

# Central Bank Digital Currency and Quantitative Easing

Martina Frascini\*    Luciano Somoza<sup>†</sup>    Tamarro Terracciano<sup>‡</sup>

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## ABSTRACT

We study how issuing a central bank digital currency (CBDC) interacts with the monetary policy, i.e., standard policy or quantitative easing. We reach three main conclusions. First, the equilibrium impact of introducing a CBDC depends on the ongoing monetary policy. Second, under both monetary policies, there exist conditions for which issuing a CBDC is neutral to the economy. Third, issuing a CBDC under quantitative easing can negatively affect the lending and might render this policy quasi-permanent. Commercial banks optimally use their excess reserves to accommodate retailers' demand for CBDC deposits, making quantitative easing tapering problematic.

Keywords: CBDC, central banking, monetary policy, QE.

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\*Corresponding author. Swiss Finance Institute and Department of Finance, HEC Lausanne, University of Lausanne. E-mail: [martina.fraschini@unil.ch](mailto:martina.fraschini@unil.ch).

<sup>†</sup>Swiss Finance Institute and Department of Finance, HEC Lausanne, University of Lausanne. E-mail: [luciano.somoza@unil.ch](mailto:luciano.somoza@unil.ch).

<sup>‡</sup>Swiss Finance Institute and University of Geneva. E-mail: [tamarro.terracciano@unige.ch](mailto:tamarro.terracciano@unige.ch).

# 1 Introduction

Most major central banks are considering introducing a retail Central Bank Digital Currency (CBDC), i.e., a digital payment instrument, denominated in the national unit of account, that is a direct liability of the central bank (BIS, 2020). Advocates of CBDC projects argue that they would strengthen monetary sovereignty, enrich monetary policy toolkits, and foster financial innovation and inclusion.<sup>1</sup> Nonetheless, the introduction of a CBDC would lead central banks into uncharted territory as they would directly compete with banks for deposits, raising concerns about financial stability as well as privacy issues (Armeliu, Guibourg, Johansson, and Schmalholz, 2020). The burgeoning literature on the topic focuses on several aspects, such as disintermediation risk, deposit competition, and optimal design (see, e.g., Fernández-Villaverde, Sanches, Schilling, and Uhlig, 2020b; Agur, Ari, and Dell’Ariccia, 2021).

However, the interaction between a CBDC and current monetary policy remains an open question (see, e.g., BOE, 2020; ECB, 2020). This is particularly relevant, as the balance sheets of central banks reached record levels after the global financial crisis and are now expanding further, due to COVID-19 relief programs. Therefore, CBDCs are likely to be introduced before central banks have fully reverted their Quantitative Easing (QE) programs, in a low-interest rate environment with a limited supply of safe assets. We address these issues by asking the following questions: Do current monetary policies matter for the introduction of a CBDC? What are the equilibrium outcomes of introducing a CBDC in a QE environment?

Our analysis reaches three main conclusions. First, the equilibrium impact of a CBDC depends on the ongoing monetary policy. Second, under both monetary policies, there exist conditions for which issuing a CBDC is neutral to the economy. When the central bank conducts QE, the introduction of a CBDC is neutral under two conditions: the cost of issuing a CBDC is equal to the interest on reserves, and the demand for CBDC deposits is smaller than the amount of excess reserves in the system.<sup>2</sup> Third, commercial banks optimally use their excess reserves to accommodate retailers’ demand for CBDC deposits. This mechanism leads to retailers replacing banks as counterparts on the liability side of the central bank’s balance sheet. As retail deposits

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<sup>1</sup>G7 Finance Ministers and Central Bank Governors’ Communiqué, Art. 17, June 5th 2021, [www.g7uk.org/g7-finance-ministers-and-central-bank-governors-communication](http://www.g7uk.org/g7-finance-ministers-and-central-bank-governors-communication).

<sup>2</sup>Excess reserves are the amount of reserves that exceeds liquidity requirement.

are typically inelastic (Chiu and Hill, 2015), tapering QE policies might become more difficult.

We obtain these results by extending the model proposed by Magill, Quinzii, and Rochet (2020). This framework features a central bank that implements two different monetary policies. The first is standard monetary policy, where the central bank holds government bonds, their interest rate is kept above the one on reserves, and liquidity requirements are binding. We take this specification as our baseline. The second is QE policy, where the central bank holds risky securities, the interest rates on treasuries and reserves are equal, and there are excess reserves in the system.

We introduce a CBDC under two main assumptions. First, the central bank holds assets to back CBDC deposits (consistently with ECB, 2020). Even if it were possible for a central bank to issue an unbacked CBDC, it would result in a decline in central bank equity and would be akin to helicopter money, which is not currently an option (BIS, 2020). Second, bank deposits and CBDC deposits are not perfect substitutes. While they can both be remunerated, they have different technological features and a plethora of complimentary services (e.g., programmability, smart contracts). It is plausible that a CBDC would rely on more efficient technology, allowing for faster, smoother digital payments, while the banking sector is better suited to providing complimentary services and is more efficient at targeting customers. A good example of such complementarity is given by the co-existence of traditional banks and numerous fintech companies, which provide deposits and payment solutions. For instance, the average PayPal user also has a bank account and keeps only a small sum in her PayPal account.<sup>3</sup> We assume that a CBDC would work in a similar way, offering better technological solutions for payments and that banks will simultaneously leverage their existing relationships, deposit rates, and commercial skills to retain depositors.

We start our analysis by considering a baseline scenario where a CBDC is launched while the central bank conducts standard monetary policy. In this scenario, the central bank has to buy additional treasuries to back CBDC deposits, indirectly passing CBDC funds to banks by influencing inter-bank funding via open market operations (i.e., changing the amount of floating treasuries in the economy). Such additional inter-bank funding fully compensates for the reduction in bank deposits due to the introduction of a CBDC. Thus, the total amount of risky loans to the economy remains the same, although the composition of bank liabilities is different. Our setting is consis-

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<sup>3</sup>Source: Demos, T. June 1st 2016, *PayPal Isn't a Bank, But It May Be the New Face of Banking*, The Wall Street Journal.

tent with the equivalence theorem between private and public money by Brunnermeier and Niepelt (2019), where the central bank lends directly to banks to compensate them for missing deposit funding. However, once we consider additional frictions, such as convenience yields, maintenance costs, or risky holdings by the central bank, we need to impose further conditions for a CBDC to be neutral.

In this baseline case, the balance sheet of the central bank increases due to the newly-issued CBDC deposits. Nonetheless, the model suggests that the central bank's profitability and risk exposure depend on the ability of the government sector to design and manage CBDC deposits (i.e., interest rate and management costs). Specifically, we find that when CBDC deposits are less onerous than bank deposits, seignorage revenues increase, and taxes decrease while the risk exposure of the central bank remains the same.

When the central bank conducts QE programs, the equilibrium outcomes are less straightforward and mainly depend on the amount of bank deposits converted into CBDC as well as on the amount of its excess reserves.<sup>4</sup> When depositors decide to convert one unit of bank deposits into one of CBDC, commercial banks will have to transfer one unit of resources to the central bank. When converting bank deposits into CBDC deposits, the commercial bank will optimally decide to reduce its excess reserves. The size of the central bank's balance sheet remains the same, as one unit of reserves is simply transferred from the commercial bank's account to the retailer's CBDC account. Thus, as long as the amount of CBDC deposits does not exceed the amount of excess reserves, the introduction of a CBDC leads to a reduction in both deposits and reserves, without real consequences for lending to the economy.

It is worth noting that if large amounts of bank deposits are converted into CBDC deposits through this mechanism, it will arguably be harder for the central bank to reverse its expansionary policies. Reverting an asset purchase program implies selling the assets back to the banking sector in exchange for central bank reserves. If the banking sector does not have excess reserves because they have been transferred to retailers that hold CBDC deposits, it would be more difficult for the central bank to taper its balance sheet. Facing financial intermediaries is not the same as facing retail depositors, as they tend to be inelastic (Chiu and Hill, 2015). In other words, the widespread adoption of a CBDC might render current quantitative easing programs quasi-permanent.

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<sup>4</sup>In the US, excess bank reserves are at an all-time high due to the COVID-19 stimulus programs. See Figure 3.

When the demand for CBDC deposits exceeds the amount of excess reserves, the introduction of a CBDC changes the equilibrium outcomes of the economy. In this case, the reduction in deposits leads to a reduction in reserves due to liquidity requirements. The central bank now has new CBDC deposits but lower reserves. However, since the central bank holds risky securities against its liabilities, the changes in its holdings do not influence the amount of floating government bonds. Thus, the central bank is not able to channel funding back to the banking sector via open market operations, and the amount of loans to the economy shrinks. The additional purchases of risky securities by the central bank increase its size and level of risk-taking. Even if seigniorage revenues are more volatile, they increase in expectation allowing the government sector to levy lower taxes.

Although our model encompasses important real-world features, such as liquidity and capital requirements, explicit and implicit deposit guarantees, and shortage of safe assets, it has some limitations. First, the state of the economy is exogenous and taken as given by the actors. Second, monetary policies, including the introduction of a CBDC, are exogenous. Third, all interest rates in the model are real rates, and thus there is no inflation from one period to the next. Our analysis is a comparative statistics exercise focused on the balance sheet effects of introducing a CBDC during QE. Providing an exhaustive theoretical account of the general equilibrium effects of introducing a CBDC is beyond the scope of this paper.

Our results directly inform the debate about CBDCs in two ways. First, our findings suggest that the decision to issue a CBDC should consider the ongoing monetary policy. While the direction of the effects can be easily determined under standard monetary policy, it is largely ambiguous under QE. Second, if a central bank launches a CBDC while pursuing QE policies, it should consider the amount of excess reserves in the banking system, as the impact on lending is neutral only insofar the demand for CBDC deposits is lower than the amount of excess reserves. Moreover, the fact that a CBDC might render the reversion of QE policies harder to implement undermines any commitment to return to a pre-QE world.

The rest of the paper is organised as follows. Section 2 reviews the literature on the topic. Section 3 describes the model setup. Sections 4 and 5 present the equilibrium conditions and the Pareto optimal allocations. Section 6 discusses the effects of introducing a CBDC while the central bank conducts standard monetary policy, and Section 7 deals with the case where CBDC is introduced while the central bank is pursuing QE. Finally, Section 8 concludes.

## 2 Literature Review

Our paper contributes to the growing literature that studies the introduction of a CBDC. To the best of our knowledge, our paper is the first to focus on the interaction between QE and a CBDC.

