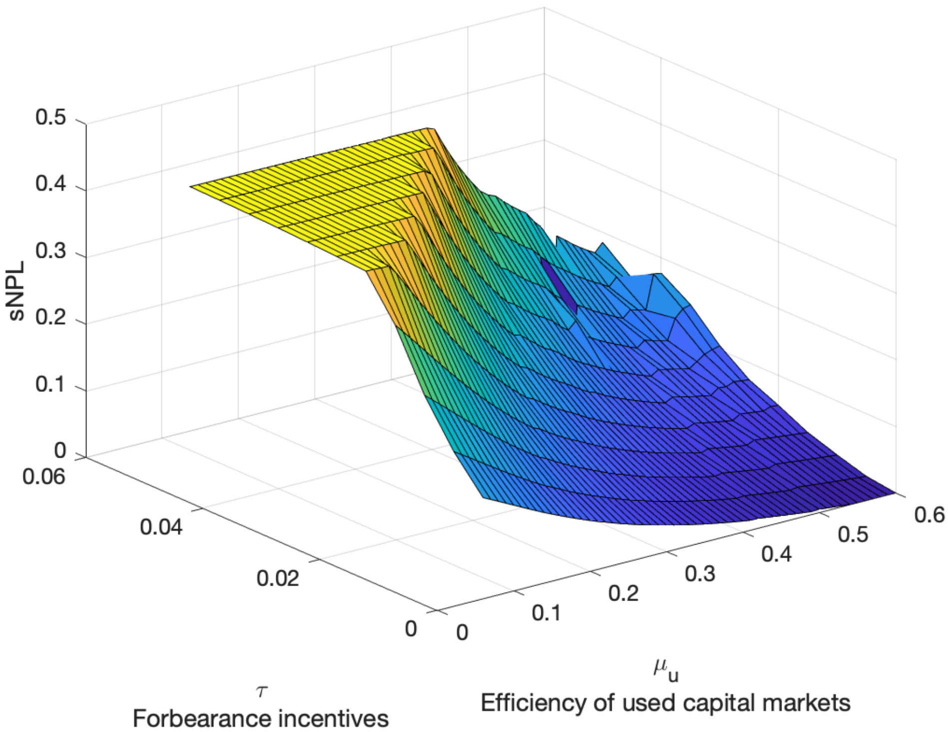

Figure 17: Comparative effects of τ and μ_u on sNPL.

Dynamic competitive equilibrium: State transitions

Law of Motion for cash:

$$X_t = (1-\delta_x)[X_{t-1} - p(\theta_{x,t-1})\psi X_{t-1}] + I_{t-1}$$

Law of Motion for capital in safe loans:

$$K_t^N = (1-\delta_k)[\pi^N K_{t-1}^N + p(\theta_{u,t-1})K_{t-1}^U + p(\theta_{x,t-1})S_{t-1} + (1-\pi^L)K_{t-1}^L]$$

Law of Motion for capital in weak loans:

$$K_t^L = (1-\delta_k)(1-H(z_{s,t}))[\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N]$$

Law of Motion for foreclosed capital:

$$K_t^U = (1-\delta_k)[H(z_{s,t})(\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N) + (1-p(\theta_{u,t-1}))K_{t-1}^U]$$

Highlighted Box 4: Competitive equilibrium: Laws of Motion

This goes hand in hand with a reduction in the aggregate return on capital as shown by figure 18. However, the amount of foreclosed capital truly decreases only with an increase in forbearance incentives as figure 19 shows. This means there is less capital on offer, which given similar demand should drive up the price of capital. In steady-state, however, agents will account for the additional cost of foreclosure making the effect of forbearance incentives for loans on capital prices neutral. Meanwhile, higher demand in more efficient markets and equal supply will mean that capital prices increase as the efficiency of used capital markets increases as shown in figure 20. This shows that the dynamics of capital prices are needed to separate forbearance incentives from the efficiency of used capital in their impact on the sNPL.

Parameter	Value	Description
β	0.99	Discount rate
ψ_x	0.05	Share of cash that can be lent
δ_k	0.06	Capital depreciation
δ_x	0.02	Real cash depreciation
κ_x	0.01	Proposal cost for new capital
κ_u	0.01	Proposal cost for used capital
m_x	0.2	Match productivity new capital
m_u	0.05	Match productivity for used capital
$\bar{z}$	1	Maximum loan productivity
γ	6	Exponential distribution parameter
α	0.35	Aggregate capital exponent
ξ	0.7	Parameter on matching function
η	0.3	Bargaining power of banks
τ	0	Possible forbearance incentives
g	0.07	Production value of foreclosed capital
π^N	0.8	Probability of a safe loan staying safe in the next period
π^L	0.8	Probability of a weak loan staying weak in the next period
ρ_a	0.9	Persistence of the aggregate process
ρ_τ	0.9	Persistence of the forbearance process
ρ_μ	0.9	Persistence of the process for used capital market efficiency
$\sigma_{a,\tau}$	0	Correlation of forbearance incentives with the aggregate process
$\sigma_{a,\mu}$	0	Correlation of used capital market efficiency with the aggregate process

