Optimal Central Bank Balance Sheets*

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Abstract

Can central bank balance sheet policies play a role away from the effective lower bound on interest rates? We extend the canonical DSGE model with financial frictions to include a fully specified central bank. We find that central bank balance sheet size and composition has a macroprudential effect on monetary policy efficacy. Optimal policy reduces bank duration risk, increasing their resilience to shocks. Short-run balance sheet policies offer no additional advantage if the optimal long-run balance sheet is in place. Our results highlight the importance of government debt maturity and bank regulation in determining optimal central bank balance sheets.

Keywords: optimal monetary policy, central bank balance sheet, government debt, reserves, financial frictions, macroprudential.

JEL Classification Numbers: E42, E44, E51, E52, G21

*The views in this article are those of the authors and should not be attributed to the BIS. All errors are our own.

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1 Introduction

Confronted with the effective lower bound on interest rates in responding to crises, central banks have adopted a number of unconventional monetary policy interventions to address financial market disruptions and to complement conventional monetary policy. Prominent among these have been balance sheet policies: asset-purchase programs and lending operations matched by an increase in bank reserves. As a result of these programs, central bank balance sheets have swelled to unprecedented levels.

As central banks unwind their balance sheets and embark on quantitative tightening (QT), there are two major questions: What is the optimal size and composition of the long-run central bank balance sheet? Is there room for balance sheet policies in the policy toolkit away from the effective lower bound on interest rates? While there is a large literature analyzing the effectiveness of QE programmes, the literature on the optimal long-run size of the central bank balance sheet is nascent.

Our paper develops a DSGE model with realistic features of the monetary system to study the optimal size and composition of the central bank balance sheet and the role for balance sheet policies away from the effective lower bound. The central bank operates with a fully specified budget constraint such that asset purchases (government bonds) must be primarily conducted by issuing reserves or cash. The model features financial and monetary frictions as well as price stickiness. The central bank implements optimal policy that maximizes welfare.

Optimal central bank balance sheet policy weighs the relative value of reserves and government bonds for the economy, and allocates duration risk optimally among agents. Our first result is if the value of reserves for banks is larger than that of (long-term) government debt in alleviating financial frictions, a large central bank balance sheet is desirable (see also Vissing-

Jørgensen, 2023) *in the long run.*Our second finding is that the long-run size of the central bank balance sheet also plays a crucial role in how the economy responds to shocks. Long-run central bank size matters over both medium- and short-term. Balance sheet policy reallocates duration risk in the economy since government debt maturity typically exceeds on average the maturity of reserves. Upon a shock, the valuation effect of the assets and liabilities held by banks have more material effects than the portfolio adjustment induced by the (balance-sheet) response of the central bank. Thirdly, we find that when the long-run central bank balance sheet is not set optimally, it is welfare-improving for the central bank to pursue active balance-sheet policies in addition to standard rate adjustments.

Taken all together, the long-run balance sheet of the central bank can be seen from a macroprudential policy perspective. For a given supply of public debt and preferences (habitat), banks can end up with a portfolio that is less resilient to aggregate shocks than what a policymaker could envisage. By supplying reserves and absorbing government debt in circulation, the central bank can induce banks to move to a less vulnerable asset structure. Once this is achieved, conventional monetary policy becomes more effective. This is because a lower duration risk implies that banks' balance-sheet valuation changes would be smaller, mitigating the financial-friction wedge and thus improving the monetary policy trade-offs. Said differently, the optimal long-run central bank balance sheet itself has desirable short-run effects in response to shocks through its macroprudential implications.

The model is based on a closed sticky-price production economy with financial intermediation à la Gertler and Karadi (2011). Our main modelling departure from the literature is fully specifying the balance sheet of the central bank, whereby the central bank purchases assets by issuing reserves and cash, facing price-sensitive demands and with limited room for variation in central bank capital. As a result, changes in the size and composition of the central bank's balance sheet has a market impact with real effects.

The key financial friction in the model is captured by banks facing a limited commitment problem as in Gertler and Kiyotaki (2010). This leads to an incentive compatibility constraint (ICC) that aligns the incentives of the banks and their creditors by limiting bank asset holdings. To study the interactions with the central bank balance sheet, we extend the baseline model used in the literature to include bank demand for reserves and for government debt. In our model, bank holdings of loans, bonds and reserves each contribute differently to the asset-limit imposed by the ICC. The difference between how reserves and government debt alleviates the ICC could be interpreted as their relative convenience yield as in Vissing-Jørgensen (2023).