As of writing, there is not yet data available for research, as only few projects are in advanced stages (Auer and Böhme, 2020). As a consequence, the literature lacks empirical contributions, and scholars rely on counterfactual and theoretical exercises (see Barrdear and Kumhof, 2016). Brunnermeier and Niepelt (2019) provide a starting point with their indifference theorem, which states that, under certain conditions, swapping private money with public money (e.g., CBDC) is neutral for equilibrium allocations. In their setting, the central bank collects retail deposits and lends them to commercial banks to compensate for missing funding. Later, Niepelt (2020) generalizes the result on the macro irrelevance between public and private money and shows that a deposit-based payment system requires higher taxes. Under certain conditions, our setting is consistent with the indifference theorem. Nevertheless, the theorem does not hold when taking frictions and QE into account.

While there is little research that directly addresses the relationship between monetary policy and CBDC, it is worth mentioning the paper by Ferrari, Mehl, and Stracca (2020). They do not focus on ongoing monetary policy regimes as we do, but rather on international spillovers of shocks. They find that a CBDC might increase international linkages and that domestic issuance of a CBDC increases asymmetries in the international monetary system by reducing monetary policy autonomy in foreign economies.

Our paper contributes to the strand of literature about CBDC design by studying the consequences of launching a CBDC while the central bank pursues QE. The choice of CBDC design has sizeable real effects on the economy in terms of technological innovation, users' privacy, and the bank's ability to intermediate. A comprehensive BIS report by Auer and Böhme (2020) studies the differences between three main architectural choices: account- vs token-based system, one- or two-tier distribution, and whether to adopt a decentralized ledger technology (see also Armelius et al., 2020). Agur et al. (2021) studies the relation between preferences over anonymity and security by developing a theoretical model where depositors can choose between cash, CBDC, and bank

deposits. They conclude that the optimal CBDC design trades off bank intermediation against the social value of maintaining diverse payment instruments. By contrast, Keister and Sanches (2020) study CBDC optimal design in a setting with financially constrained banks and with a liquidity premium on bank deposits. They highlight an important policy trade-off: while a digital currency tends to promote efficiency, it may also crowd out bank deposits, raise banks' funding costs, and decrease investment. They also find that despite these effects, introducing a CBDC often increases welfare.

Furthermore, we contribute to the literature related to the disintermediation risk of the banking sector due to the introduction of a CBDC. Specifically, we show the extent to which the banking sector would welcome the issuance of a CBDC under QE. Fernández-Villaverde, Sanches, Schilling, and Uhlig (2020a) and Fernández-Villaverde et al. (2020b) focus on these issues by using a modified version of the model by Diamond and Dybvig (1983), where a central bank engages in large-scale intermediation by competing with private financial intermediaries for deposits and investing in long term projects. They find that the set of allocations achieved with private financial intermediation is also achieved with a CBDC and that, during a run, the central bank is more stable than the commercial banking sector. For this reason, they conclude that the central bank would arise as a deposit monopolist. Chiu, Davoodalhosseini, Jiang, and Zhu (2020) focus on banks' market power and show that when banks have no market power, issuing a CBDC would crowd out private banking. However, when banks have deposit market power, a CBDC with a reasonable interest rate would encourage banks to pay higher interests or offer better services to keep their customers (see also Andolfatto, 2018). In the same spirit of our analysis, Böser and Gersbach (2020) study how the introduction of a CBDC interferes with central bank collateral requirements and conclude that in the medium-term tight collateral requirements will undermine the functioning of the banking sector.<sup>5</sup>

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<sup>5</sup>See also Williamson (2019) on this aspect.

## 3 The Model

### 3.1 Introduction

For our analysis, we extend the model developed by Magill, Quinzii, and Rochet (2020) by adding a one-tier interest-bearing CBDC. The model has two periods and an economy with a private and a public sector. The private sector consists of agents and a representative commercial bank, whereas the public sector consists of a central bank (CB) and a fiscal authority, which are treated as a single actor, the government.

Agents are depositors, investors, and institutional cash pools. Depositors and cash pools are infinitely risk-averse and only lend to banks if they are sure of having their funds returned. Deposits are explicitly insured (e.g., DGS in the Eurozone or FDIC in the US). In addition to the deposit rate, depositors benefit from the payment services provided by the banks. Cash pools invest indifferently in public and bank debt and consider the latter to be implicitly insured by the government. This belief was essentially confirmed in 2008 when the government bailed out most failing financial institutions or provided relief by purchasing assets through the central bank. Because of the public insurance on the bank liabilities, there is no possibility of bank runs. On the other hand, investors are willing to accept risk and therefore invest in bank equity. We do not explicitly model entrepreneurs' decision-making. Banks have a unique technology that allows them to invest in risky ventures and perform maturity transformation. They channel funds from savers to entrepreneurs and allow savers to transfer funds from period 0 to period 1. We do not explicitly model the bank's screening process, and we assume that banks invest in productive ventures. The government regulates banks, bails them out of bankruptcy when needed, issues debt to fund its spending, and collects taxes from investors to repay its debt.

In this setting, we include a CBDC, by which depositors have the option to deposit their funds at the central bank. CBDC deposits pay an interest and provide payment services.

We discuss in more detail the characteristics and decisions of each of the three types of agents (depositors, cash pools, and investors), the representative commercial bank, and the government sector.



### 3.2 Depositors

The representative depositor is infinitely risk averse and receives an endowment  $w_{d0}$  at time 0 and no endowment in period 1. The depositor can place her funds either in a commercial bank (as a standard bank deposit) or in the central bank (as a CBDC deposit) to transfer them to time 1 for consumption. She also benefits from the payment services provided by the bank and the central bank. The agent's utility derives from the consumption stream  $x_d$ , which consists of  $x_{d0}$  at time 0 and the random consumption  $\tilde{x}_{d1}$  at time 1. We consider  $\tilde{x}_{d1} = \tilde{x}_{d1}^d + \tilde{x}_{d1}^h$ , where  $\tilde{x}_{d1}^d$  is the consumption that derives from the returns on bank deposits, and  $\tilde{x}_{d1}^h$  is the consumption that derives from the ones on CBDC funds. The total utility is given by:

$$u_d(x_{d0}) + \min \tilde{x}_{d1} + \rho_d \min \tilde{x}_{d1}^d + \rho_h \min \tilde{x}_{d1}^h, \quad (1)$$

where  $u_d$  is a concave increasing function,  $\min \tilde{x}_{d1}$  represents the depositor's infinite risk aversion and  $\rho_d$  and  $\rho_h$  capture the convenience yields obtained from the bank and central bank transaction services at time 1. We assume that the convenience yields are linear. If  $R^d$  denotes the deposit interest paid by banks, a bank deposit  $d$  generates a consumption  $x_d^d = (-d, R^d d)$ . If  $R^h$  denotes the deposit interest paid by the central bank,  $h$  worth of CBDC deposit generates a consumption  $x_d^h = (-h, R^h h)$ . The total consumption is therefore  $x_d = (w_{d0} - d - h, R^d d + R^h h)$  and the depositors utility is  $u_d(w_{d0} - d - h) + (1 + \rho_d)R^d d + (1 + \rho_h)R^h h$ .

If in time 0 the utility function of depositors  $u_d$  satisfies the Inada conditions  $\frac{\partial u_d}{\partial d}(x_{d0}) \rightarrow \infty$  as  $x_{d0} \rightarrow 0$  and  $\frac{\partial u_d}{\partial h}(x_{d0}) \rightarrow \infty$  as  $x_{d0} \rightarrow 0$ , then the solutions to the maximization problem are characterized by the following first-order conditions:

$$\frac{\partial u_d}{\partial d}(w_{d0} - d - h) = (1 + \rho_d)R^d, \quad (2)$$

$$\frac{\partial u_d}{\partial h}(w_{d0} - d - h) = (1 + \rho_h)R^h. \quad (3)$$

**PROPOSITION 1.** *If the utility function of depositors  $u_d$  satisfies the Inada conditions, then positive funds allocations in bank and CBDC deposits,  $(d, h) > 0$ , are guaranteed if and only if*

$$(1 + \rho_d)R^d = (1 + \rho_h)R^h. \quad (4)$$

*Proof.* Using Leibniz's notation,  $\frac{\partial u_d}{\partial d} = \frac{\partial u_d}{\partial x_{d0}} \frac{\partial x_{d0}}{\partial d}$  and  $\frac{\partial u_d}{\partial h} = \frac{\partial u_d}{\partial x_{d0}} \frac{\partial x_{d0}}{\partial h}$ . In this model, it holds that  $\frac{\partial x_{d0}}{\partial d} = \frac{\partial x_{d0}}{\partial h}$  and, therefore, that  $\frac{\partial u_d}{\partial d} = \frac{\partial u_d}{\partial h}$ . Applying this result to (2) and (3), it follows (4).  $\square$

In other words, there is no corner solution for depositors if the unitary utilities, considering interest rates and convenience yields, for deposits in bank and deposits in CBDC are the same.

### 3.3 Cash Pools

The cash pools agents represent the wholesale money market, which includes money market funds, wealth managers, and the like. Just like depositors, cash pools only invest in safe and liquid assets. During the 2008 financial crisis, the actions by the central bank and the treasury prevented runs and confirmed the perception that these bank liabilities are implicitly assured by the government. The representative cash pool has an endowment  $w_{c0}$  only at time 0, it is infinitely risk averse, and it has a utility function  $u_c(x_{c0}) + \min \tilde{x}_{c1}$ , where  $u_c$  is an increasing concave function that captures the opportunity cost of the cash pool funds. Potentially, they could invest in CBDC deposits, bank deposits, government debt, and bank debt. Nevertheless, they only choose between government and bank debt as they do not benefit from the convenience yield.<sup>6</sup> Since cash pools invest only in safe assets, they can choose between government and bank liabilities, which have to be interpreted as short-term debt, either loans or bonds. Since treasuries are not enough to satisfy the demand of cash pools, part of their savings is absorbed by the bank ( $c_b$ ). The representative cash pool will choose how much to invest ( $c$ ) in order to maximize  $u_c(w_{c0} - c) + R^c c$ , where  $R^c$  is the interest received by the bank or the government.