Table 3: Baseline calibration

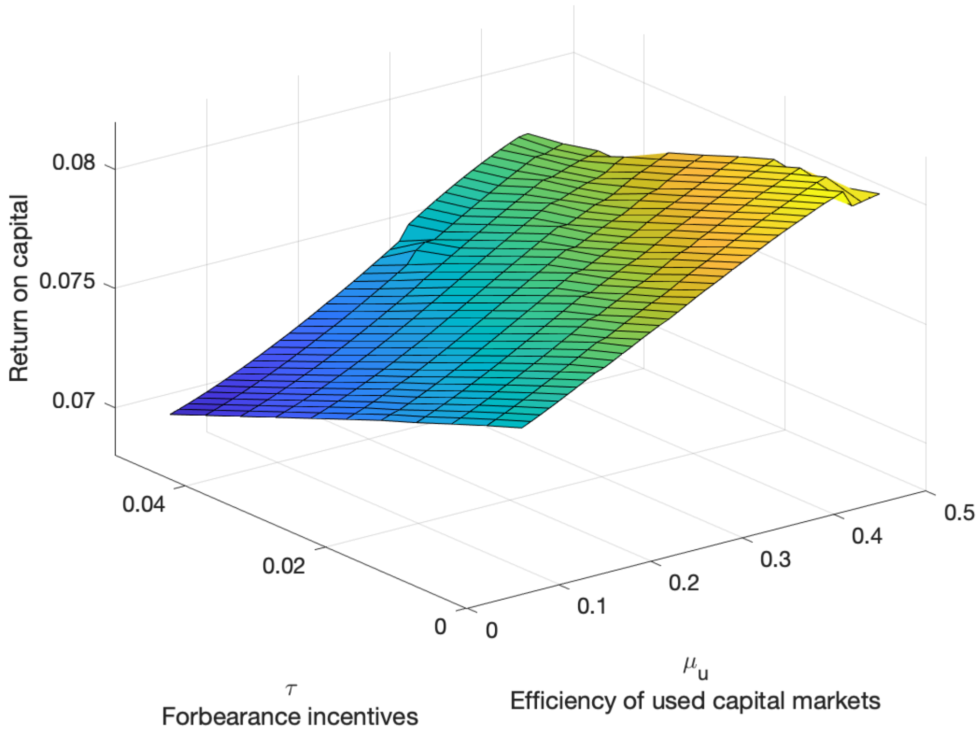

Figure 18: Comparative effects of τ and μ_u on the return on capital

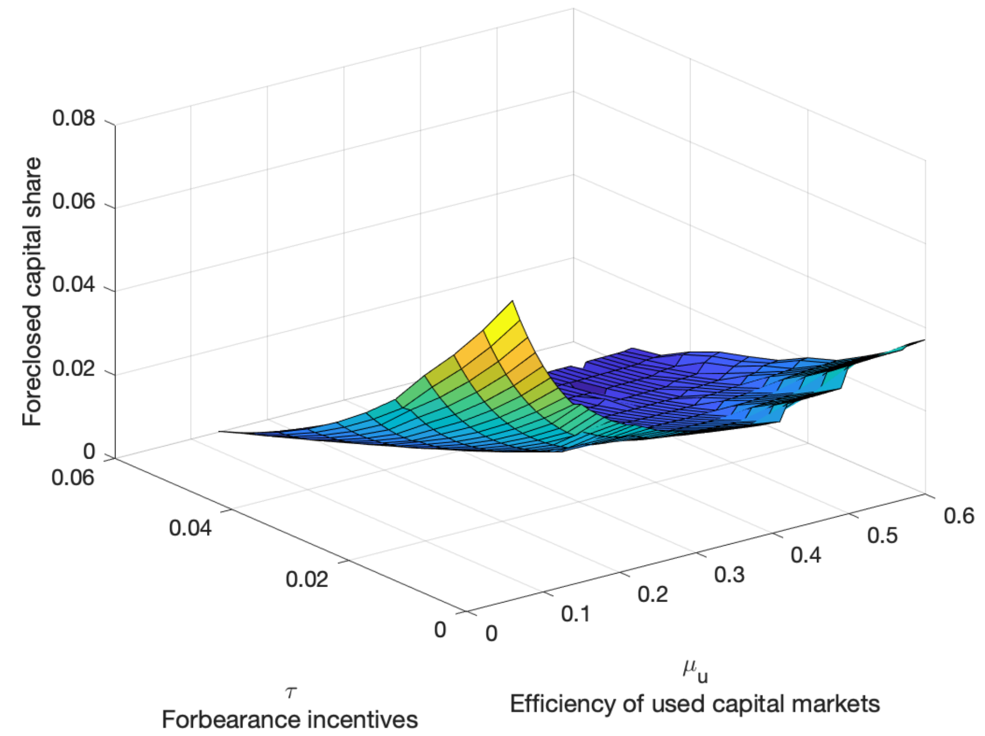

Figure 19: Comparative effects of τ and μ_u on the state of foreclosed capital.

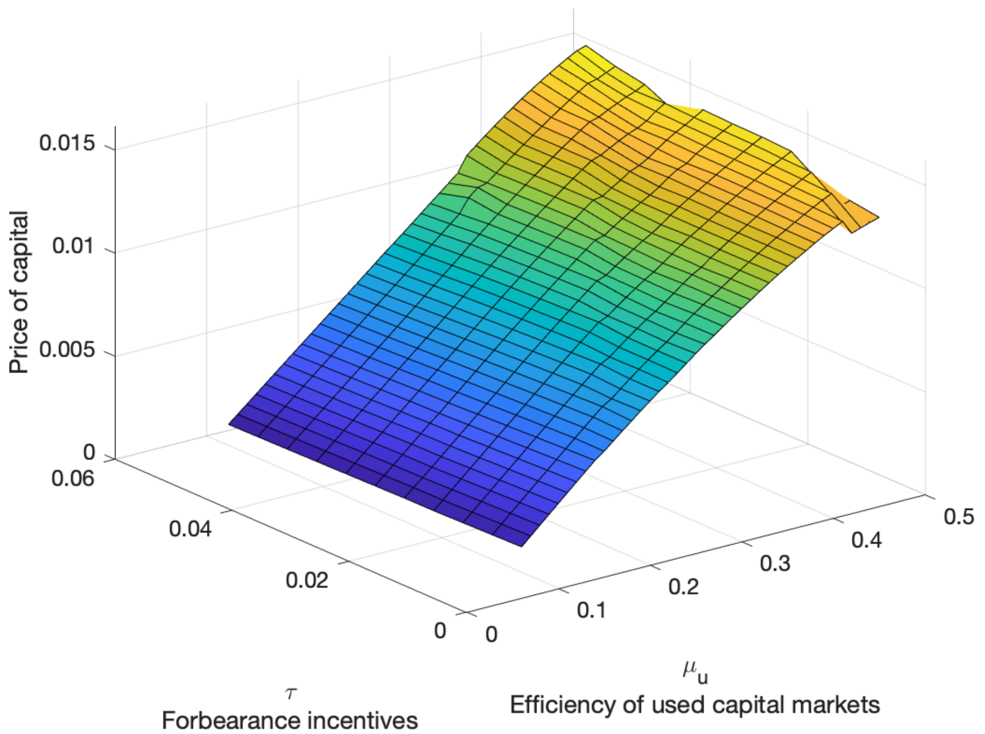

Figure 20: Comparative effects of τ and μ_u on capital prices.

3.11 Dynamics

The model is simulated from the steady-state via perturbation. In a next step the parameters of the model will be estimated with this simulation via impulse response function matching, however, the simulated impulse responses here show the directions an increase of forbearance incentives and a reduction in used capital market efficiency have on the sNPL. The simulated impulse response functions in figure 21 show that the model can replicate the general correlations of non-performing loans and the business cycle. A negative TFP shock will drive down the real return on capital. At the same time, the sNPL will increase, while investment in fresh unspecified capital and consumption fall. The mean value of a weak non-foreclosed loan will decline, meaning that banks accept not to foreclose more loans with a lower value even though the idiosyncratic productivity cutoff in this calibration becomes more restrictive as shown in figure 23. In the current calibration, the sNPL is aimed at a one-to-one response to the aggregate process, but this may change as the parameters are varied.

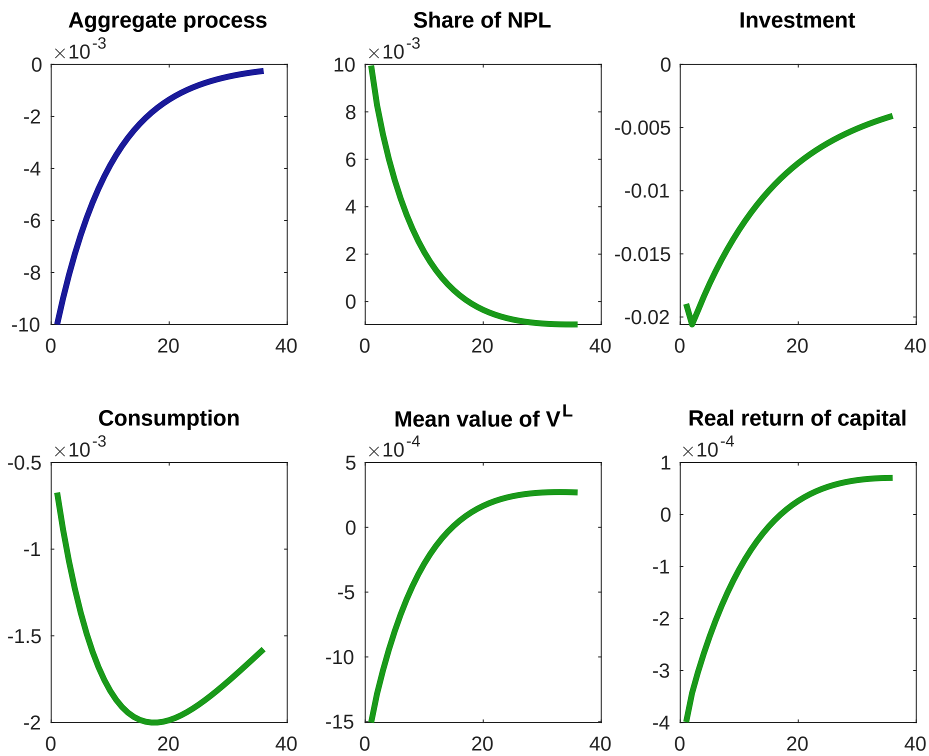

Figure 21: Impulse response to a 1% aggregate shock

Figure compmutau shows that the sNPL responds similarly if the aggregate shock has no effect on $\mu_{u,t}$ and $\hat{\tau}_t$ and only the steady-state value is varied. In both cases, higher forbearance incentives or lower used capital market efficiency will lead to a higher increase of the sNPL in response to a negative aggregate shock. Nevertheless, banks get more restrictive as shown in figure 23 in the type of weak loan they accept, but this doesn't make up for the amount of loans that fall below the performing level as the productivity of all loans shifts down.