Even away from the effective lower bound (ELB) of policy rates, balance sheet policies can have real effects. This is true if reserves and government debt play a special role in financial intermediation, for example, captured by commanding a convenience yield as a result of banks' payment or collateral needs or compliance with regulations that assign differential weights to certain assets. If reserves carry a higher convenience yield for banks than bonds, the central bank absorbs more government bonds and issue more reserves to alleviate the financial frictions banks face, resulting in a larger central bank balance sheet in the long-run. In addition, since government bonds have greater duration than reserves, the structure of the central bank balance sheet also has implications for the allocation of duration risk in the economy. Both the size and composition of the central bank balance sheet jointly affect bank decisions which flow to the real economy. Evaluating the model at empirically most plausible parameter values for the relative role of reserves and government bonds, we find that the central bank should maintain a large balance sheet. Under this configuration, commercial banks are less exposed to duration risk (by holding less government debt and more reserves) and are thus less subject to valuation changes.

Next, we consider the economic impact of various shocks as the central bank i) operates at its long-run balance sheet and ii) conducts balance sheet policies along the business cycle. We find that the long-run structure of the central bank balance sheet is the key factor determining the response of the economy to shocks. Moreover, if the central bank operates at its long-run optimal balance sheet, the real effects of cyclical balance sheet policies are negligible.

We illustrate these results by considering the following counterfactual exercises. In the first set of exercises, we set the central bank balance sheet to its long-run optimal level and track the impulse responses of productivity and financial shocks in two different scenarios. One in which the central bank only conducts optimal standard monetary policy and the other in which the central bank conducts both optimal monetary and balance sheet policies. These impulse responses look similar suggesting a negligible role for cyclical balance sheet policies. However, if the central bank balance sheet is not set to its long-run optimal level prior to the shock, a central bank which implements only the optimal monetary policy reaches significantly sub-optimal outcomes compared to the fully optimal case: the size of the balance sheet matters. Getting the long-run balance sheet right matters.

We also consider debt demand shocks in which banks' preference for holding government bonds changes. In this regard, our model suggests that indeed, sudden changes in the relative demand for liquid assets will be associated with spikes in spreads (albeit modest in our set-up). These can be minimized if the central bank is able and willing to accommodate changes in market preferences quickly and amply enough. This was indeed the case during the September 2019 repo spike, March 2020 dash-for-cash, March 2023 banking turmoil in the US and the UK gilt crisis in September 2022.

Our results have several important implications. First, bank regulation and government debt maturity play a key role in determining optimal balance sheet policy, by affecting the value of reserves versus government debt and duration risk in bank balance sheets, respectively. Second, taking government debt supply as given, our results suggest that absent a large central bank balance sheet, the problems in the banking sector during the March 2023 banking turmoil would have been exacerbated.

While our conclusion – that a large central bank balance sheet is likely to be welfare improving and that QE policies can work away from the ELB – is consistent with those of a number of recent papers (see literature review below), it is subject to important qualifications. A large central bank could reduce market discipline on government debt. We rule out governmental moral hazard by assuming fiscal policy is 'passive.' Ultimately, the crucial element of both fiscal and central bank policies in our analysis is that they are not discretionary but governed by rigid rules. Whether a similar institutional design should and could be in place in the real world depends strongly on political economy considerations that transcend the analysis we can carry out with our model. Importantly, in most jurisdictions, deficit spending is now regulated by law, including in many emerging economies.[1] Second, while QE-type of policies are effective in our model even away from the effective lower bound, their benefits become negligible when the long-run CB balance sheet is set optimally. We believe that the latter should be seen as the policy recommendation emerging from our analysis.

[1]While the existing rules might not exclude fiscal dominance in all countries, they indicate that there is awareness about the need to frame the policy space within legally sanctioned rules. Under well-designed rules, the amount of government debt held by the central bank should not affect the 'dominance' of monetary policy.

Finally, the rapid increase of interest rates after the pandemic has produced balance sheet losses for many central banks with large exposure to duration risk. These losses have attracted considerable attention from the press and policymakers (See e.g. Bell et al., 2023). From a purely economic point of view, losses are not different from fiscal deficits. In our model, for example, losses would (temporarily) affect the distribution of wealth between banks and households.[2] While these losses could lead to political economy issues, we abstract from this dimension, as it would require a different modelling strategy.

Related literature. Our paper contributes to the literature on the effectiveness of balance sheet policies. While the literature on the effectiveness of QE policies is too ample to be fully summarized here, representative contributions, pertinent to our analysis, include the following papers. Dell'Ariccia et al. (2018), CGFS (2019) and CGFS (2023) review the experience with asset purchases of a number of countries and elaborate on the evidence of side effects. As an early attempt to estimate directly the macroeconomic effects see Baumeister and Benati (2013) and Weale and Wieladek (2016). Others have followed a more indirect-effect analysis based on measuring the response of yields: among the first Vissing-Jorgensen and Krishnamurthy (2011) and D'Amico et al. (2012). Identification of the effects, even in this more limited scope, is daunting. Rostagno et al. (2021) provide an attempt in this direction based on ECB interventions. Hamilton (2018) review the Fed experience and highlights the problems of identification of QE effectiveness and Greenlaw et al. (2018) raise some doubts on the effects of QE on yields, especially their persistence. More positive are the findings by Swanson (2021), including on persistence. Rodnyansky and Darmouni (2017) show empirically that the type of assets purchased matters for the