If in time 0 the utility function of cash pools  $u_c$  satisfies the Inada conditions  $\frac{\partial u_c}{\partial c}(x_{c0}) \rightarrow \infty$  as  $x_{c0} \rightarrow 0$ , then the solution to their maximization problem is characterized by the first-order condition:

$$\frac{\partial u_c}{\partial c}(w_{c0} - c) = R^c. \quad (5)$$

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<sup>6</sup>In equilibrium  $(1 + \rho_x)R^x = R^c = R^B$ , which means  $R^c = R^B > R^x$ , for  $x = d, h$ .

### 3.4 Investors

Investors play two roles in the model. They are long-term investors who take risks, and they act as taxpayers. We better describe taxes in section 3.6. Investors receive an endowment in both periods  $w_i = (w_{i0}, w_{i1})$  and are risk neutral. Their utility function is  $u_i(x_{i0}) + \mathbb{E}(\tilde{x}_{i1})$ , where  $u_i$  is an increasing concave function that satisfies the Inada conditions. Investors can place their funds in government bonds, bank debt, and bank equity. The payoff of bank equity is  $V(y)$  per unit of equity, where  $y$  is the realization of the random payoff  $\tilde{y}$  per unit of investment in risky projects. If they invest in bank debt, they receive the same return  $R^c$  as cash pools. If the funds invested in safe assets are denoted by  $c_i$  and  $e$  denotes the amount invested in bank equity, then the investor problem is to choose  $(c_i, e)$  to maximize

$$u_i(w_{i0} - c_i - e) + \mathbb{E}(w_{i1} - t(y) + V(y)e + R^c c_i), \quad (6)$$

where  $t(y)$  is a lump-sum tax due to the government at time 1. Given the expected return on equity  $R^E = \mathbb{E}[V(\tilde{y})]$ , we exclude the case where  $R^c > R^E$  for which  $c_i > 0$  and  $e = 0$ , since banks must have positive equity in equilibrium. We assume that when  $R^E = R^c$ , investors choose to invest only in equity. Finally, when  $R^E > R^c$ , investors prefer to invest only in equity and  $c_i = 0$ .

Therefore, the first-order condition that characterizes the solution of the investor maximization problem is:

$$\frac{\partial u_i}{\partial e}(w_{i0} - e) = R^E. \quad (7)$$

### 3.5 Commercial Bank

The banking sector is modeled with a representative commercial bank that can either store funds in reserves ( $M$ ) at the central bank or invest ( $K$ ) in a productive risky technology that delivers  $\tilde{y}$  at time 1. The distribution of returns is characterized by the density function  $f(y)$  on  $\mathbb{R}_{\geq 0}$ , and it is different from zero for  $\tilde{y} > \underline{y} > 0$ .<sup>7</sup> To finance its assets, the bank collects deposits ( $d$ ), obtains financing from cash pools ( $c_b$ ), and issues equity ( $E$ ). Hence, it holds that  $M + K = d + c_b + E$ .

<sup>7</sup>As in Magill et al. (2020), we assume that all shocks are perfectly correlated and, due to the law of large numbers, we can treat  $\tilde{y}$  as an aggregate shock for the economy.

The commercial bank offers bank deposits with a series of complimentary services and faces a unitary cost  $\mu_d$  at time 1, which represents the cost of maintenance of the infrastructure, managing of accounts, and so forth. In light of what occurred in the aftermath of the 2008 crisis, our model encompasses two kinds of insurances. The first one is explicit and refers to the depositors, featuring the deposit guarantee schemes of major economies. The second one is implicit and applies only to cash pools, who believe that, in case of crisis, the government would bail out the banking sector following the too-big-to-fail argument.<sup>8</sup> The bank is the only one that can perform risk and maturity transformation: it borrows short safe deposits and lends long risky loans to entrepreneurs.

Our model also incorporates current banking regulations with liquidity and capital requirements. The bank is forced to store at least  $\delta$  of its deposits in reserves to satisfy the liquidity requirement and finance at least  $\bar{\alpha}$  of the risky projects with equity for the capital requirement. The central bank pays an interest rate  $R^r$  on reserves.

The representative bank optimally chooses the items of its balance sheet  $(d, c_b, E, M, K)$  taking as given the interest rates in the economy  $(R^r, R^d, R^c, R^E)$  and maximizes the shareholders' expected profit:

$$\max_{d, c_b, E, M, K} \int_{\hat{y}}^{\infty} [Ky + MR^r - dR^d(1 + \mu_d) - c_bR^c]f(y)dy - R^E E, \quad (8)$$

subject to

$$d + c_b + E = K + M, \quad (9)$$

$$M \geq \delta d, \quad (\text{liquidity requirement}) \quad (10)$$

$$E \geq \bar{\alpha}K, \quad (\text{capital requirement}) \quad (11)$$

where  $\hat{y}$  is the minimum return on the risky technology that allows the bank to repay its creditors, i.e.,  $K\hat{y} + MR^r = dR^d(1 + \mu_d) + c_bR^c$ . This means that the bank is solvent for  $y > \hat{y}$ .

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<sup>8</sup>It is worth mentioning that cash pools receive  $yK$  as collateral from the bank. Thus, in case of default, they are only interested in the fact that the government would repay them the difference between what they lent out and the collateral value.

### 3.6 Government

We consider the fiscal authority and the central bank as a single entity (i.e., the government) that conducts guarantee, prudential, interest rate, and balance sheet policies. To finance its expenditure  $G$ , the government issues bonds ( $B = G$ ) at time 0, on which it pays an interest rate  $R^B$  at time 1. The central bank can influence this interest rate via open market operations, namely repos and reverse-repos with cash pools.<sup>9</sup> The interest rate takes different values according to the monetary policy regime. At time 1, the government levies taxes on the investors to service its bonds. We make the strong assumption that prices are fully rigid as it allows us to work with a real variable model.

As mentioned before, the government provides explicit and implicit insurance to depositors and cash pools to avoid runs, and it sets the liquidity ( $\delta$ ) and capital ( $\bar{\alpha}$ ) requirements.

The central bank manages the funds coming from reserves ( $M$ ) and CBDC deposits ( $h$ ) by deciding the compositions of its assets. Hence, it either invests in government bonds ( $B^{CB}$ ) or in risky securities ( $E^{CB}$ ), which in our model are represented by the bank's equity. We define a baseline *standard policy* setting where the central bank holds government bonds against reserves and a *quantitative easing (QE) policy* setting where the reserves are backed by risky assets (i.e., bank equity, which is the only risky asset in the model). It is worth noting that purchasing distressed assets from the banking sector is economically equivalent to recapitalize banks by injecting equity.

In standard policy, the liquidity requirement is always binding ( $M = \delta d$ ), and the interest rate on government bonds is larger than the one reserves,  $R^B > R^r$ . In a QE world, the amount of reserves usually exceeds the liquidity requirement ( $M \geq \delta d$ ), and the banking sector holds excess reserves ( $M - \delta d$ ) at the central bank. In our setting, the amount of excess reserves can be considered as exogenous to the banking sector, as it is solely due to the asset purchase programs of the central bank. Finally, under QE, the interest rate on reserves is equal to the one on government bonds,  $R^B = R^r$ .

We assume that the central bank will be consistent with its ongoing monetary policy when introducing a CBDC. Therefore, it will hold treasuries against CBDC deposits under standard policy and risky assets under QE policy. The central bank could also decide on a hybrid strategy, in which

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<sup>9</sup>We consider only two periods, so we interpret  $B$  as very short-term bonds.

it could hold risky securities against CBDC deposits under standard policy and treasuries under QE policy. While we focus only on the first two cases for our main analysis, we comprehensively discuss all scenarios in Appendix A and Appendix B. From a theoretical standpoint, investing CBDC deposits in risky assets could be justified by two main reasons. The first is that, in response to the 2008 crisis, major central banks have taken up massive amounts of risks on their balance sheet, and it is not yet clear if and when they will do full tapering. Second, there might not be sufficient safe assets (i.e., government bonds) to fully absorb the overall demand.

Under quantitative easing, the commercial bank prefers to swap its excess reserves into CBDC deposits on the central bank's balance sheet. Once it runs out of excess reserves, i.e., the liquidity requirement is binding, the commercial bank is forced to liquidate assets in favor of the central bank to transfer depositors' savings. We denote with  $\bar{h}$  the maximum demand for CBDC deposits for which the liquidity requirement is not binding. Therefore, when the demand for CBDC deposits is greater than this amount ( $h > \bar{h}$ ), the central bank's balance sheet increases its size by  $\tilde{h} = h - \bar{h}$ . This mechanism is explained in detail in Section 7.

Finally, when the commercial bank is solvent ( $y > \hat{y}$ ), the tax is equal to the amount needed to repay bondholders minus the net seignorage revenue ( $\theta$ ). In case the bank goes bankrupt ( $y \leq \hat{y}$ ), the tax also includes the bank's liabilities, as it has to repay depositors and cash pools (explicit and implicit guarantee), minus the value of bank assets. Thus, we define the bankruptcy costs as  $\phi = (1 + \mu_d)R^d d + R^c c_b - (Ky + MR^r)$ . The taxes are given by:

$$t = R^B B - \theta - \phi \mathbb{1}_{y \leq \hat{y}}. \quad (12)$$

## 4 Equilibrium

This section is organized as follows. Subsections 4.1 and 4.2 report the assumptions and conditions common to all monetary policy regimes. The remaining subsection 4.3 focuses on the peculiarities of each scenario.

### 4.1 Assumptions

**ASSUMPTION 1.** *Investors.*

- (a) Investors do not prefer to consume all their endowment at time 0 as they are better off in investing in bank equity:  $\frac{\partial u_i}{\partial w_{i0}}(w_{i0}) < \mathbb{E}[\tilde{y}]$ .
- (b) Investors have enough endowment at date 1 to afford the tax when  $y \leq \hat{y}$ , even in the limit case in which at date 0 depositors store all their endowment in deposits, and cash pools invest all their endowment in bank debt:  $w_{i1} > (w_{d0}(1 + \mu_d) + w_{c0}) \mathbb{E}[\tilde{y}]$ .

**ASSUMPTION 2.** *Cash pools.*

- (a) Cash pools prefer to buy both government bonds and bank debt rather than having only treasuries:  $\frac{\partial u_c}{\partial w_{c0}}(w_{c0} - B) < R_B \leq \mathbb{E}[\tilde{y}]$ .

**ASSUMPTION 3.** *Depositors.*

- (a) Depositors would buy treasuries if they had no other choice:  $\frac{\partial u_d}{\partial w_{d0}}(w_{d0}) < \frac{\partial u_d}{\partial w_{d0}}(w_{d0} - B)$ .
- (b) Depositors prefer to hold their savings in CBDC and bank deposits than in treasuries:  $\rho_d > \mu_d$ ,  $\rho_h > \mu_h$ .