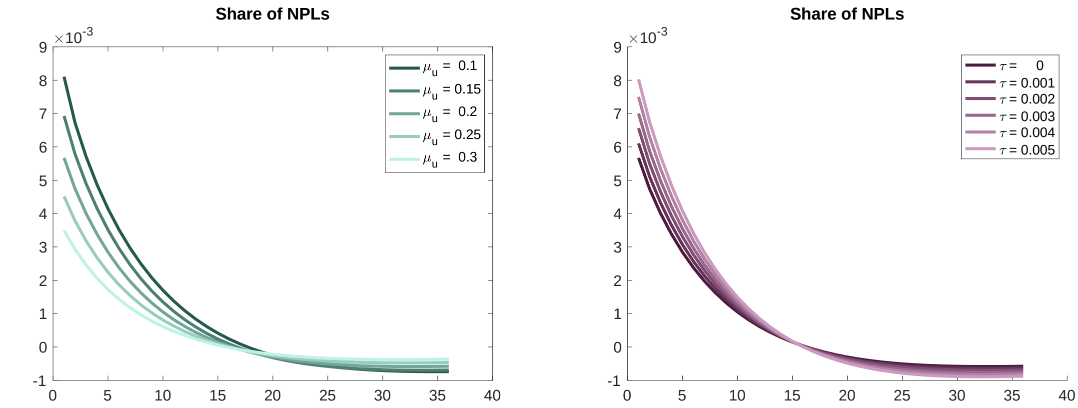

Figure 22: Dynamic responses to a negative 1% TFP shock (top row) and varying measures of μ_u (left) and $\hat{\tau}$ right.

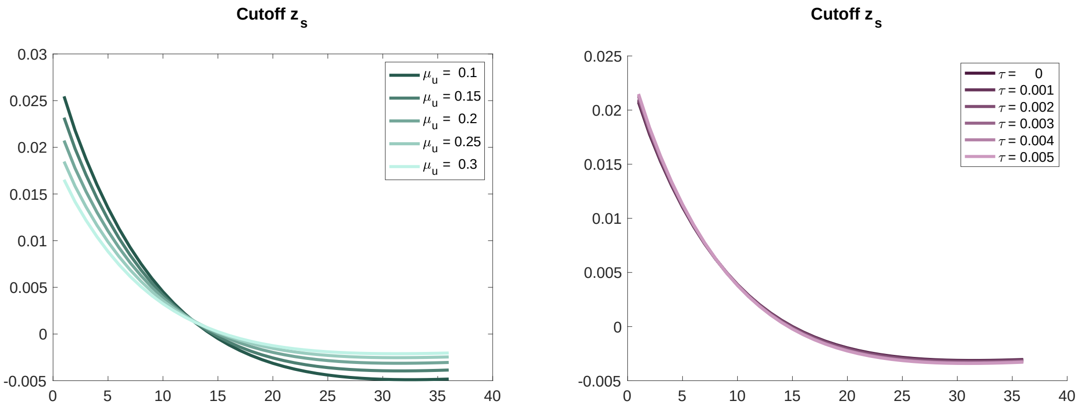

Figure 23: Dynamic responses to a negative 1% TFP shock (top row) and varying measures of μ_u (left) and $\hat{\tau}$ right.

The purpose of figure 24 to 26 is to show that when varying towards $\sigma_{a,\tau} < 0$ or $\sigma_{a,\mu_u} > 0$ it is clear that a negative shock is likely to be paired with a shock to the efficiency of used capital markets if the price of capital is supposed to fall as observed in the data. The model shows in figure 24 that even small increases in forbearance incentives or changes to the efficiency of used capital markets can have very large effects on the response of the sNPL. However, figure 25 shows that even a relatively small increase in forbearance incentives may drive the price of capital up, as less used capital is supplied to the market. Figure 26 shows this decrease in the supply of used capital to the market is not there when μ_u is decreasing, but only when $\hat{\tau}$ is increasing as foreclosure is stopped via incentives not to foreclose and not via congestion in capital markets driving down the value of foreclosed capital. This shows that a low efficiency in used capital markets and a decline in recessions is a driver the sNPL more congruent with the observed response of the sNPL and capital prices.

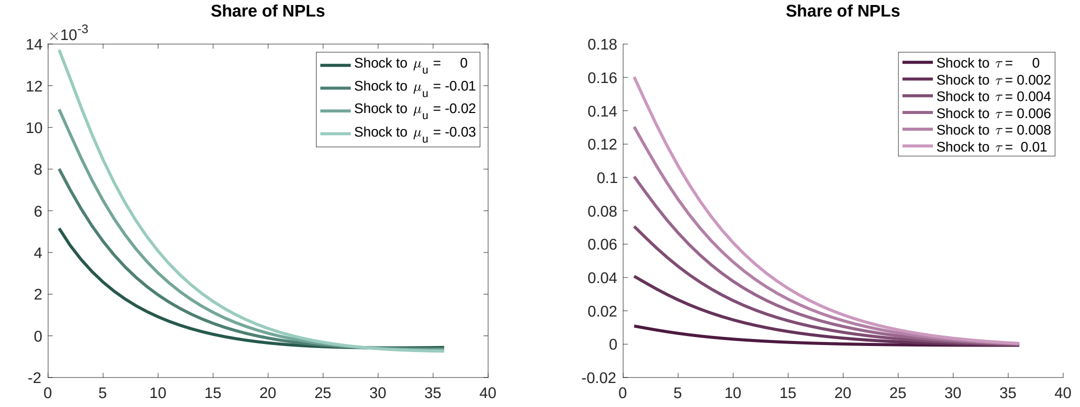

Figure 24: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the sNPL.

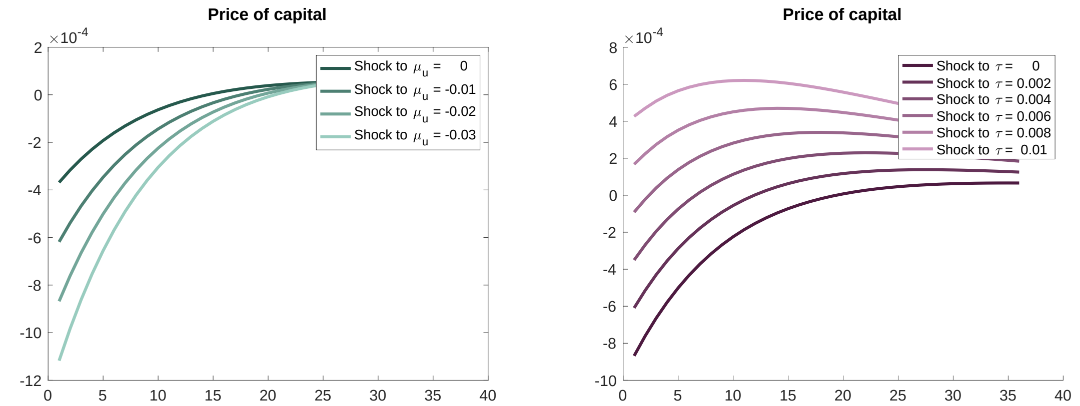

Figure 25: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the price of capital.