[2] In our model we assume that the consolidated public revenue is rebated lump-sum to households.

response of credit. This point is studied in a DSGE model by Gertler and Karadi (2018). Among the side effects of QE, it has been argued that financial intermediaries might lose profitability due to excessively low interest rate and to the flattening of the yield curve.[3] Altavilla et al. (2018) address the profitability issue as for ECB asset purchases. A number of papers addresses more specifically the apparently paradoxical observation that QE might damage financial intermediaries by further reducing interest margins (flattening the yield curve) especially in combination with negative short-term rates, e.g. Brunnermeier and Koby (2018) and the empirical analysis by Heider et al. (2019). This paradox is also the source of the "addictiveness" of QE highlighted by Karadi and Nakov (2021) in a paper closely related to ours. Our paper differs from Karadi and Nakov (2021) in many respects. First, we emphasise the fact that the central bank purchases assets (government debt) mainly by issuing reserves, which are held by banks. This characterization of asset purchases highlights the maturity transformation of QE interventions. Second, while they consider "normal times" and "crisis scenarios", when the policy rate hits its lower bound, we discuss only times away from the lower bound. Our analysis complements the existing literature exactly in showing that the *average size* of the central bank's balance sheet matters *also* during normal times. Third, we abstract from the possibility that banks might become unconstrained under certain shocks. While we believe that non-linearities can be important during certain turbulent times, we don't find the "unconstrained" scenario a particularly appealing baseline. In our model, and to first order of approximation, this unconstrained world would imply zero spreads. Moving from zero spreads to positive might not be the key turning point in a finan-

[3] It should be noted that this channel is not specific of QE: standard monetary policy delivers certainly delivers lower interest rates. What QE might do is to bring this mechanism to an extreme, going beyond the limits of interest rate policies especially concerning the slope of the yield curve.

cial crisis. In most occasionally-binding constraint analyses, beyond that turning point no further non-linearity in financial conditions is assumed. This would be the case in our analysis too. A number of papers have shown that, at least theoretically, balance sheet policies can be effective also away from the ELB (e.g. De Fiore and Tristani, 2019 and Karadi and Nakov, 2021, among others), as long as some degree of market segmentation concerning central bank's assets and/or liabilities (partial substitutability due for example to preferred habitat etc.) is in place.

The literature on the implications of large long-run (average) central banks' balance sheet is relatively new. Among the recent contributions Afonso et al. (2022) study the effects of reserve supply given an empirically estimated demand for reserves. Lagos and Navarro (2023) provide structural estimates of the aggregate demand for reserves. Afonso et al. (2023) studies the optimal supply of reserves under uncertainty. Lopez-Salido and Vissing-Jorgensen (2023) study the demand for reserves in relation to the spread between market rates and the interest rate on reserves, banks' liquidity needs, and bank balance sheet costs. Vissing-Jørgensen (2023) argues that central bank balance sheet should be adjusted such that the convenience yield of reserves and Treasuries should be equalized. Our paper shares a similar view. In our case, convenience yields are captured by the beneficial effects of reserves and Treasuries on the bank's agency problem: other things equal, they relax the bank's borrowing constraint relative to holding only private risky loans. In this case too, though, the optimal supply of reserves must take into account the general-equilibrium effect and the interaction with standard monetary policy. In a similar vain, Reis et al. (2016) also argues that the Fed's balance sheet should remain as large as "saturating" the market for reserves. In this way, the policy rate (on reserves) would suffice to control inflation, while the balance sheet could be used to target other (e.g. financial) goals. Greenwood et al. (2019)

argue that a large central bank balance sheet is desirable as to reduce the private supply of short-term liquid assets by leveraged (and "runnable") institutions (mainly banks).[4][5]

Our paper relates to these findings in the following way. We model the role of public debt in the economy as in Gertler and Karadi (2018) and Karadi and Nakov (2021). This captures the portfolio-balance channel (effect) of asset purchases. Banks are not indifferent concerning the composition of their portfolio. Thus changes in the quantity of the available assets changes the price (return) at which they are willing to purchase them. This is turn affects other yields via arbitrage. Contrary to these papers, we fully model the balance-sheet of the central bank including reserves and cash. The portfolio-balance effect depends thus on the relative degree of substitutability between sovereign debt and reserves and also relate it to "twisting" mechanisms embedded in the "duration extraction" that such operation implies (Swanson, 2011). We capture "duration risk" by modelling government debt as a long-maturity debt. Furthermore, we show that modelling pubic debt as commanding a premium over deposits – i.e. its holding reduces the agency problem of the banks, at least up to a certain amount – can imply quite different effects of QE (QT). This mechanism allows us to talk about the "scarcity" effect of asset purchases, albeit in a very abstract term (see also Ferrari et al., 2017).[6] Under these

[4]We find that central banks balance sheets should be optimally large even without the fire-sale externality. Including this would serve to reinforce our result. See also Goodhart et al. (2017) for a review of the key issues at stake.