## 4.2 Common Equilibrium Conditions

In this section, we describe the conditions that are common to all monetary policy regimes.

The first condition for the equilibrium is given by the scarcity of the safe assets in the economy (i.e., treasuries), which are not enough to satisfy the demand of cash pools. Therefore, in equilibrium, it must hold that

$$R^c = R^B \tag{13}$$

to make bank debt attractive enough to cash pools. This way, the central bank can influence the cost of bank funding by setting  $R^B$ .

The second condition allows the bank to use both deposits and cash pools as a source of funding. Because of the liquidity requirement in equation (10), 1 unit of deposits is equivalent to  $(1 - \delta)$  unit of cash pools loans for the bank investment. They must have the same opportunity costs for the bank to use them both. It follows that  $(1 + \mu_d)R^d - \delta R^r = (1 - \delta)R^B$ , which translates in

$$R^d = \frac{(1 - \delta)R^B + \delta R^r}{1 + \mu_d}. \tag{14}$$

We define the investable debt of the bank as all the debt fundings that can be invested in the risky technology, but the reserves:

$$D = d + c_b - M. \quad (15)$$

Replacing (15) into the bank's balance sheet constraint (9), the following equation holds:

$$K = E + D. \quad (16)$$

At equilibrium, the capital requirement (11) is always binding:

$$E = \bar{\alpha}K. \quad (17)$$

Then, by substituting the capital requirement (17) in condition (16), we derive that  $D = (1 - \alpha)K$ . Consequently,

$$E = \frac{\bar{\alpha}}{1 - \bar{\alpha}}D. \quad (18)$$

We can prove that  $Ky + MR^r - dR^d(1 + \mu) - c_bR^B = Ky - DR^B = K(y - (1 - \bar{\alpha})R^B)$ . The threshold  $\hat{y}$  for bankruptcy is defined as the payoff that nullifies this equation. Hence,  $K\hat{y} = DR^B$  and  $\hat{y} = (1 - \bar{\alpha})R^B$ . The bank's maximization problem is then reduced to the choice of  $(\alpha, E)$  that maximizes  $E \left( \frac{1}{\alpha} \int_{(1-\alpha)R^B}^{\infty} [y - (1 - \alpha)R^B] f(y)dy - R^E \right)$ . As in Magill et al. (2020), this problem has solution if and only if the zero profit condition is satisfied. This implies that:

$$R^E = \frac{1}{\bar{\alpha}} \int_{(1-\bar{\alpha})R^B}^{\infty} [y - (1 - \bar{\alpha})R^B] f(y)dy. \quad (19)$$

Finally,  $R^h$  is such that it satisfies equation (4).

### 4.3 Equilibrium

The following definitions characterize the equilibrium conditions under the two monetary policy regimes. In particular, conditions (a) are the common ones discussed above. Condition (b) specifies the agents' optimal choices. Condition (c) refers to whether the liquidity requirement is binding or not. Condition (d) derives from the dynamics of the money market, in which cash pools invest in (short-term) government bonds, and they lend the remaining part to the bank. Finally, condition



(e) imposes market clearing for bank equity.

### 4.3.1 Standard Policy

**DEFINITION 1.** *Given the central bank standard monetary policy  $(R^B, R^r, \delta, \bar{\alpha})$ , with interest rate policy  $R^B > R^r$  and balance sheet policy  $(B^{CB}, E^{CB}) = (M + h, 0)$ , the banking equilibrium consists of rates of return  $(R^d, R^h, R^c, R^E)$  and choices  $(d, h, c, e, E, D, M, K)$  such that:*

- (a) *Conditions (4), (13), (14), (15), (16), (18), (19) hold;*
- (b)  *$(d, h)$  is optimal for depositors, given  $(R^d, R^h)$ ;  $c$  is optimal for cash pools, given  $R^c$ ;  $e$  is optimal for investors, given  $(R^B, R^E)$ ;*
- (c)  *$M = \delta d$ ;*
- (d)  *$c_b = c - (B - M - h)$ ;*
- (e)  *$e = E$ .*

### 4.3.2 Quantitative Easing Policy

**DEFINITION 2.** *If the demand for CBDC deposits is such that  $h > \bar{h}$ , given the central bank quantitative easing policy  $(R^B, R^r, \delta, \bar{\alpha})$ , with interest rate policy  $R^B = R^r$  and balance sheet policy  $(B^{CB}, E^{CB}) = (0, M + \tilde{h})$ , then the banking equilibrium consists of rates of return  $(R^d, R^h, R^c, R^E)$  and choices  $(d, h, c, e, E, D, M, K)$  such that:*

- (a) *Conditions (4), (13), (14), (15), (16), (18), (19) hold;*
- (b)  *$(d, h)$  is optimal for depositors, given  $(R^d, R^h)$ ;  $c$  is optimal for cash pools, given  $R^c$ ;  $e$  is optimal for investors, given  $(R^B, R^E)$ ;*
- (c)  *$M \geq \delta d$ ;*
- (d)  *$c_b = c - B$ ;*
- (e)  *$e + M + \tilde{h} = E$ .*

## 4.4 Balance Sheets: Summary

Figure 1 summarizes the model set-up by depicting the balance sheets of all economic actors at equilibrium under standard monetary policy at time 1.

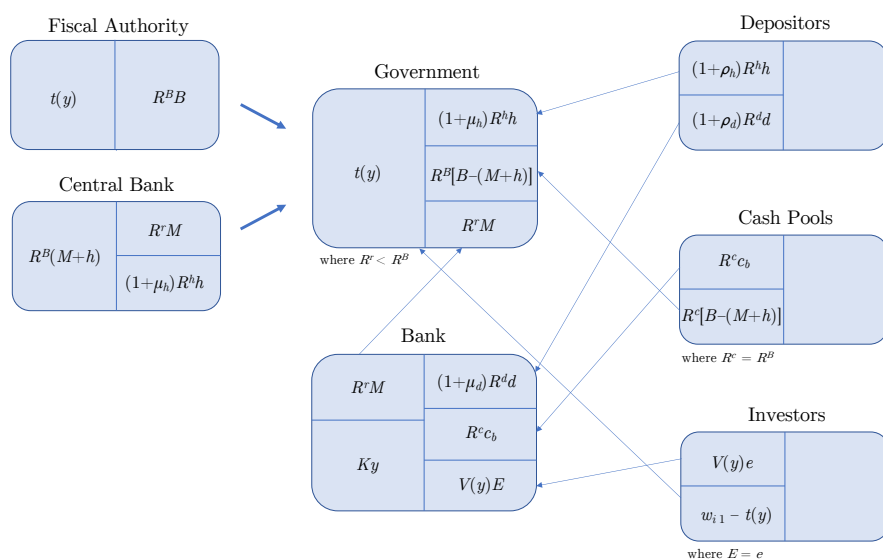


Figure 1: Actors' balance sheets and relationships when the government implements standard monetary policy at time 1.

Similarly, Figure 2 depicts the balance sheets at equilibrium under quantitative easing policy at time 1.

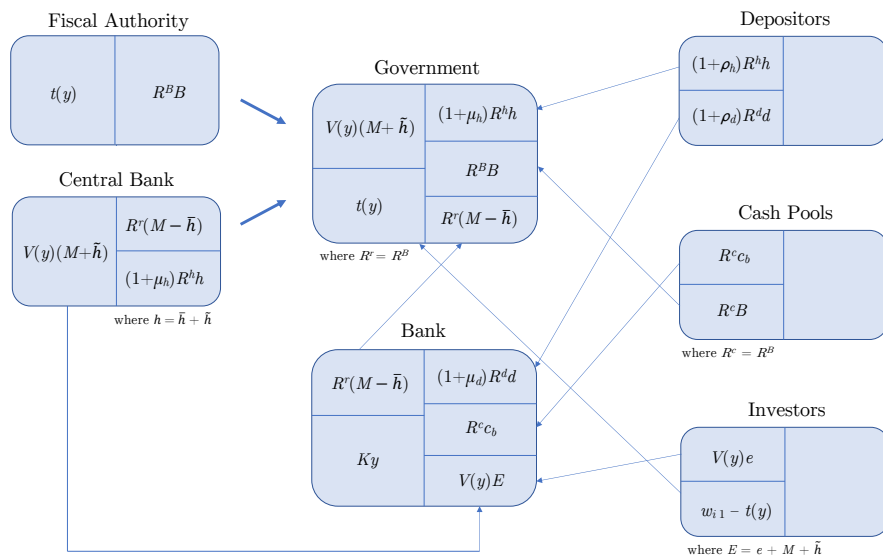


Figure 2: Actors' balance sheets and relationships when the government implements quantitative easing policy at time 1.

## 5 Pareto Optimal Allocations

The maximization of social welfare determines the optimal allocations of resources at time 0 and the optimal weight of each agent. The Pareto problem can be written as:

$$\begin{aligned} \max_{x_{d0}, x_{d1}^d, x_{d1}^h, x_{c0}, x_{c1}, x_{i0}, \{x_{i1}(y)\}_{y \in Y}, K} & \beta_d \left[ u_d(x_{d0}) + (1 + \rho_d)x_{d1}^d + (1 + \rho_h)x_{d1}^h \right] + \\ & + \beta_c \left[ u_c(x_{c0}) + x_{c1} \right] + \\ & + \beta_i \left[ u_i(x_{i0}) + \int_0^\infty x_{i1}(y)f(y)dy \right] \end{aligned} \quad (20)$$

subject to

$$x_{d0} + x_{c0} + x_{i0} + K + G = w_{d0} + w_{c0} + w_{i0}, \quad (21)$$

$$(1 + \mu_d)x_{d1}^d + (1 + \mu_h)x_{d1}^h + x_{c1} + x_{i1}(y) = w_{i0} + Ky, \quad (22)$$

$$x_{d1} = x_{d1}^d + x_{d1}^h, \quad (23)$$

where  $(\beta_d, \beta_c, \beta_i) > 0$  are the relative weights of the agents, and equations (21) and (22) represent the resource constraints at time 0 and 1, respectively. Substituting  $x_{i1}(y)$ <sup>10</sup> in the maximization problem and computing the first order conditions with respect to  $x_{d1}^d$ ,  $x_{d1}^h$  and  $x_{c1}$ , we find that a solution exists only if

$$\beta_c = \beta_i = \frac{1 + \rho_d}{1 + \mu_d} \beta_d = \frac{1 + \rho_h}{1 + \mu_h} \beta_d. \quad (24)$$

Interestingly, equation (24) shows that, at Pareto optimum, the ratio between the benefits and the costs of CBDC and bank deposits have to be the same, i.e.,  $\frac{1 + \rho_d}{1 + \mu_d} = \frac{1 + \rho_h}{1 + \mu_h}$ .