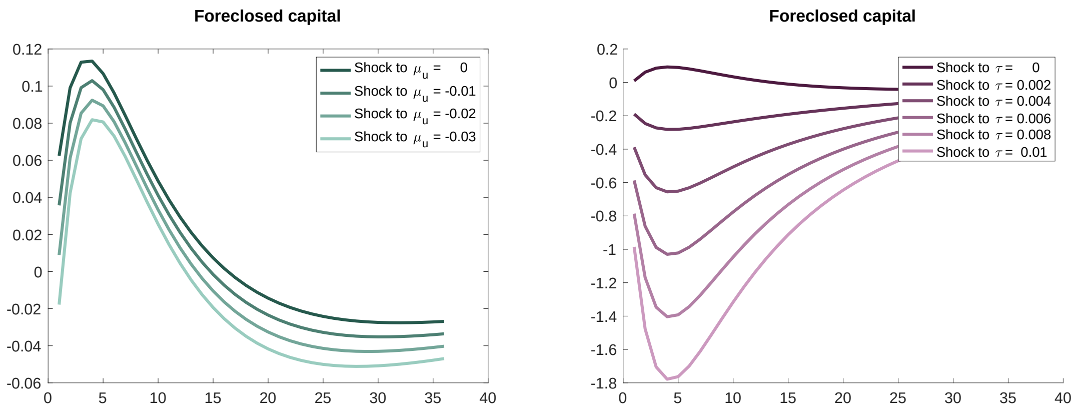

Figure 26: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the state of foreclosed capital seeking re-matching.

To summarise, the model predicts that increasing sNPL go hand in hand with reduced capital market reallocation. This is a testable prediction. Looking at cross-country changes in countries' capital reallocation activity in 9 it seems that the prediction holds some value.

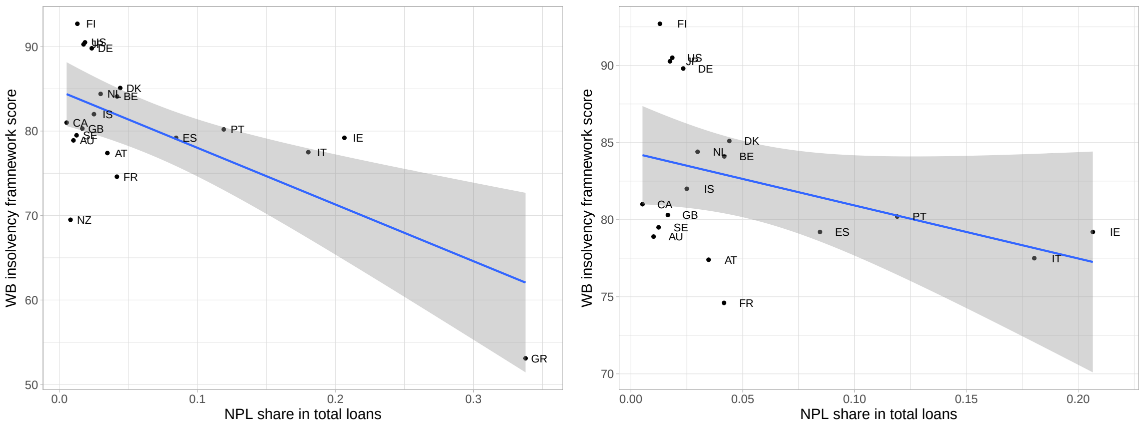

Figure 27: NPL shares and latest insolvency framework scores

The model concludes from the response of the price of capital that frictions in used capital markets are a more likely driver than forbearance incentives for banks in the sNPL observed in recent years. A further figure providing credit to the theoretical result shows the world bank insolvency scores from the ease of doing business indicators plotted against non-performing loans share for the sample of countries. We can see the difficulty in realising an insolvency as due to the low prices realised for foreclosed capital and the high matching frictions. The correlation is negative meaning that a higher ease of winding up a company, which delivers a higher outside value to banks will lead to lower NPL shares. Both figures are based on the same data, but the second figure removes outliers by only plotting countries with an insolvency score larger than 70.

4 Conclusion

This paper presents a structural model where non-performing loans matter for investment and are the flip-side of capital misallocation. The model can capture the empirical relations shown in section 2 well. It shows that a random search model for re-matching capital markets can provide insights on NPL drivers, i.e. used capital market efficiency. It can combine recent advancements in the capital misallocation literature with empirical studies regarding the sNPLs. The theoretical relations in the model imply that resolving non-performing loans is a matter of the efficiency of capital reallocation. This is defined by an economy's ability to bring used specified capital to productive uses, hence the efficiency of used capital markets. The model is simple enough to estimate parameters from the data of various countries and to evaluate policy changes, which will be done in a next step.

The model is kept simple but is set up in such a way that it allows for tractable persistent heterogeneity extensions via a dynamic directed search block recursive equilibrium. The simple model can be expanded to allow for dealing with heterogeneity via a block recursive solution of the credit market as developed in (Menzio and Shi, 2010a). A block recursive solution of the credit market has been developed before in (Boualam, 2018) in a' model focusing on borrower lender ”relationship capital”. A solution is possible in this general equilibrium setup because the aggregate return of a unit of capital remains uninfluenced by the distribution of states of capital in employed or unemployed capital linked to non-performing loans. With the block recursion persistent heterogeneous loan relationships, different types of firms and banks, refinancing, endogenous bankruptcies, and different policies to encourage solutions to non-performing loans may be handled with relative ease given the complexities involved within this model setup.

References

Accornero, M., P. Alessandri, L. Carpinelli, and A. M. Sorrentino (2017): "Non-performing loans and the supply of bank credit: evidence from Italy," *Bank of Italy Occasional Paper*.

Acharya, V. V., M. Crosignani, T. Eisert, and C. Eufinger (2020): "Zombie credit and (dis-) inflation: evidence from Europe," Tech. rep., National Bureau of Economic Research.

Ari, A., S. Chen, and L. Ratnovski (2020): "The dynamics of non-performing loans during banking crises: a new database," .

Balgova, M., M. Nies, and A. Plekhanov (2016): "The economic impact of reducing non-performing loans," .

Beaubrun-Diant, K. E. and F. Tripier (2015): "Search frictions, credit market liquidity and net interest margin cyclicality," *Economica*, 82, 79–102.

Beck, R., P. Jakubik, and A. Piloiu (2013): "Non-performing loans: What matters in addition to the economic cycle?" .

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of macroeconomics*, 1, 1341–1393.

Bertola, G. and R. J. Caballero (1994): "Irreversibility and aggregate investment," *The Review of Economic Studies*, 61, 223–246.

Blanchard, O. J. and D. Quah (1988): "The dynamic effects of aggregate demand and supply disturbances," .

Boualam, Y. (2018): "Credit markets and relationship capital," Tech. rep., Working paper, University of North Carolina.

Bredl, S. (2018): "The role of non-performing loans for bank lending rates," .

Caballero, R. J., T. Hoshi, and A. K. Kashyap (2008): "Zombie lending and depressed restructuring in Japan," *American economic review*, 98, 1943–77.

Cao, M. and S. Shi (2017): "Endogenously procyclical liquidity, capital reallocation, and q," *Capital Reallocation, and q (July 2017)*.

Cui, W. and S. Radde (2020): "Search-based endogenous asset liquidity and the macroeconomy," *Journal of the European Economic Association*, 18, 2221–2269.

Eisfeldt, A. L. and A. A. Rampini (2006): "Capital reallocation and liquidity," *Journal of monetary Economics*, 53, 369–399.

Eisfeldt, A. L. and Y. Shi (2018): "Capital reallocation," *Annual Review of Financial Economics*.

Gertler, M. and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of monetary Economics*, 58, 17–34.

Gopinath, G., Ş. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017): "Capital allocation and productivity in South Europe," *The Quarterly Journal of Economics*, 132, 1915–1967.

Hosios, A. J. (1990): "On the efficiency of matching and related models of search and unemployment," *The Review of Economic Studies*, 57, 279–298.

JASSAUD, N. AND M. K. KANG (2015): *A strategy for developing a market for nonperforming loans in Italy*, 15-24, International Monetary Fund.

KASINGER, J., J. P. KRAHNEN, S. ONGENA, L. PELIZZON, M. SCHMELING, AND M. WAHRENBURG (2021): "Non-performing loans-new risks and policies? NPL resolution after COVID-19: Main differences to previous crises," Tech. rep., SAFE White Paper.

KERMANI, A. AND Y. MA (2020): "Asset specificity of non-financial firms," Tech. rep., National Bureau of Economic Research.