[5]Several other related papers also study the central bank balance sheet, e.g. Arce et al. (2020), Bigio and Sannikov (2021) and d'Avernas et al. (2024) among others. Arce et al. (2020) focus on the implementation of monetary policy in a New Keynesian model with an interbank market and show that a large central bank balance sheet delivers more policy space relative to the effective lower bound than a leaner one. Bigio and Sannikov (2021) highlight a trade-off between insurance across agents and across states which affects the countercyclical use of the central bank balance sheet. Finally, d'Avernas et al. (2024) show that the size of and adjustments to the central bank balance sheet can explain observed Treasury market dynamics.

[6]In the literature the "scarcity" effect or channel have been referred to to indicate two aspects of QE. On the one hand, QE is intended to create a relative scarcity of one

assumptions, QE (and thus QT) are not bound to have positive effects by design. In extreme cases QE could be totally detrimental for the economy by further disrupting the financial transmission channel. At a minimum, we view these extensions as a way to study the presence and interaction of some "side effects" of balance sheet policies.

Relative to the literature concerned with the long-run size of the balance sheet, discussed above, our paper has a number of commonalities and points of difference. Similarly to Afonso et al. (2022) we conclude that, in the most empirically likely scenario, central banks should maintain a large balance sheet on average. Yet, whereas their argument is solely based on the shape of the demand schedule for reserves, our argument is based on the general-equilibrium effects of mitigating financial frictions. As in Reis (2019), we discuss under which circumstances the balance sheet can play a role that is independent of policy rates. We emphasize that, under the assumptions of our model, reserves (and government debt) have a portfolio-balance effect on banks even when the reserve market is not "saturated". This is possible thanks to variable spreads between rates. If rates were assumed proportional to each other, it would not be possible for the central bank to decide both on quantities and prices. In other words, our model implies that monetary policy works also through spreads (besides operating through inflation expectations as in the typical case). Finally, as in Greenwood et al. (2019), the supply of reserves affects the size and composition (and thus leverage) of commercial bank's balance sheet, thereby affecting financial risk. In this way the balance sheet of the central banks affects the sensitivity to shocks of financial variables and the economy. More in general, relative to this literature, our general equilibrium approach allows us

asset (e.g. long-term government debt) thus increasing its price. On the other hand, when government debt is instrumental to perform certain operations within the banking sector, e.g. as collateral in repo operations, excessive scarcity can hinder the healthy functioning of financial intermediation (see also CGFS, 2023).

to assess the benefits of a larger balance sheet within the classical monetary policy problem.

Finally, Poole's model of banks' demand for reserves (Poole, 1968) has been used in a large number of papers (Woodford, 2001b, Afonso et al. (2022), Lopez-Salido and Vissing-Jørgensen (2023)) and remains a workhorse model in the analysis of central bank's operating frameworks. The key tenet of the model is that banks lend and borrow funds under uncertainty (about late-day in/out payments), while the central bank offers lending (e.g. discount window or marginal lending facility) and deposit facilities (reserves deposits). The rates asked and offered by the central bank on these two facilities (respectively) approximately determine the upper and lower bounds of funds rates. Payments' uncertainty – under general distributional assumptions – results in a downward-sloping demand for reserves between the extremes (where it becomes flat due to arbitrage). Depending on the supply of reserves, the inter-bank market rate will lie between the two extremes (corridor system). When reserves are sufficiently abundant, the inter-bank market rate will coincide with the remuneration of reserves (floor system), the prevalent case among advanced economies since the Global Financial Crisis. Our model captures some of the features of Poole's model, while remaining at a higher level of abstraction. First, we model central banks reserves as facilitating financial intermediation: holding them mitigates the agency problem. In Poole's model this "facilitation" would emerge from smoothing banks' idiosyncratic payment shocks. Second, while we do not strictly have an interbank rate alike the federal-funds rate, we have multiple rates and can thus discuss how the rate on reserves varies in relation to the other market rates. Yet, consistently with the received wisdom, the remuneration of (one of the) central bank's liabilities, offers to the central bank an additional policy instrument.[7] A

[7] In the classical monetarist model, the ability of remunerating cash would offer the

central contribution of our paper is the discussion of the extent to which this additional instrument (i.e. the size of the central bank balance sheet in normal times) increases the effectiveness of monetary policy.

The rest of the paper is organized as follows. Section 2 presents the model and in Section 3 we calibrate. Section 4 shows, by way of a simple ad-hoc policy experiment, that even in normal times, combining conventional monetary policy with balance sheet policies can strongly affect the response of the economy to shocks. Section 5 discusses the properties of the economy under three optimal policy regimes. The first consists of a jointly optimal monetary and balance-sheet policy. The second and third consists of an optimal monetary policy without the counter-cyclical use of the balance sheet. The difference between these two regimes consists of the long-run size of the balance sheet. In one case we assume that the central bank operates with a balance sheet of the same size as that under the fully optimal policy. In the other case we assume that the size of the balance sheet is only a small fraction of that under the fully optimal policy. Section 6 concludes.