The necessary and sufficient conditions for a Pareto optimal equilibrium can be summarized by

$$\frac{1 + \mu_d}{1 + \rho_d} \frac{\partial u_d}{\partial x_{d0}}(x_{d0}) = \frac{\partial u_c}{\partial x_{c0}}(x_{c0}) = \frac{\partial u_i}{\partial x_{i0}}(x_{i0}) = \mathbb{E}[\tilde{y}], \quad (25)$$

and the resource constraints (21) and (22).<sup>11</sup>

<sup>10</sup>We derive the equation for  $x_{i1}(y)$  from the resource constraint (22).

<sup>11</sup>Equation (25) derives from the first order conditions with respect to  $x_{d0}$ ,  $x_{c0}$ ,  $x_{i0}$ , and  $K$ .

Furthermore, the implicit contributions  $(d^*, h^*, c^*, e^*)$  of all the agents are given by:

$$\begin{aligned}\frac{\partial u_d}{\partial d^*}(w_{d0} - d^* - h^*) &= \frac{1 + \rho_d}{1 + \mu_d} \mathbb{E}[\tilde{y}], \\ \frac{\partial u_d}{\partial h^*}(w_{d0} - d^* - h^*) &= \frac{1 + \rho_h}{1 + \mu_h} \mathbb{E}[\tilde{y}], \\ \frac{\partial u_c}{\partial c^*}(w_{c0} - c^*) &= \mathbb{E}[\tilde{y}], \\ \frac{\partial u_i}{\partial e^*}(w_{i0} - e^*) &= \mathbb{E}[\tilde{y}].\end{aligned}$$

**PROPOSITION 2.** *In any Pareto optimal allocation, the implicit rates of return are:*

$$(1 + \mu_d) R^d = (1 + \mu_h) R^h = R^c = R^E = \mathbb{E}[\tilde{y}]. \quad (26)$$

*Proof.* It follows from the combination of the Pareto optimal allocations and the first order conditions of the single agents' maximization.  $\square$

## 6 Baseline: Standard Policy

This section discusses the introduction of a CBDC when the central bank conducts standard monetary policy. Under standard monetary policy, the central bank issues reserves (to meet liquidity needs of the banking sector), acquires treasuries to influence the amount of wholesale funding to the banking sector via open-market operations, and allows the interest rate on reserves to be lower than the one on government bonds. We refer to this case as the baseline scenario.

### 6.1 Equivalence Theorem

**DEFINITION 3.** *We define the introduction of a CBDC as neutral for equilibrium economic allocations when it has no impact both on the bank's lending ( $\Delta_K = 0$ ) and on taxes ( $\Delta_t = 0$ ).*

Under standard policy, the central bank indirectly channels funds back to the commercial bank via open-market operations. Since the new CBDC deposits increase the amount of liabilities on its balance sheet, when the central bank holds treasuries against CBDC deposits, it decreases the amount of safe assets available to cash pools. This mechanism allows the commercial bank to receive part of the cash pools' savings in the form of debt funding. Thus, when the central bank

only holds treasuries on its asset side of the balance sheet, its pass-through policy is complete as the increase in cash pools funding can fully compensate for the reduction in bank deposits. For this reason, the bank's lending to the economy is not affected by the introduction of a CBDC. However, to have an introduction fully neutral for the economy, the cost of issuing CBDC deposits for the central bank must be equal to the cost of issuing bank deposits for the commercial bank. In fact, let  $\Delta_t^{sB} = [(1 + \mu_h)R^h - (1 + \mu_d)R^d]h$  be the change in taxes given by the introduction of a CBDC under standard policy. We have that it is null when  $(1 + \mu_d)R^d = (1 + \mu_h)R^h$ .

**THEOREM 1.** *Under standard policy, introducing a CBDC is neutral for equilibrium economic allocations when:*

- *the cost of issuing CBDC deposits for the central bank is equal to the cost of issuing bank deposits for the commercial bank:*

$$(1 + \mu_d)R^d = (1 + \mu_h)R^h.$$

*Proof.* See Appendix B for  $\Delta_K^{sB} = 0$  and  $\Delta_t^{sB} = [(1 + \mu_h)R^h - (1 + \mu_d)R^d]h$ , under standard policy. Therefore,  $\Delta_t^{sB} = 0$  when  $(1 + \mu_d)R^d = (1 + \mu_h)R^h$ .  $\square$

Brunnermeier and Niepelt (2019) pinpoint the conditions under which the introduction of a CBDC does not change the equilibrium allocations in the economy. Their theorem states that the equivalence can be obtained only through liquidity and span neutral open-market operations with compensating transfers and a corresponding central bank pass-through policy. In our model, the CBDC design assures liquidity and span neutrality since CBDC deposits have the same liquidity properties as bank deposits and the same payoffs of a portfolio of existing securities. Moreover, under standard policy, the central bank can implement an indirect but complete pass-through policy. If there are no convenience yields ( $\rho_d = \rho_h = 0$ ) and no maintenance costs ( $\mu_d = \mu_h = 0$ ), we also find that  $\Delta_K^{sB} = 0$  and  $\Delta_t^{sB} = 0$ . This setting is consistent with the equivalence presented in Brunnermeier and Niepelt (2019). However, we need to impose further conditions once we consider convenience yields or maintenance costs for deposits or quantitative easing policies. Finally, it is worth noting that without liquidity requirement ( $\delta = 0$ ), the introduction of a CBDC has no impact on the size of the banking sector ( $\Delta_S^{sB} = 0$ ).

## 6.2 Effects on Banks and Government

The main mechanism driving our results is the reduction in bank deposits (as in Klein, Gross, and Sandner, 2020; Kumhof and Noone, 2018).

The introduction of a CBDC under standard monetary policy leads to a decline in deposits by the amount of depositors' savings placed in CBDC ( $h$ ). Since in equilibrium the liquidity constraint is binding, the bank reserves held at the central bank decline by  $\delta h$ , and the size of the bank's balance sheet ( $S$ ) shrinks. Furthermore, since net liabilities shrink and equity remains unchanged, the commercial bank's leverage declines.<sup>12</sup> The central bank's treasuries holding increases by  $h$  and declines by  $\delta h$ , as the reduction in bank deposits is followed by a decrease in central bank reserves ( $M$ ). This additional demand for treasuries  $((1 - \delta)h)$  from the central bank (to back CBDC deposits) crowds out cash pools that cannot buy as many treasuries as they desire. Consequently, cash pools compensate by investing  $(1 - \delta)h$  more in bank debt. The amount of investable funds  $D$  for the bank does not change, as the decrease in deposits is fully compensated by the reduction in reserves and the increase in cash pool funding. Therefore, the bank does not change the amount invested in risky loans ( $K$ ). Bankruptcy costs ( $\phi$ ) remain the same.

The effect on the government sector depends on the cost of issuing CBDC deposits, namely interest rate ( $R^h$ ) and management cost  $(1 + \mu_h)$ . The impact on seignorage revenues is determined by the difference between the cost of deposits for the central bank,  $(1 + \mu_h)R^h$ , and the commercial bank,  $(1 + \mu_d)R^d$ . When the cost of deposits for the central bank is higher than for the commercial bank (i.e.,  $(1 + \mu_h)R^h > (1 + \mu_d)R^d$ ), seignorage revenues decrease, and taxes increase. Vice versa when  $(1 + \mu_h)R^h < (1 + \mu_d)R^d$ .

## 7 Quantitative Easing Policy

### 7.1 Institutional Settings

When conducting quantitative easing policies, the central bank creates new reserves and uses them to purchase assets. The result is an increase in the central bank's balance sheet size and an abundance of reserves in the banking system (Joyce, Miles, Scott, and Vayanos, 2012). Such policies

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<sup>12</sup>We define leverage as bank liabilities divided by the size of the balance sheet, i.e.,  $(d + c_b)/(d + c_b + E)$ .

aim to support the financial system in periods of distress and to ease the pressure on governments and banks. Furthermore, banks are subject to liquidity requirements, and the abundance of reserves should help to boost lending. However, in the US, the launch of quantitative easing programs in 2008 has led to a significant amount of excess reserves, i.e., reserves above liquidity requirements. Figure 3 shows the evolution of the FED’s balance sheet size and the amount of excess reserves in the system between 2006 and 2021.

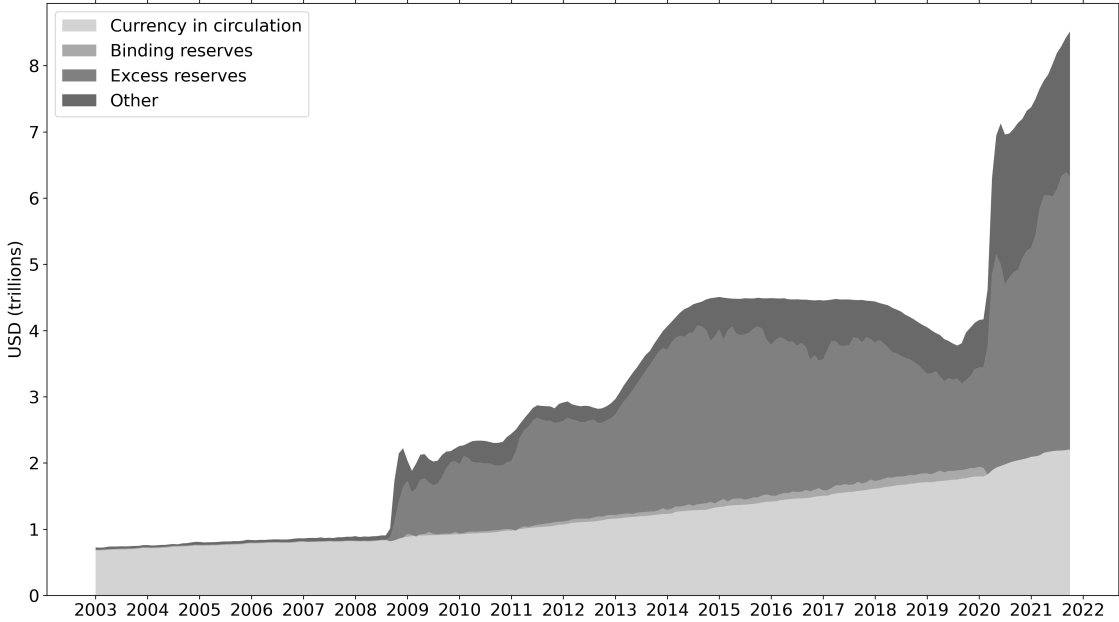


Figure 3: FED’s total liabilities decomposition. Source: FRED, Federal Reserve Bank of St. Louis, December 2021.