KLEIN, N. (2013): *Non-performing loans in CESEE: Determinants and impact on macroeconomic performance*, 13-72, International Monetary Fund.

LANTERI, A. (2018): "The market for used capital: Endogenous irreversibility and reallocation over the business cycle," *American Economic Review*, 108, 2383–2419.

LOUZIS, D. P., A. T. VOULDIS, AND V. L. METAXAS (2012): "Macroeconomic and bank-specific determinants of non-performing loans in Greece: A comparative study of mortgage, business and consumer loan portfolios," *Journal of Banking & Finance*, 36, 1012–1027.

MENZIO, G. AND S. SHI (2010a): "Block recursive equilibria for stochastic models of search on the job," *Journal of Economic Theory*, 145, 1453–1494.

——— (2010b): "Directed search on the job, heterogeneity, and aggregate fluctuations," *American Economic Review*, 100, 327–32.

MORTENSEN, D. T. AND C. A. PISSARIDES (1994): "Job creation and job destruction in the theory of unemployment," *The review of economic studies*, 61, 397–415.

OTTONELLO, P. (2017): "Capital unemployment," *Work. Pap., Dep. Econ., Columbia Univ., New York Google Scholar Article Locations: Article Location A*.

SEDLÁČEK, P. (2014): "Match efficiency and firms' hiring standards," *Journal of Monetary Economics*, 62, 123–133.

Appendix A

Exogenous foreclose

The steady-state can be solved around the steady-state of the exogenous processes where $a = 1$ and $\lambda_t = \lambda$.

$$\delta K = I \tag{44}$$

$$K_E = \lambda^{-1}[\frac{\delta}{1-\delta} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda)]K_U \tag{45}$$

Define $\psi = \lambda^{-1}[\frac{\delta}{1-\delta} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda)]$ so $K_E = \psi K_U$

$$\Pi = K_E[(1-\eta)[K_E + K_U]^{\alpha-1} - \eta p(1-\delta)(1-\lambda)\beta V_E)] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}} K_U \tag{46}$$

$$\Pi = K_E[(1-\eta)K_E^{\alpha-1}[1+\psi^{-1}]^{\alpha-1} - \eta p(1-\delta)(1-\lambda)\beta V_E)] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}\psi^{-1}K_E \tag{47}$$

$$C = \Pi + (\rho - \delta)D = \Pi + (\beta^{-1} - 1)D \tag{48}$$

$$\beta^{-1} + \delta - 1 = \rho \tag{49}$$

$$r = (1+\psi^{-1})[\beta^{-1} + \delta - 1] \tag{50}$$

$$V_E = \frac{(1-\eta)[1+\psi^{-1}]^{\alpha-1}}{1-\beta(1-\delta)(1-\lambda)(1-\eta p)}K_E^{\alpha-1} \tag{51}$$

$$C = K_E^{\alpha}[1+\psi^{-1}]^{\alpha-1} - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}\psi^{-1}K_E - \delta K_E(1+\psi^{-1}) \tag{52}$$

$$r = \eta[[1+\psi^{-1}]^{\alpha-1}K_E^{\alpha-1} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}\beta\delta(1-\lambda)V_E] \tag{53}$$

$$\beta^{-1} + \delta - 1 = r\frac{1}{1+\psi^{-1}} \tag{54}$$

Use r to solve for K_E. With K_E solve for the other variables.

$$K_E^{1-\alpha} = \frac{\eta[1+\psi^{-1}]^{\alpha-1}}{r}[1 + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}\beta(1-\delta)(1-\lambda)\frac{1-\eta}{1-\beta(1-\delta)(1-\lambda)(1-\eta p)}] \tag{55}$$

Endogenous foreclose

Steady states are solved around the aggregate process $A = 1$

$$1 = \beta[1 - \delta_d + \rho] \tag{56}$$

Creation condition

$$\frac{\kappa_x}{q_x(\theta_x)}(\frac{1}{\beta(1-\delta_k)} - \pi_N) = (1-\eta)[\bar{z}K^{\alpha-1} - g] - \eta\kappa\theta_u + \beta(1-\delta_k)(1-\pi^N)\hat{V}^E \tag{57}$$

$$\hat{V}^E = \frac{1 - H(z_s)}{1 - \beta(1-\delta_k)(1-H(z_s))\pi^L(1-\frac{\hat{z}}{z_s})}([(1-pa.eta)\hat{z}K^{\alpha-1} - g + \hat{\tau}] - \eta\kappa\theta_u + (1-\pi^L)\frac{\kappa_x}{q_x(\theta_x)}) \tag{58}$$

$$[\delta_x + p(\theta_x)\psi_x(1-\delta_x)]X = I \tag{59}$$

Interest to banks

$$r_{\bar{z}} = \eta[\bar{z}K_t^{\alpha-1} + \kappa\theta_u] + (1-\eta)g \tag{60}$$

$$r_{\hat{z}} = \eta[\hat{z}K_t^{\alpha-1} + \kappa\theta_u] + (1-\eta)(g - \hat{\tau}) \tag{61}$$

State steady states

K^L:

$$K^L = \frac{(1-\pi^N)(1-\delta_k).(1-H(z_s))}{1-(1-\delta_k)\pi^L(1-H(z_s))}X = \nu_1 X \tag{62}$$

K^U:

$$K^U = \frac{H(z_s)(1-\delta_k)(\nu_1\pi^L + (1-\pi^N))}{1-(1-\delta)_k)(1-p_u(\theta_u))}X = \nu_2 X \tag{63}$$

K^N:

$$K^N = \frac{(1-\delta_k)p_x(\theta_x)\psi_x}{1-(1-\delta_k)(\pi^N + (1-\pi^L)\nu_1 + \nu_2 p_u(\theta_u))}X = \nu_3 X \tag{64}$$

Foreclosure decision

$$z_s = K^{1-\alpha}[g + \frac{\eta}{1-\eta}\kappa_u\theta_u - \frac{1}{1-\eta}((1-\pi^L)\frac{\kappa_u}{q_u(\theta_u)} + \pi^L\hat{V}^E) - \hat{\tau}] \tag{65}$$

$$K^{\alpha-1} = [g + \frac{\eta}{1-\eta}\kappa_u\theta_u - \frac{1}{1-\eta}((1-\pi^L)\frac{\kappa_u}{q_u(\theta_u)} + \pi^L\hat{V}^E) - \hat{\tau}]\frac{1}{z_s} \tag{66}$$

δ_d"

$$\delta_d = \delta_k \frac{\nu_3 + \nu_3\nu_2 + \nu_3\nu_1 + p(\theta_x)\psi}{1 + \nu_3 + \nu_3\nu_2 + \nu_3\nu_1} + \delta_x \frac{1 - p(\theta_x)\psi}{1 + \nu_3 + \nu_3\nu_2 + \nu_3\nu_1} \tag{67}$$

$$\rho = \frac{\nu_3 r_{\bar{z}} + \nu_3\nu_1 r_{\hat{z}} + \nu_3\nu_2 g}{1 + \nu_3 + \nu_3\nu_2 + \nu_3\nu_1} \tag{68}$$

Appendix B

Series underlying the correlations table in section 2.

Developed countries with higher NPLs have lower returns on capital. This relation is strong since the NPL divergence with Great Recession.