2 Model

The model is based on a closed sticky-price production economy with financial intermediation à la Gertler and Karadi (2011). Our main modelling departure from the existing literature consists of fully specifying the balance sheet for the central bank. In particular, we impose a tighter budget constraint to the central bank, whereby assets (government debt) are purchased by issuing cash and reserves, with limited room to variations in central bank capital. While still stylized, this tighter specification of the budget constraint comes closer to the the real-world trade-offs and degrees

central bank an additional instrument (e.g. Woodford, 2003). This argument is also central in the debate of digitalization of currencies.

of freedom faced by central banks.[8] In issuing cash and reserves, the central bank faces well-specified price-sensitive demands, from households and banks, respectively. This implies that changes in the size and composition of the central bank's balance sheet has a market impact with real effects, while strengthening the interdependence of conventional monetary policy and balance-sheet policies.

2.1 Households

In each period households decide how much to consume (C_t), how many hours of work to supply to firms (H_t) at real wage W_t, how many deposits to purchase from banks (D_t) and how much cash to hold (real balances, M_t). For calibration purposes, households hold a constant proportion of total capital that is not intermediated by banks (K_H). Household income consists of labor income, revenue from deposits (at gross rate $R_{D,t}$), return on equities (at gross rate $R_{K,t}$) and lump-sum transfers (T_t). Lump-sum transfers consist of net profits from capital producers, net worth of exiting private banks net of transfers to start-up banks, and lump-sum taxation ($\mathcal{T}_t$).

The representative household has CRRA preferences and solves the following problem:

$$\max_{C_t,H_t,M_t,D_t,M_t} \mathcal{W}_0 \tag{2.1}$$

$\mathcal{W}_0$ is time-zero welfare defined as

$$\mathcal{W}_0 := E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} \left(C_t\right)^{1-\sigma} - \frac{\chi \, H_t^{1+\psi}}{1+\psi} \right) \tag{2.2}$$

[8]The related literature, spurred by balance-sheet interventions to respond to the Global Financial Crisis, typically model only the ability of the central bank to purchase assets, or inject capital into banks, with funds borrowed from the government. The role of reserves, and its tight link with asset purchases, is typically neglected.

subject to the budget constraint

$$
\begin{aligned}
&(1 + s(v_{M,t}))\, C_t + D_t + M_t + Q_t K_H \\
&= W_t H_t + \frac{R_{D,t-1}}{\pi_t}\, D_{t-1} + \frac{R_{M,t-1}}{\pi_t}\, M_{,t-1} + R_{K,t} Q_{t-1} K_H + T_t,
\end{aligned}
\tag{2.3}
$$

where $R_{M,t} = 1\ \forall t$ implies that M_t is cash with zero nominal return. Lump-sum transfers are defined as

$$
T_t = (1-\theta)\ N_{i,t} - Q_t\, I_t - I_t \left(1 + \frac{\eta}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) - \delta_T K_{F,t-1} - \mathcal{T}_t,
\tag{2.4}
$$

and $s(\cdot)$ denotes a transactions costs incurred in purchasing consumption. Define $v_t := C_t/M_t$ as the velocity of cash. Households particularly value the money-ness features of cash (M_t) for transactions (without which there would be zero demand). We choose a transactions cost function, $s(v)$ which satisfies the following properties. Define $\underline{v} = \{\underline{v} \in \mathbb{R}^+ | s(\underline{v}) = 0\}$, then have that $s'(\underline{v}) = 0$. The general assumptions about $s(v)$ are as in Schmitt-Grohé and Uribe (2010):

a) $s(v) \geq 0$

b) $s(\underline{v}) = s'(\underline{v}) = 0$ (demand for money can be finite even at zero nominal rate)

c) $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$ (money velocity always greater or equal the satiation level)

d) $2s'(v) + vs'' > 0\ \forall\ v \geq \underline{v}$ (demand for money decreasing function of the interest rate [differential])

Specifically, the transactions cost function takes the form à la Schmitt-

Grohé and Uribe (2010):[9]

$$s(v_t) = A_M v_t + B_M / v_t - 2\sqrt{A_M B_M}. \tag{2.5}$$

The first-order conditions to the household problem, denoting the Lagrange multiplier on the budget constraint by λ_t, are:

$$C_t : \qquad (C_t)^{-\sigma} = \lambda_t (1 + s(v_t) + s'(v_t) v_t), \tag{2.6}$$

$$H_t : \qquad \lambda_t W_t = \chi H_t^{\varphi} \tag{2.7}$$

$$D_t : \qquad E_t \Lambda_{t+1|t} \frac{R_{D,t}}{\pi_{t+1}} = 1, \tag{2.8}$$

$$M_t : \qquad E_t \Lambda_{t+1|t} \frac{R_{M,t}}{\pi_{t+1}} = 1 - s'(v_t)(v_t)^2, \tag{2.9}$$

where the stochastic discount factor is $\Lambda_{t+1|t} := \beta \left(C_{t+1}/C_t \right)^{-\sigma}$.