The chart clearly shows the strong link between quantitative easing and excess reserves. When central banks want to taper quantitative easing policies and decrease the size of their balance sheets, they sell assets to the market and cancel outstanding reserves in exchange. Reducing the size of the balance sheet implies reducing both assets and liabilities at the same time.

### 7.2 Equivalence Theorem

Under QE policy, the central bank holds risky assets, and it sets the interest rate on treasuries equal to the one on bank reserves (i.e.,  $R^B = R^r$ ). With the introduction of a CBDC, part of depositors’ savings is transferred from bank deposits into CBDC deposits. Therefore, the com-

mercial bank needs to transfer resources to the central bank. When the liquidity requirement is binding ( $M = \delta d$ ), the commercial bank can only liquidate part of its assets in favor of the central bank. In this case, the size of the central bank's balance sheet increases by the amount of CBDC deposits. However, under QE policy, when the liquidity requirement is not binding ( $M > \delta d$ ), the commercial bank can also swap excess reserves into CBDC deposits. The central bank's balance sheet does not change in size, as one type of liabilities (excess reserves) is transformed into another (CBDC deposits).

To better understand this mechanism, consider a depositor withdrawing cash from her bank deposit. The bank has to purchase banknotes from the central bank, and their face value is deducted from the bank's reserves at the central bank. The size of the commercial bank's balance sheet decreases by the amount of cash withdrawn. The operation is neutral for the central bank's balance sheet size, as one liability is transformed into another, i.e., reserves into banknotes. A CBDC would work in the same way from an accounting perspective. When a depositor converts one unit of bank deposit into CBDC, the commercial bank reduces its reserve by one unit, and the central bank converts one unit of bank reserves into CBDC deposits. This mechanism is neutral as far as the demand for CBDC deposits is lower than the amount of excess reserves in the system. If the demand for CBDC deposits is higher, the commercial bank will need to liquidate assets in favor of the central bank, increasing the size of its balance sheet. Figure 4 provides a graphical representation of the mechanism described.



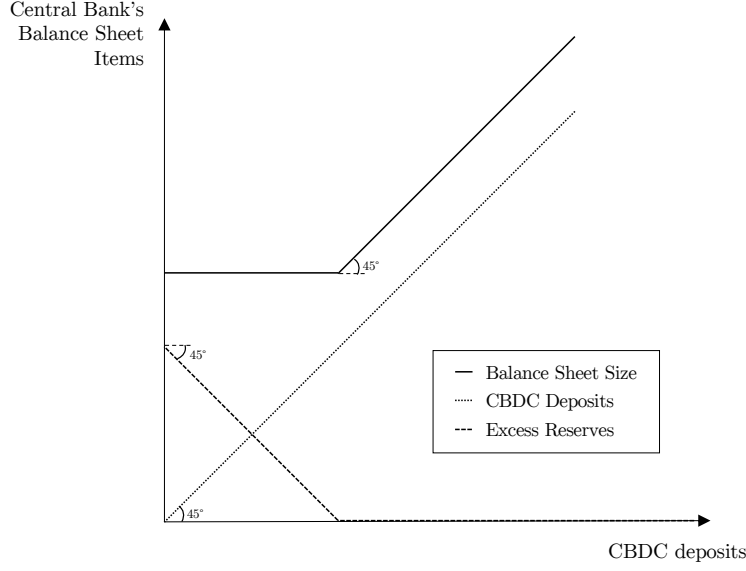


Figure 4: Relationships between CBDC deposits, excess reserves, and central bank's balance sheet size. If the liquidity requirement is not binding, the commercial bank swaps excess reserves for CBDC deposits. In this case, the size of the central bank does not change as one type of liability is simply transformed into another. Once the liquidity requirement is met, the commercial bank liquidates assets in favor of the central bank, increasing its size.

When possible, the commercial bank prefers to liquidate reserves rather than other assets. In our model, when the commercial bank decides to transfer assets in favor of the central bank, its expected profits change:

$$\Delta\pi' = h \left[ (1 + \mu_d)R^d - \int_{\hat{y}}^{\infty} yf(y)dy \right].$$

Conversely, when the commercial bank reduces its excess reserves to convert its deposits into CBDC ones, the difference in expected profits is

$$\Delta\pi'' = h \left[ (1 + \mu_d)R^d - R^r \right].$$

By assumption,  $R^r < \int_{\hat{y}}^{\infty} yf(y)dy$  as an incentive for the commercial bank to invest in risky projects. This implies that  $\Delta\pi'' > \Delta\pi'$ , meaning that the commercial bank prefers to reduce its excess reserves when it maximizes its expected profits. However, the commercial bank can reduce its reserves only until the liquidity requirement is binding. After that point, the commercial bank

has no choice but to liquidate its assets in favor of the central bank.

When the liquidity requirement is not binding, and the commercial bank can reduce its excess reserves, the reduction in bank deposits is fully compensated by a decrease in reserves. This mechanism leaves the amount of lending to the economy unchanged. However, since the central bank transforms one type of liabilities into another, there is no impact on taxes only if the cost of both these types of liabilities is equal. We define  $\bar{h}$  as the maximum demand for CBDC deposits for which the commercial bank can swap excess reserves. Therefore,  $\bar{h}$  is such that the liquidity requirement is binding,  $M - \bar{h} = \delta(d - \bar{h})$ , i.e., the maximum amount for which the reduction in reserves fully compensates for the reduction in deposits. We have:

$$\bar{h} = \frac{M - \delta d}{1 - \delta}. \quad (27)$$

If the demand for CBDC deposits exceeds this threshold ( $h > \bar{h}$ ), then the commercial bank has to liquidate assets in favor of the central bank. We define  $\tilde{h} = h - \bar{h}$  as the reduction in bank deposits. Since the liquidity requirement is now binding, the bank reserves decrease by an additional  $\delta\tilde{h}$  on top of  $\bar{h}$ .

**THEOREM 2.** *Under QE policy, the introduction of a CBDC can be neutral for equilibrium economic allocations when:*

- *the demand for CBDC deposits is lower than the amount of excess reserves:*

$$h < \bar{h};$$

- *the cost of reserves for the central bank is equal to the cost of CBDC deposits:*

$$R^r = (1 + \mu_h)R^h.$$

*Proof.* If the demand for CBDC deposits is lower than the amount of excess reserves, the commercial bank can swap excess reserves for CBDC deposits. In this way, the amount of lending to the economy remains unchanged because the reduction in reserves fully compensates for the reduction in deposits ( $\Delta_K^q = 0$ ). Since the central bank transforms one type of liabilities into another, the impact on

taxes is given by:  $\Delta_t^q = [(1 + \mu_h)R^h - R^r] h$ . This is null only when  $R^r = (1 + \mu_h)R^h$ .  $\square$

It is worth noting that if the commercial bank converted its excess reserves into CBDC deposits, it would be much harder for the central bank to revert QE programs. Having a large number of small depositors as counterparts is not the same as having a limited number of financial institutions. Depositors would use a CBDC for payments and savings and would probably be much less elastic than financial institutions. It is reasonable to assume that the CBDC deposits' elasticity would be similar to bank deposits' one, which tends to be low (Chiu and Hill, 2015). Rolling back quantitative easing policies means selling assets on the one side and canceling liabilities on the other. An inelastic liability side would render quantitative easing policies semi-permanent.

**OBSERVATION 1.** *The introduction of a CBDC under QE policy might render QE quasi-permanent, or at least very hard to roll back.*

### 7.3 Effects on Banks and Government

The rest of the discussion is under the hypothesis that demand for CBDC exceeds excess reserves, i.e.,  $h > \bar{h}$ . In this case, the liquidity requirement for the commercial bank is binding, and thus a reduction in deposits is not neutral. Depositors switch from bank deposits to CBDC, and the central bank holds risky securities against them. The commercial bank loses deposits, which are a cheap source of funding, and receives equity injections, which are a more costly one. The result is a reduction in lending.

To analyze the impact on the government sector, we need to introduce an additional concept. As proven by Magill et al. (2020), no equilibrium is Pareto optimal under standard policy. However, the central bank can implement a Pareto optimal equilibrium under IR-QE policy by setting the capital requirement above a certain threshold  $\alpha_c$ . For  $\bar{\alpha} > \alpha_c$ , the bank has enough capital to absorb the losses even when  $\tilde{y}$  is  $\underline{y}$ , its lowest possible realization. With such macroprudential policy, there are no bankruptcies, and the equilibrium is Pareto optimal. The central bank holds riskier assets on its balance sheet, with higher expected seignorage revenues. Seignorage volatility increases as the central bank holds more risky assets on its balance sheet. Consequently, taxes are lower in expectation but more volatile. When  $\bar{\alpha} < \alpha_c$ , the impact on the government sector depends on the relative levels of  $R^B$ ,  $R^h$ , and  $V(y)$ . In this case, the impact on seignorage is ambiguous.

## 8 Conclusion

When central banks issue a CBDC, the equilibrium effects on the economy largely depend on the ongoing monetary policy. In this paper, we investigate and compare two illustrative cases, the first where the central bank pursues standard monetary policy and the second where it implements QE. Realistically, central banks will conduct mixed policies; however, our paper sheds light on the key equilibrium mechanisms that affect the bank and government sectors.

First, we find that the economic effects do indeed differ depending on the interaction between the ongoing monetary policy and the kind of CBDC introduced. For instance, a CBDC can reduce lending to the economy under QE but is neutral under standard monetary policy. This fact can be regarded as a warning that the debate over CBDCs cannot be held in a vacuum, as a CBDC will interact with the other central bank policies.