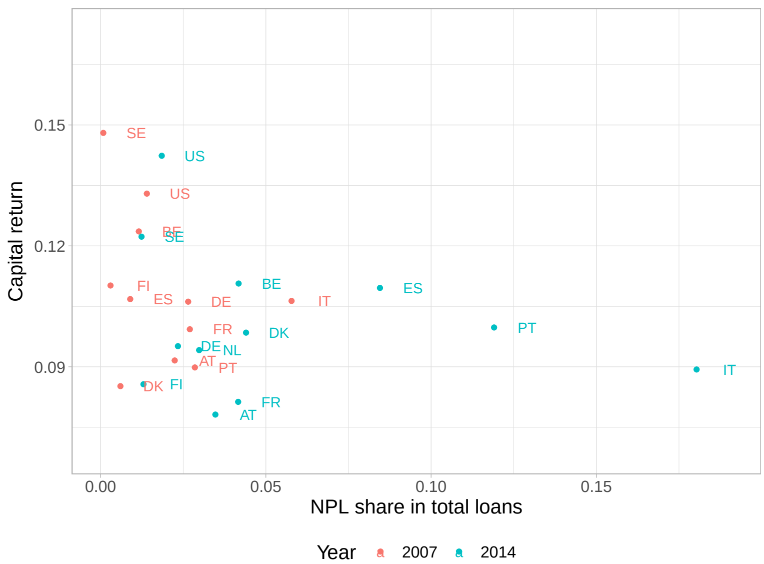

Figure 29: Returns on capital calculated from the KLEMS database, NPL ratios from World bank and IMF data

(Balgova et al., 2016) show that countries that reduce NPLs with asset management companies experience real investment and output growth following such periods compared to countries that don't.

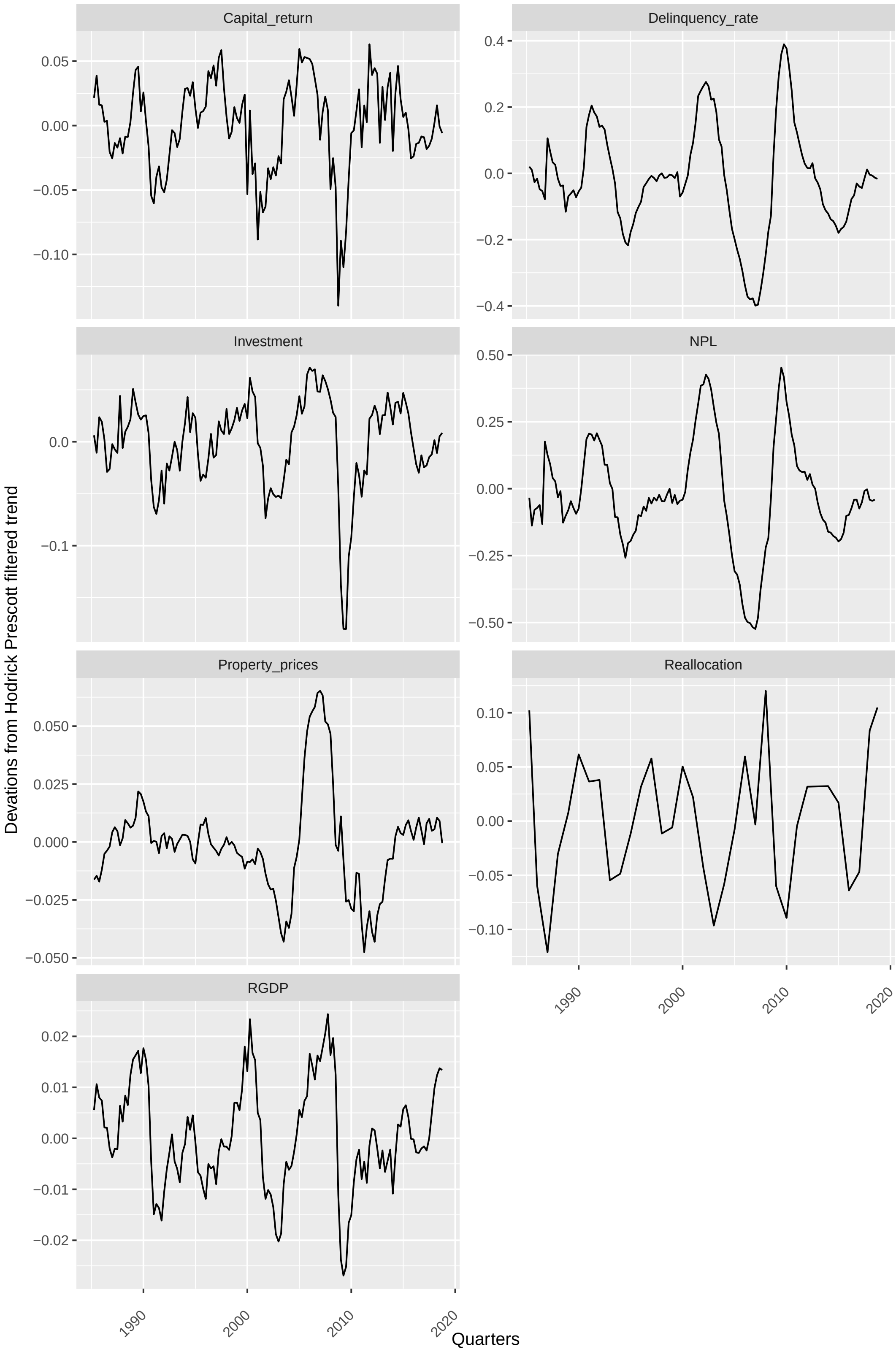

Figure 28: HP filtered US series

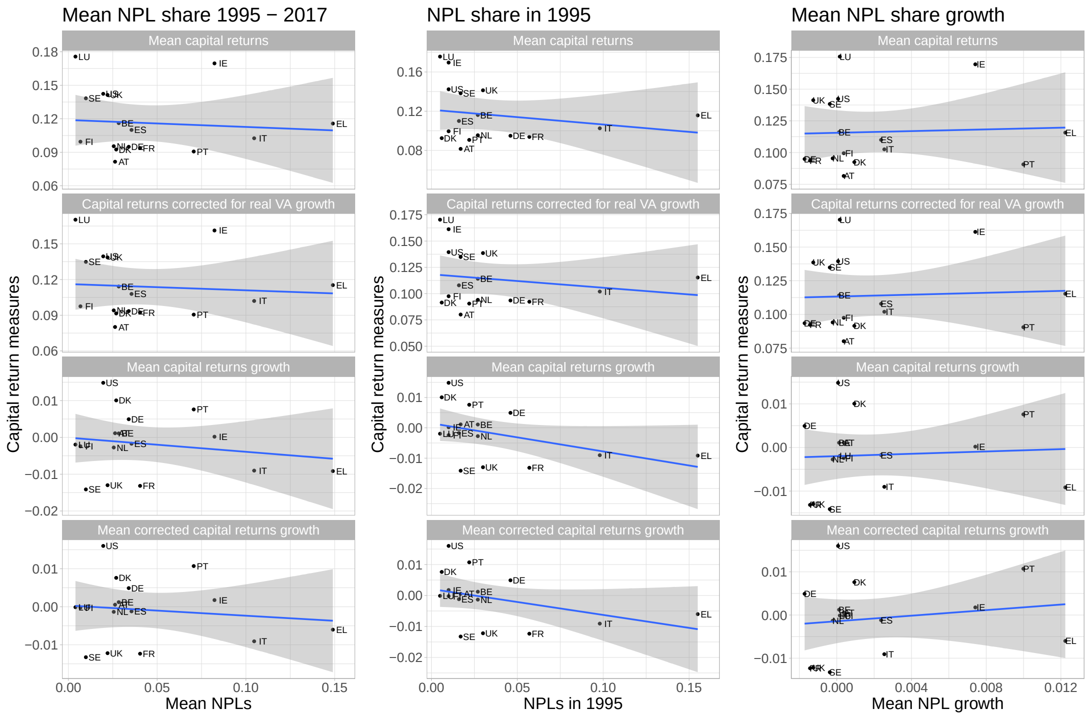

Figure 31: Mean, start and NPL share growth correlated with capital returns, capital return growth, and capital reuturns corrected for VA growth to proxy for TFP changes