2.2 Government

We assume that in each period the government issues $B_{N,t}$ new quasi-perpetuities at the price $P_{B,t}$ in units of the consumption basket that have a fixed probability $(1 - \delta_p)$ to expire in each period after payment of the coupon.[10] We assume that these bonds pay a fixed coupon $\bar{r}_p$. We refer to these assets as long-term bonds and assume they are traded in secondary markets such that an unexpired bond is valued at $P_{B,t}$. The gross return on bonds is thus:[11]

$$R_{B,t} = \frac{\delta_p P_{B,t} + (1 - \delta_p) + \bar{r}_p}{P_{B,t-1}}. \tag{2.10}$$

[9] This implies Lucas timing. Svensson (1985) timing would instead use lagged values of cash.

[10] This assumption can be seen as a variant of the perpetuity assumed by Woodford (2001a). See also Chen et al. (2012) for an application of this modelling assumption.

[11] Note that if $\delta_p = 0$, the perpetuity becomes a one-period bond.

The total real stock of these perpetuities outstanding at the beginning of each period is

$$\mathcal{B}_t = \sum_{s=0}^{\infty} \frac{\delta_p^s}{\prod_{j=0}^{s-1} \pi_{t-j}} B_{N,t-s} = B_{N,t} + \frac{\delta_p}{\pi_t} \mathcal{B}_{t-1}, \tag{2.11}$$

where if $s - 1 < j$ then $\prod_{j=0}^{s-1} \pi_{t-j} = 1$. Long-term bonds are held by banks and the central bank in quantities $\mathcal{B}_H$, $\mathcal{B}_{B,t}$ and $\mathcal{B}_{CB,t}$ respectively. Market-clearing implies that these allocations equal the total supply set by the government

$$P_{B,t}\mathcal{B}_t = P_{B,t}\mathcal{B}_H + P_{B,t}\mathcal{B}_{CB,t} + P_{B,t}\mathcal{B}_{B,t}. \tag{2.12}$$

We model government spending, G_t, as a purely exogenous AR(1) process. Taxation, $\mathcal{T}$, is lumpsum so the government budget constraint is

$$\frac{(1 - \delta_p) + r_p}{\pi_t} \mathcal{B}_{t-1} + G_t = P_{B,t} B_{N,t} + \mathcal{T}_t + \mathcal{T}_{CB,t} \tag{2.13}$$

where $\mathcal{T}_{CB,t}$ references the lump-sum transfer from the central bank to the government, derived in the next section.

We assume the government maintains a constant real stock of debt, $\mathcal{B}$, by only issuing new quasi-perpetuities to cover expiry and inflation:

$$B_{N,t} = \left(\mathcal{B}_t - \frac{\delta_p}{\pi_t} \mathcal{B}_{t-1} \right). \tag{2.14}$$

2.3 Banks

There is an infinite number of identical banks (financial intermediaries) indexed by the subscript i.[12] Banks sell deposits to households and use them, together with own net-worth ($N_{i,t}$), to purchase: claims on private capital from firms ($Q_t K_{i,F,t}$); long-term bonds ($P_{B,t}\mathcal{B}_{i,B,t}$); and central bank

[12] Our abstraction from non-bank participants amounts to assuming that banks are the arbitrageurs in the bond market. Consistent with this, Eren et al. (2023) and Jansen (2023) find banks to be the most price elastic investor group in the United States and in the Netherlands, respectively.

reserves ($B_{i,F,t}$). Banks face a limited commitment problem as in Gertler and Kiyotaki (2010) which leads to an incentive compatibility constraint (ICC) that aligns the incentives of the banks and the depositors.

Relative to most of the related literature, we extend the baseline model to include a demand for reserves. In the real world, the demand for central bank's (excess) reserve deposits reflects banks' payment needs – in large part linked to the amount of deposits issued by banks, the relative return on reserve deposits, regulatory requirements and the related market discipline.[13] We capture all this is a stylized manner, assuming that reserves facilitate financial intermediation by reducing the stringency of the ICC relative to other assets. We assume a similar role for government debt holdings.