Second, the impact of introducing a CBDC while the central bank is conducting QE depends on the amount of excess reserves in the system. Banks optimally transfer excess reserves to depositors when creating new CBDC accounts. Therefore, a CBDC has no impact on the banking sector as long as the demand for CBDC does not exceed excess reserves. Above this threshold, introducing a CBDC is problematic as banks lose a cheap source of funding, which is not replaced. Furthermore, it is worth noting that substituting banks with depositors on the liability side of the central bank's balance sheet is not without consequences. Depositors tend to be inelastic, so it would be difficult for the central bank to reduce the size of its balance sheet when reverting QE policies. In this sense, introducing a CBDC might render QE quasi-permanent.

These findings are relevant for policymakers in charge of designing future digital currencies. CBDCs have the potential to radically change monetary policy transmission, and central banks should have a comprehensive approach that considers the interaction with current monetary policies.

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## A Hybrid Monetary Policies

### A.1. Standard Policy with CBDC backed by Risky Securities

**DEFINITION 4.** *Given the central bank standard policy  $(R^B, R^r, \delta, \bar{\alpha})$ , with interest rate policy  $R^B > R^r$  and balance sheet policy  $(B^{CB}, E^{CB}) = (M, h)$ , the banking equilibrium consists of rates of return  $(R^d, R^h, R^c, R^E)$  and choices  $(d, h, c, e, E, D, M, K)$  such that:*

- (a) *Conditions (4), (13), (14), (15), (16), (18), (19) hold;*
- (b)  *$(d, h)$  is optimal for depositors, given  $(R^d, R^h)$ ;  $c$  is optimal for cash pools, given  $R^c$ ;  $e$  is optimal for investors, given  $(R^B, R^E)$ ;*
- (c)  $M = \delta d$ ;
- (d)  $c_b = c - (B - M)$ ;
- (e)  $e + h = E$ .

The introduction of a CBDC leads to a decline in deposits by the amount of depositors' savings placed in CBDC ( $h$ ). Since the liquidity constraint is binding in equilibrium, the bank reserves decline by  $\delta h$ , and the size of the bank ( $S$ ) shrinks. In this scenario, the cash pools do not compensate for the reduction in deposits, and the bank reduces the investment in risky loans. As a consequence, bankruptcy costs ( $\phi$ ) decrease. The bank's leverage declines as there is a reduction in the net liabilities while the equity remains unchanged.

Since CBDCs are backed by risky securities, the effect on seignorage is ambiguous. The central bank loses  $(R^B - R^r)\delta h$  as commercial bank reserves decline, while it gains  $(V(y) - (1 + \mu_h)R^h)h$  from issuing CBDC deposits. Nonetheless, we can argue that seignorage revenues increase for reasonable values of liquidity requirement and interest rates. Seignorage revenues are more volatile, as the returns on the central bank's assets partially depend on  $\tilde{y}$ .

### A.2. Quantitative Easing Policy with CBDC backed by Treasuries

**DEFINITION 5.** *If the demand for CBDC deposits is such that  $h > \bar{h}$ , given the central bank quantitative easing policy  $(R^B, R^r, \delta, \bar{\alpha})$ , with interest rate policy  $R^B = R^r$  and balance sheet policy  $(B^{CB}, E^{CB}) = (h, M - \bar{h})$ , the banking equilibrium consists of rates of return  $(R^d, R^h, R^c, R^E)$  and choices  $(d, h, c, e, E, D, M, K)$  such that:*

- (a) Conditions (4), (13), (14), (15), (16), (18), (19) hold;
- (b)  $(d, h)$  is optimal for depositors, given  $(R^d, R^h)$ ;  $c$  is optimal for cash pools, given  $R^c$ ;  $e$  is optimal for investors, given  $(R^B, R^E)$ ;
- (c)  $M \geq \delta d$ ;
- (d)  $c_b = c - (B - h)$ ;
- (e)  $e + M - \bar{h} = E$ .

Under QE policy, there is a positive amount of excess reserves in the system due to the asset-purchase programs. Thus, the liquidity requirement is not binding. When the central bank decides to hold treasuries against CBDC deposits, the amount of risky investments in the economy ( $K$ ) increases but not the size of the bank ( $S$ ). This happens because the decrease in deposits ( $d$ ) is fully offset by increased cash pools' funding ( $c_b$ ) as there are  $h$  less bonds available in the economy. At the same time, the reduction in deposits allows the commercial bank to further decrease its reserves ( $M$ ), increasing the amount of investable debt ( $D$ ). Thus, the commercial bank has more funding to allocate in risky loans ( $K$ ). The difference with the standard policy setting is that, in this scenario, bank reserves are not backed by treasuries but by bank equity. Therefore, a reduction in bank reserves has no impact on the treasury market, and it does not allow cash pools to purchase more treasuries. We find that the bank's leverage decreases and that the larger investable debt increases bankruptcy costs ( $\phi$ ).

In this scenario, since some bank reserves are swapped into CBDC deposits, and some are simply reduced, the central bank asset side is less risky. Therefore, the introduction of CBDCs backed by treasuries, under QE policy, reduces the seigniorage volatility. Consequently, the economy benefits from more stable taxes.

## B CBDC Equilibrium Effects - Proofs

### B.0. Nomenclature

The superscripts  $s$  and  $q$  denote the standard policy and the QE policy scenarios, respectively, without the CBDC. In this section, we always consider the QE policy when the amount of CBDC deposits exceeds the amount of excess reserves in the economy ( $h > \bar{h}$ ) and the liquidity requirement is binding. With the introduction of a CBDC, a  $B$  superscript indicates when the central bank



decides to hold government bonds against CBDC deposits and a  $E$  superscript when the CBDC is backed by bank equity. The  $\Delta_x^{sB}$  is defined as the difference between the generic variable  $x$  in the case of standard policy with CBDC backed by treasuries and the same variable in a scenario with the same policy but no CBDC:  $\Delta_x^{sB} = x^{sB} - x^s$ . Similarly, the differences  $\Delta_x^{sE} = x^{sE} - x^s$ ,  $\Delta_x^{qB} = x^{qB} - x^q$ , and  $\Delta_x^{qE} = x^{qE} - x^q$  illustrate the variation with the respective baseline scenarios.

## B.1. Agents' optimal choices

We assume that the monetary policy interest rates ( $R^r, R^B$ ), the amount of treasuries in the economy ( $B$ ), the unit cost of deposits ( $\mu_d$ ), and their convenience yields ( $\rho_d$ ) do not change with the introduction of a CBDC. If we also assume that the initial endowments of the agents do not change, it implies that the optimal amounts of savings for depositors and cash pools remain the same with the introduction of a CBDC.

## B.2. Bank deposits and reserves

In scenarios without the CBDC, bank deposits are the same:  $d^s = d^q$ . With the introduction of a CBDC, we always have that part of the depositors' savings goes to the central bank and, therefore, bank deposits decrease:

$$\begin{aligned} d^{sB} &= d^{sE} = d^s - h, \\ d^{qB} &= d^{qE} = d^q - h, \end{aligned}$$

with  $\Delta_d^{sB} = \Delta_d^{sE} = \Delta_d^{qB} = \Delta_d^{qE} = -h < 0$ .

The amount of bank reserves in standard policy is given by  $M^{sB} = M^{sE} = \delta(d^s - h) = M^s - \delta h$ , because the liquidity requirement is binding. Under QE policy, the commercial bank swaps  $\bar{h}$  excess reserves into CBDC deposits. After this point, the liquidity requirement is binding, and at each further unit of bank deposits reduction corresponds  $\delta$  units of reserves reduction. We have that  $M^{qB} = M^{qE} = M^q - \bar{h} - \delta\tilde{h}$ , where  $\tilde{h} = h - \bar{h}$ . We obtain  $\Delta_M^{sB} = \Delta_M^{sE} = -\delta h < 0$ , and  $\Delta_M^{qB} = \Delta_M^{qE} = -h + (1 - \delta)\tilde{h} < 0$ .

### B.3. Cash pool funding

The cash pool funding is given by the cash pool demand of savings, minus all the available government bonds in the economy. The amount of treasuries available for cash pools is given by the amount of bonds issued by the government minus the ones bought by the central bank. In standard policy  $c_b^s = c - (B - M^s)$ , while under QE policy the central bank does not hold any bond:  $c_b^q = c - B$ .

With the introduction of a CBDC backed by treasuries in standard policy, the cash pool funding becomes  $c_b^{sB} = c^s - (B^s - M^{sB} - h)$ , which translate in an increase of  $\Delta_{c_b}^{sB} = c_b^{sB} - c_b^s = (1 - \delta)h > 0$ . When the CBDC deposits are backed by equity, the mechanism is similar to before, i.e.,  $c_b^{sE} = c^s - (B^s - M^{sE})$ , which corresponds to a decline of  $\Delta_{c_b}^{sE} = c_b^{sE} - c_b^s = -\delta h < 0$ , given by the decrease in the reserves.

Under QE policy, the bank's cash pool funding when the central bank holds bonds against CBDC deposits is  $c_b^{qB} = c^q - (B^q - h)$ , with an increase of  $\Delta_{c_b}^{qB} = c_b^{qB} - c_b^q = \tilde{h} > 0$ . The funding does not change if the central bank decides to hold only equity:  $c_b^{qE} = c^q - B^q$ , with  $\Delta_{c_b}^{qE} = c_b^{qE} - c_b^q = 0$ .

### B.4. Investable debt, bank equity and risky investment

As in equation (15), we define the investable debt of the bank as all the debt fundings that can be invested in the risky technology, but the reserves. In all scenarios, the investable debt is determined by:

$$D = d + c_b - M.$$

Under standard policy with CBDC backed by treasuries, there is no difference with the baseline:  $\Delta_D^{sB} = 0$ . However, if the central bank decides to allocate these funds in bank equity, then the investable debt declines by  $\Delta_D^{sE} = -h < 0$ . On the other hand, under quantitative easing policy, the CBDC investment in the safe asset translates in an increase in the debt that the banks can use to fund the risky technology,  $\Delta_D^{qB} = h - (1 - \delta)\tilde{h} > 0$ , while an investment in bank equity decreases it,  $\Delta_D^{qE} = -(1 - \delta)\tilde{h} < 0$ .

Let's define  $\gamma = \frac{\bar{\alpha}}{1 - \bar{\alpha}}$  for simplicity in the notation. At equilibrium, as in equation (18), the

amount of bank equity is fixed at:

$$E = \gamma D,$$

and, because of condition (16), the risky investment is always given by:

$$K = (1 + \gamma)D.$$

For both equity and risky investment the results are the same as for the investment debt, but scaled by  $\gamma$  and  $1 + \gamma$ , respectively.