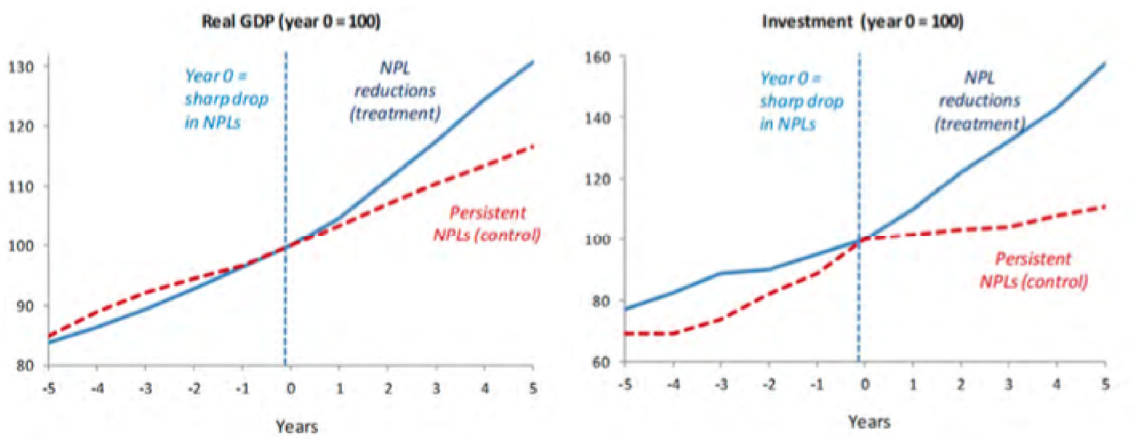

Figure 30: Source: (Balgova et al., 2016)

A cross-country comparison between OECD economies shows that non-performing loans tend correlate negatively with average real capital returns.

Appendix C - Alternative model with exogenous loan foreclosure

The model with exogenous loan foreclosure is presented here to provide some straightforward intuition about the impact of sudden rises in unemployed capital on other aggregate variables such as investment consumption or capital productivity. Unemployed capital can be interpreted in this model both as rises in unmatched capital and exogenous rises in non-performing loans. The model provides a structural explanation for the empirical observations in (Balgova et al., 2016) and other policy papers investigating the impact of non performing loan increases or reductions on the real economy. It shows that an increase in misallocated capital, which can here be interpreted as both non-performing loans and foreclosed loans, can have large and persistent real effects on the economy.

Aggregate output

There are two types of capital stocks in the economy:

- Employed or productive capital K_t^E linked to performing loans or non-performing loans.
- Unemployed capital meaning non-performing or foreclosed capital K_t^U.

Both together form the aggregate capital stock in the economy $K_t = K_t^E + K_t^U$. Unemployed capital provides a negative externality for employed capital as aggregate output is given by:

$$Y_t = A_t K_{E,t}(K_{E,t} + K_{U,t})^{\alpha-1} \tag{69}$$

A is a stochastic TFP process. We commonly assume that $\alpha < 1$. Note that this function simplifies to a standard RBC function when $K_{U,t} = 0$. [4] [5]

Aggregate output in this economy can be consumed, used for investment in further capital, or used for setting up firms with provided capital.

A is an exogenous auto-regressive process describing productivity.

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \tag{71}$$

Savings problem of the individual household

The economy is populated by a unit mass of identical households. Each household has some initial deposit wealth D_0. Each household takes credit and capital markets as given when making the individual saving decision. Thus the saving decision is a result of expected interest rates paid on deposits leading to a typical savings problem leading and a conventional Euler equation defining the savings decision.

$$\max_{C_t, I_t} E_t\left(\sum_{s=0}^{\infty} \frac{C_{t+s}^{1-\sigma}}{1-\sigma}\right) \tag{72}$$

[4] Then $Y_t = A_t K_{E,t}^{\alpha}$

[5] It is also straightforward to generalise the output function to foreclosed loans also producing

$$Y_t = [K_{E,t} + \zeta K_{U,t}] A_t (K_{E,t} + K_{U,t})^{\alpha-1} \tag{70}$$

where $\zeta < 1$ and small enough that the value of a unit of performing loan capital exceeds the value of a non-performing loan to the bank.

subject to a budget constraint:

$$C_t + I_t = \Pi_t + \rho_t D_t \tag{73}$$

Here Π are profits by firms, ρ is the interest rate paid by banks on deposits, D is the stock of deposits and I is investment.

$$D_t = (1 - \delta)D_{t-1} + I_{t-1} \tag{74}$$

Deposits depreciate in value with δ in this simple model as they are identical to the value of the underlying collateral of loans. In the expanded model this will not be the case.

Solving the maximisation problem it is straightforward to find the inter-temporal Euler equation.

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta + \rho_{t+1})] \tag{75}$$

One can also summarize the stochastic discount factor for firms and banks in the following sections as μ.

$$\mu_{t+1} = \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \tag{76}$$

Financial market

For simplicity deposits are assumed to equal lent out capital in this model $D_t = K_t$, but this is relaxed in the next section. Introducing matching frictions for new capital as well leads to a state of matched loans, a state of foreclosed loans, and a state of deposits.

All capital is assumed to be held by banks and equals the value of deposits by the household. Employed capital is used by firms, which pay interest r_t on the capital borrowed. New investment builds the capital stock. Investment of fresh capital works without frictions. However, once the firm that first received the capital defaults the capital will become unemployed, something that can be interpreted as a non-performing loan.

Foreclosed capital has to be re-matched to an entrepreneur willing to transform it for her purposes for a cost. This argument is similar to the argument in (Lanteri, 2018), who has shown that it helps provide a micro-foundation for RBC models with capital adjustment costs, which are needed to match the pro-cyclical capital reallocation observed in the data. This heterogeneous process of matching new entrepreneurs with old capital underlying non-performing loans is modelled in reduced form via a matching function.

In order to set up a firm from used capital agents need to spend κ to present a business plan to a bank and agree on an interest rate. The business proposals B_t will be matched with cash deposits and available unemployed capital. This is a frictional process, which will be summed up by the matching function:

$$M_t = m B_t^{1-\xi} K_{U,t}^{\xi} \tag{77}$$

The cost of creating a new unit of employed capital from output in the simple model without matching frictions for fresh capital can be normalised to 1. Thus as long as there is positive investment in the economy the cost of creating a unit of employed capital from unemployed capital must equal the cost of new.

$$\kappa\theta^{\xi} = m \tag{78}$$

$$B_t = (\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} \tag{79}$$

The cost of reemploying capital must be equal to the cost of creating fresh capital to the representative household. Substituting 79 in the matching function yields.

$$M_t = (\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t} \tag{80}$$

This means $\theta_t = \theta$ in the simple model due to the fixed cost of creating fresh capital.

Benefit of an average foreclosed unemployed or an employed capital unit

The expected benefit of a single capital unit invested in the banking sector is defined by the current level of employed and unemployed capital, and aggregate investment. In the following equations, δ is the depreciation rate of capital, while λ is the exogenous probability of the firm exiting production and the capital becoming a non-performing loan.

$$V^U(K_t) = p(1-\delta)(1-\lambda_t)E_t(\beta_{t+1}[V^L(IK_{t+1})-V^U(K_{t+1})])+(1-\delta)E_t[\beta_{t+1}V^U(K_{t+1})] \tag{81}$$

$$V^L(K_t) = r_t+(1-\delta)(1-\lambda_t)E_t(\beta_{t+1}[V^L(K_{t+1})-V^U(K_{t+1})])+(1-\delta)E_t[\beta_{t+1}V^U(I_{t+1}, K_{t+1})] \tag{82}$$

In surplus notation familiar from (Mortensen and Pissarides, 1994)

$$V^L(K_t) - V^U(K_t) = r_t + (1-\delta)(1-\lambda_t)(1-p)E_t(\beta_{t+1}[V^L(K_{t+1}) - V^U(K_{t+1})]) \tag{83}$$

Entrepreneurs incur a cost κ for creating a business plan and will enter the market for used capital until the benefits of entering equal the cost of entering.