The representative bank's optimal value is

$$J_{i,t}(N_{i,t}) = \max_{[K_{i,F,t}\mathcal{B}_{i,B,t},B_{i,F,t},D_{i,t}]} E_t\Lambda_{t+1|t}\left[(1-\theta)N_{i,t+1} + \theta J_{i,t+1}(N_{i,t+1})\right], \tag{2.15}$$

where $(1-\theta)$ is the probability of exiting the banking industry and bank net worth evolves according to

$$N_{i,t} = R_{K,t}Q_{t-1}K_{i,F,t-1} + \frac{R_{B,t}}{\pi_t}P_{B,t-1}\mathcal{B}_{i,B,t-1} + \frac{R_{F,t-1}}{\pi_t}B_{i,F,t-1} - \frac{R_{D,t-1}}{\pi_t}D_{i,t-1}, \tag{2.16}$$

subject to the balance sheet

$$N_{i,t} + D_{i,t} = Q_tK_{i,F,t} + P_{B,t}\mathcal{B}_{i,B,t} + B_{i,F,t}, \tag{2.17}$$

[13] The Basel III Accord stipulates that banks should hold a sufficient amount of high-quality liquid assets (HQLA) "to allow them to survive a period of significant liquidity stress lasting 30 calendar days" (BIS, 2013). The ratio between HQLA and the "total net cash outflows over the next 30 days" is called the Liquidity Coverage Ratio (LCR) and has a minimum regulatory value of 100%. Reserves are listed among the eligible liquid assets. Most banks in countries implementing this regulation have been well above the minimum (in many cases above 200%), due to the abundant supply of reserves, which thus account for most of HQLA (EBA, 2023). In such an environment the regulatory constraint is not binding for most banks.

Following Gertler et al. (2012), we assume that different assets can be absconded to different extents (they have different recovery rates), i.e.

$$J_{i,t} \geq \kappa_K Q_t K_{i,t} + \kappa_{B,t} P_{B,t} \mathcal{B}_{i,B,t} + \kappa_F B_{i,F,t}. \tag{2.18}$$

with κ_K and κ_F positive parameters. $\kappa_{B,t}$ is assumed to be stochastic (see below).

The riskiness of firm-lending, bonds and reserves increases linearly, parameterized by κ_K, $\kappa_{B,t}$ and κ_F respectively. The relative value of $\kappa_{B,t}$ and κ_F determines the extent to which an "operation twist" can have the desired effects.

The bank FOCs are:

$$K_{i,t} : E_t \Omega_{t+1|t} \left(R_{K,t} - \frac{R_{D,t-1}}{\pi_t} \right) = \kappa_t \gamma_{i,t} \tag{2.19}$$

$$\mathcal{B}_{i,B,t} : E_t \Omega_{t+1|t} \frac{R_{B,t} - R_{D,t-1}}{\pi_t} = \kappa_{B,t} \gamma_{i,t} \tag{2.20}$$

$$B_{i,F,t} : E_t \Omega_{t+1|t} \frac{R_{F,t-1} - R_{D,t-1}}{\pi_t} = \kappa_F \gamma_{i,t} \tag{2.21}$$

where $\gamma_{i,t}$ is the Lagrange multiplier associated to the ICC (2.18) and $\Omega_{t|t-1} := \Lambda_{t|t-1} \left[(1-\theta) + \theta J'_{i,t} \right]$ is the discount factor of the bank where $J'_{i,t}$ is the bank's marginal value.[14]

The envelope condition is:

$$N_{i,t} : \qquad J'_{i,t} \left(1 - \gamma_t\right) = E_t \Omega_{t+1|t} \frac{R_{D,t}}{\pi_{t+1}} \tag{2.22}$$

2.4 Central bank

The central bank carries out monetary and balance-sheet policy in order to reduce specific inefficiency wedges, namely nominal, financial and monetary

[14] Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) show that under the assumption underlying the bank problem, the value function J_t is linear in net-worth. This allows aggregation across agents and implies that $J_{i,t} = J'_{i,t} N_{i,t}$. We verify that the linearity assumption holds for our formulation of the ICC.

frictions.

The role of the central bank in the economy consists of pinning down the process of inflation, provide cash and regulate the net-supply of government debt, i.e. "free float". The ultimate goal of the central bank is assumed to be the level of households' welfare.

The central bank operates with own capital $N_{CB,t}$ obtained from its revenue, net of the cost of serving its liabilities and of the lump-sum transfers to the government, $T_{CB,t}$. The balance sheet of the central bank is:

$$P_{B,t}\mathcal{B}_{CB,t} + Q_t K_{CB,t} = M_t + B_{F,t} + N_{CB,t} - T_{CB,t} \tag{2.23}$$

where central bank net worth $N_{CB,t}$ evolves according to:

$$N_{CB,t} = R_{K,t} Q_{t-1} K_{CB,t-1} + \frac{R_{B,t}}{\pi_t} P_{B,t-1}\mathcal{B}_{CB,t-1} - \frac{R_{M,t-1}}{\pi_t} M_{t-1} - \frac{R_{F,t-1}}{\pi_t} B_{F,t-1}. \tag{2.24}$$