### B.5. Bank size

We measure the bank size as the sum of all its liabilities or all its assets:

$$S = d + c_b + E = M + K.$$

The introduction of a CBDC in standard policy always leads to a decline in the bank size. In fact,  $\Delta_S^{sB} = -\delta h < 0$  and  $\Delta_S^{sE} = -(1 + \delta + \gamma)h < 0$ . Instead, in a QE policy setting, we have that  $\Delta_S^{qB} = \gamma[h(1 - \delta)\tilde{h}] > 0$  and  $\Delta_S^{qE} = -h - \gamma(1 - \delta)\tilde{h} < 0$ .

### B.6. Bankruptcy costs

Let  $\hat{y}$  be the minimum return on the risky technology that allows the bank to repay its creditors. It follows that  $\hat{y}$  is such that  $K\hat{y} + MR^r = dR^d(1 + \mu_d) + c_bR^c$  and the bank is solvent for  $y > \hat{y}$ . The bankruptcy costs are then given by:

$$\phi = dR^d(1 + \mu_d) + c_bR^c - MR^r - Ky,$$

when  $y \leq \hat{y}$ . At equilibrium, it holds that  $R^c = R^B$  as in (13),  $R^d(1 + \mu_d) = (1 - \delta)R^B + \delta R^r$  for condition (14), and  $D = d + c_b - M = \frac{K}{(1 + \gamma)}$  as defined in section B.3. This implies that  $\phi = DR^B - Ky$  and  $\hat{y} = \frac{R^B}{1 + \gamma}$ . Hence, the bankruptcy costs can be written as:

$$\phi = D[R^B - (1 + \gamma)y].$$

For this reason, all the results are the same as for the investable debt  $D$ , but scaled by  $[R^B - (1 + \gamma)y]$ , that is always positive in bankruptcy because  $y \leq \hat{y}$ .

## B.7. Seignorage

The seignorage is defined as the profit made by the government. In standard policy, this profit is given by  $\theta^s = (R^B - R^r)M^s$ , while under quantitative easing policy we have  $\theta^q = (V(y) - R^B)M^q$ . With the introduction of a CBDC, there is an additional term that depends on what the central bank decides to hold against the new funds. If CBDC deposits are backed by bonds, then the seignorage has an additional profit of  $(R^B - (1 + \mu_h)R^h)$  per unit of CBDC. Instead, if they are backed by bank equity, then the additional profit per unit of CBDC becomes  $(V(y) - (1 + \mu_h)R^h)$ .

Therefore, with the introduction of the CBDC in the standard policy we have that  $\theta^{sB} = (R^B - R^r)M^{sB} + (R^B - (1 + \mu_h)R^h)h$ , and  $\theta^{sE} = (R^B - R^r)M^{sE} + (V(y) - (1 + \mu_h)R^h)h$ , with a difference from the baseline of  $\Delta_\theta^{sB} = [(1 + \mu_d)R^d - (1 + \mu_h)R^h]h$ , and  $\Delta_\theta^{sE} = -(R^B - R^r)\delta h + (V(y) - (1 + \mu_h)R^h)h$ , respectively.

Similarly, under quantitative easing policy the seignorage is computed as  $\theta^{qB} = (V(y) - R^B)M^{qB} + (R^B - (1 + \mu_h)R^h)h$  in the scenario with a CBDC backed by safe assets, and as  $\theta^{qE} = (V(y) - R^B)M^{qE} + (V(y) - (1 + \mu_h)R^h)h$  for equity held against the CBDC. The differences with the baseline scenario are respectively  $\Delta_\theta^{qB} = (R^B - (1 + \mu_h)R^h)h - (V(y) - R^B)(h - (1 - \delta)\tilde{h})$ , and  $\Delta_\theta^{qE} = (R^B - (1 + \mu_h)R^h)h + (V(y) - R^B)(1 - \delta)\tilde{h}$ . In the quantitative easing policy, Pareto-optimum can be achieved. As  $\mathbb{E}[V(y)] = R^E$  by definition,  $R^c = R^B$  at the banking equilibrium, and  $R^E = R^c = (1 + \mu_h)R^h$  at Pareto-optimum, it follows that:

$$\mathbb{E}[\Delta_\theta^{qB}] = \mathbb{E}[\Delta_\theta^{qE}] = 0.$$

It is worth noting that whenever the central bank decides to invest in bank equity, the seignorage is no more deterministic because it depends on the realization of the payoff of the risky technology. Therefore, the only scenarios in which the seignorage volatility is null are standard policy without CBDC and with CBDC backed by bonds:  $\sigma_\theta^s = \sigma_\theta^{sB} = 0$ . If the central bank decides to hold equity against CBDC deposits, we have that  $\sigma_\theta^{sE} = h\sigma_{V(y)}$ , where  $\sigma_{V(y)}$  is the volatility of the equity payoff. Under quantitative easing policy, the seignorage is always volatile and, specifically,

we have that  $\sigma_\theta^q = M^q \sigma_{V(y)}$ . Introducing a CBDC has opposite effects to the seigniorage volatility depending on where the central bank decides to invest the funds. If the CBDC deposits are backed by treasuries, then  $\sigma_\theta^{qB} = M^{qB} \sigma_{V(y)}$ , reducing the volatility:  $\Delta_{\sigma_\theta}^{qB} = -(h - (1 - \delta)\tilde{h}) \sigma_{V(y)} < 0$ . On the other hand, holding bank equity increases the volatility of the seigniorage, as  $\sigma_\theta^{qE} = M^{qE} \sigma_{V(y)}$ , and  $\Delta_{\sigma_\theta}^{qE} = (1 - \delta)\tilde{h} \sigma_{V(y)} > 0$ .

## B.8. Taxes

Taxes are defined in Section 3.6:

$$t(y) = \begin{cases} R^B B - \theta, & \text{if } y > \hat{y} \\ R^B B - \theta + \phi, & \text{if } y \leq \hat{y} \end{cases} = R^B B - \theta + \phi \mathbb{1}_{y \leq \hat{y}}.$$

For this reason, all the differences in all scenarios can be determined as  $\Delta_t = \Delta_\phi \mathbb{1}_{y \leq \hat{y}} - \Delta_\theta$ .

## B.9. Summary tables

Table 1 and Table 2 summarize the effects of the introduction of a CBDC under consistent and hybrid policies, respectively.

Table 1: Summary of the effects of the introduction of a CBDC with consistent policies. In standard policy, the central bank holds treasuries against CBDC deposits, while under quantitative easing policy, it holds risky securities. The effects are defined as first-order differences with the economy's outcomes before the introduction of the CBDC under the same monetary policy.

	$\Delta^{sB}$	$\Delta^{qE}$
<i>Deposits</i> ( $d$ )	$-h$	$-h$
<i>Bank reserves</i> ( $M$ )	$-\delta h$	$-h + (1 - \delta)\tilde{h}$
<i>Cash pool funding</i> ( $c_b$ )	$(1 - \delta)h$	$0$
<i>Investable debt</i> ( $D$ )	$0$	$-(1 - \delta)\tilde{h}$
<i>Equity</i> ( $E$ )	$0$	$-\gamma(1 - \delta)\tilde{h}$
<i>Risky investment</i> ( $K$ )	$0$	$-(1 + \gamma)(1 - \delta)\tilde{h}$
<i>Bank size</i> ( $S$ )	$-\delta h$	$-h - \gamma(1 - \delta)\tilde{h}$
<i>Bankruptcy costs</i> ( $\phi$ )	$0$	$-[R^B - (1 + \gamma)y](1 - \delta)\tilde{h}$
<i>Seigniorage</i> ( $\theta$ )	$[(1 + \mu_d)R^d - (1 + \mu_h)R^h]h$	$(R^B - (1 + \mu_h)R^h)h + (V(y) - R^B)(1 - \delta)\tilde{h}$
<i>Seigniorage volatility</i> ( $\sigma_\theta$ )	$0$	$(1 - \delta)\tilde{h} \sigma_{V(y)}$
<i>Taxes</i> ( $t$ )	$\Delta_\phi^{sB} \mathbb{1}_{y \leq \hat{y}} - \Delta_\theta^{sB}$	$\Delta_\phi^{qE} \mathbb{1}_{y \leq \hat{y}} - \Delta_\theta^{qE}$

Table 2: Summary of the effects of the introduction of a CBDC with hybrid policies. In standard policy, the central bank holds risky securities against CBDC deposits, while under quantitative easing policy, it holds treasuries. The effects are defined as first-order differences with the economy's outcomes before the introduction of the CBDC under the same baseline monetary policy.

	$\Delta^{sE}$	$\Delta^{qB}$
<i>Deposits</i> ( $d$ )	$-h$	$-h$
<i>Bank reserves</i> ( $M$ )	$-\delta h$	$-h + (1 - \delta)\tilde{h}$
<i>Cash pool funding</i> ( $c_b$ )	$-\delta h$	$h$
<i>Investable debt</i> ( $D$ )	$-h$	$h - (1 - \delta)\tilde{h}$
<i>Equity</i> ( $E$ )	$-\gamma h$	$\gamma(h - (1 - \delta)\tilde{h})$
<i>Risky investment</i> ( $K$ )	$-(1 + \gamma)h$	$(1 + \gamma)(h - (1 - \delta)\tilde{h})$
<i>Bank size</i> ( $S$ )	$-(1 + \delta + \gamma)h$	$\gamma(h - (1 - \delta)\tilde{h})$
<i>Bankruptcy costs</i> ( $\phi$ )	$-[R^B - (1 + \gamma)y]h$	$[R^B - (1 + \gamma)y](h - (1 - \delta)\tilde{h})$
<i>Seigniorage</i> ( $\theta$ )	$-(R^B - R^r)\delta h + (V(y) - (1 + \mu_h)R^h)h$	$(R^B - (1 + \mu_h)R^h)h - (V(y) - R^B)(h - (1 - \delta)\tilde{h})$
<i>Seigniorage volatility</i> ( $\sigma_\theta$ )	$h \sigma_{V(y)}$	$-(h - (1 - \delta)\tilde{h}) \sigma_{V(y)}$
<i>Taxes</i> ( $t$ )	$\Delta_\phi^{sE} \mathbb{1}_{y \leq \hat{y}} - \Delta_\theta^{sE}$	$\Delta_\phi^{qB} \mathbb{1}_{y \leq \hat{y}} - \Delta_\theta^{qB}$