The benefit of a matched unit of capital to the firm is:

$$V^E(K_t) = A_t K_t^{\alpha-1} - r_t + (1-\delta)(1-\lambda_t)E_t(\beta_{t+1}V_{E,t+1}]) \tag{84}$$

Solving for the interest rate paid by firms to banks

The interest rate for lent capital can be found by assuming Nash bargaining between firms and banks, with the bank's bargaining power being η. The equilibrium interest rate is then the result of the bargain.

$$r_t = \eta[A_t K_t^{\alpha-1} + p\beta_t(1-\delta)(1-\lambda_t)E_t(V_{E,t+1})] \tag{85}$$

This means the pooled interest rate ρ_t paid by the bank to the household for depositing a capital unit is defined by the share of productive capital in the overall capital stock, and the level of marginal returns per capital induced by the total capital stock.

$$\rho_t = r_t \frac{K_t^E}{K_t} \tag{86}$$

Transition equations for capital states

The transition equations follow from the household investment decision, and the exogenous firm destruction shock λ as well as the re-matching frictions for the market of used capital measured by non-performing loans/

$$K_{E,t} = (1-\delta)(1-\lambda_t)[K_{E,t-1} + (\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t-1}] + I_{t-1} \tag{87}$$

$$K_{U,t} = (1-\delta)[\lambda_t K_{E,t-1} + (1-p(\theta_{t-1})(1-\lambda_t))K_{U,t-1}] \tag{88}$$

Solving the model

The dynamics of this simple model can be simulated by perturbing the model around the steady-state. This can be done with ease and allows for a quick enough simulation to estimate parameters of the model such as match efficiency, which may be useful when comparing the impact of aggregate shocks and winding up of non-performing loans of different countries.

Exogenous processes Assume for now an exogenous aggregate productivity process and an exogenous default rate. The default rate can be straightforwardly endogenised. Both processes are assumed to be auto-regressive.

The aggregate productivity process is:

$$\log(a)_t = \log(a)_{t-1}\rho_a + \epsilon_{a,t} \tag{89}$$

And the default rate:

$$\lambda_t = \lambda(1-\rho_\lambda) + \lambda_{t-1}\rho_\lambda + \epsilon_{\lambda,t} \tag{90}$$

Equilibrium equations The following are equilibrium equations and constraints describing the dynamics of the economy.

- Budget constraint:
$$C_t + D_{t+1} = \Pi_t + [1-\delta+\rho_t]D_t \tag{91}$$

- Euler Equation
$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1-\delta+\rho_{t+1})] \tag{92}$$

- Marginal input cost of investment equality
$$B_t = (\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} = \theta K_{U,t} \tag{93}$$

- The value of a fresh capital unit [6]
$$V^E(K_t) = (1-\eta)A_t K_t^{\alpha-1} + (1-\delta)(1-\lambda_t)(1-\eta p)E_t(\beta_{t+1}V_{E,t+1}]) \tag{94}$$

- Profits from firms passed on to the households
$$\Pi_t = K_{E,t}[(1-\eta)A_t K_t^{\alpha-1} - \eta p(1-\delta)(1-\lambda_t)\beta_{t+1}V_{E,t+1})] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} \tag{95}$$

- Transition equation of employed capital
$$K_{E,t} = (1-\delta)(1-\lambda_t)[K_{E,t-1} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t-1}] + I_{t-1} \tag{96}$$

- Transition equation of unemployed capital
$$K_{U,t} = (1-\delta)[\lambda_t K_{E,t-1} + (1 - m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda_t))K_{U,t-1}] \tag{97}$$

- The transition function for the capital stock, which is in the simple version assumed to equal deposits.
$$K_t = (1-\delta)K_{t-1} + I_{t-1} \tag{98}$$

- The interest rate paid by banks to households
$$\rho_t = r_t \frac{K_t^E}{K_t} \tag{99}$$

- The interest rate charged by banks to firms per lent capital unit
$$r_t = \eta[A_t K_t^{\alpha-1} + p\beta_t(1-\delta)(1-\lambda_t)E_t(V_{E,t+1})] \tag{100}$$

[6] It is assumed that $V^E(K_t) > 1$, which is achieved with appropriate calibration. This ensures that investment is always positive, i.e. $I > 0$. For moderate shocks the evolving non-differentiability can be ignored or alternatively a penalty function for low investment can be introduced to ensure more accurate estimation.

The steady-state equations can be found in Appendix A.

Simulated model

The model is simulated with $\beta = 0.99$, $\alpha = 0.66$, $m = 0.4$, $\xi = 0.8$, $\eta = 1 - \xi$, $\delta = 0.05$, $\lambda = 0.05$, and $\kappa = 0.1$. While the parameters may certainly need more calibration the dynamics of the model are nevertheless interesting. The simulated model shows that a brief rise in the default rate leads to a persistent decline of consumption and productive loans. Interest rates barely rise leading to a long recovery as investment does not rise to levels to recover destroyed productive capital due to the negative impact of non-performing loans remaining in the economy. This seems a compelling story for countries struggling with high foreclosed loan levels following financial crisis.

Meanwhile, the effect of aggregate productivity shocks is mooted due to the assumptions taking for the secondary capital market and non-endogenised defaults. These effects would change once endogenous loan calling is introduced, and search frictions are also introduced for fresh capital as the share of non-performing loans would not remain ”essentially” flat and hidden. Once an endogenous decision is introduced the evidently costly foreclosure result is endogenised. Banks will decide on foreclosure and thereby also decide to forbear foreclosing decisions leading to non performing loans.

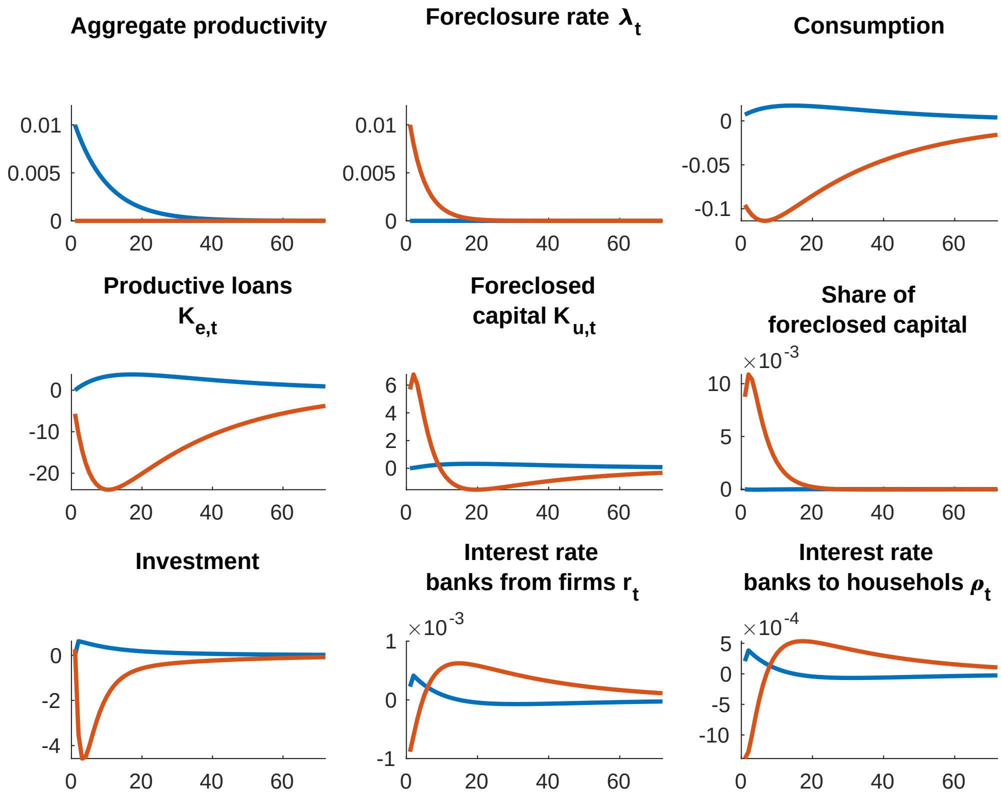

Figure 32: Impulse response functions for a 1% shock to aggregate productivity a_t (blue), and a 1% shock to the default rate λ_t (red)