We model the transfer $T_{CB,t}$ in two parts. The first is the steady state component $\tilde{T}_{CB}$. We model this to ensure that all revenue is transferred to the government in the deterministic steady state. The second, $T_{CB,t} - \tilde{T}_{CB}$, is a stability rule (equation 2.25) ensuring that the fiscal dimension of the central bank balance sheet policy is "passive" (Leeper, 1991).

$$T_{CB,t} - \tilde{T}_{CB} = \gamma_{CB}\left(\mathcal{A}_t - \tilde{\mathcal{A}}\right) \tag{2.25}$$

where net assets are defined as $\mathcal{A}_t := P_{B,t}\mathcal{B}_{CB,t} + Q_t K_{CB,t} - M_t - B_{F,t}$ and $\gamma_{CB} \in (0,1)$. It should be noted that imposing this rule restricts also the possibility of the central bank to issue liabilities and/or purchase assets by changing the transfers from/to the government. This constraint sets our analysis apart from most of the literature on asset purchases.[15]

[15] The difference is a matter of focus. The literature on QE/QT is mainly concerned with the effects of asset purchases and less with the ability of the central bank to carry them out. That approach implies a particular coordination between central bank and government that does not take place in our model.

We assume that the central bank can credibly commit to its policy strategy. Furthermore, following Woodford (2003) we assume that the central bank commits to a "timeless" policy, i.e. that the policy at the time of its first implementation is identical to that in all subsequent periods.[16] The central bank chooses its strategy as to maximize households' welfare $\mathcal{W}_0$ under the constraint that all decentralized markets clear given the resource constraints, the agents budget constraints and banks' ICC. The resulting policy is a "targeting rule" (Woodford, 2003).[17]

We consider two possible optimal policy strategies. The first amounts to the standard optimal monetary policy widely discussed in the literature (Woodford, 2003). In this case we impose an ad-hoc central bank's balance sheet (size and composition). The central bank has thus only one degree of freedom to steer the economy. The second strategy allows the central bank to choose also the size and liability-composition of its balance sheet, both in the short- and long-run.[18] In this case the central bank can avail of two degrees of freedom, i.e. two implicit instruments. Whether the additional policy instrument has a lot or little traction on the general economy, depends first on the elasticity of the demand for central bank's liabilities to the additional policy instrument and, second, on the role these liabilities play in the economy.

Define the vector-function of length n of all decentralized market clearing conditions, resource and budget constraints as

$$E_t F\left(\boldsymbol{Z}_{t+1}, \boldsymbol{Z}_t, \boldsymbol{Z}_{t-1}; \boldsymbol{\mu}_{t+1}, \boldsymbol{\mu}_t, \boldsymbol{\mu}_{t-1}\right) = \mathbf{0}, \tag{2.26}$$

[16] The latter commitment suffices to make this policy time-consistent.

[17] Giannoni and Woodford (2003) discuss the limits of transforming this class of rules into "instrument rules", i.e. feedback rules linking the policy rate to structural variables of the model. See Devereux et al. (2020) for an approximate derivation of simple implementable rules extracted from policy problems of the kind we use in our model.

[18] To prevent extreme deviations, we impose a small adjustment cost to central bank bond purchases.

where $\boldsymbol{Z}_t$ is the length-$m < n$ vector of all endogenous variables, $\boldsymbol{\mu}_t$ is the vector of all exogenous variables and $\mathbf{0}$ is a length-n vector of zeros.[19] If $m - n = 1$ the central bank has one degree of freedom (the MP regime); if $m - n = 2$ the central bank has two degrees of freedom (the MP+BSP regime). What varies in the problem is the length n, as we include an ad-hoc rule for the central bank's balance sheet in the MP problem and exclude it in the BSP+MP problem.

Then we can write the general central bank optimization problem in Lagrangian form as

$$\max_{\boldsymbol{Z}_t, \boldsymbol{\lambda}_{\mathcal{W},t}} \left[\mathcal{W}_0 - E_0 \sum_{t=0}^{\infty} \beta^t \boldsymbol{\lambda}_{\mathcal{W},t} F\left(\boldsymbol{Z}_{t+1}, \boldsymbol{Z}_t, \boldsymbol{Z}_{t-1}; \boldsymbol{\mu}_{t+1}, \boldsymbol{\mu}_t, \boldsymbol{\mu}_{t-1}\right) \right], \tag{2.27}$$

where $\boldsymbol{\lambda}_{\mathcal{W},t}$ is the length-n vector of Lagrange multipliers. The resulting set of $n + m$ first order conditions (FOCs) allows us to solve for the $n + m$ variables ($\boldsymbol{Z}_t$ and $\boldsymbol{\lambda}_{\mathcal{W},t}$).

2.5 Shocks

We consider four shocks: TFP, government spending, net-worth (showck to θ_t), a shock to banks' demand for government debt (a shock to $\kappa_{B,t}$) and, only in the calibration, a monetary policy shock. The first two shocks can be seen as real-economy shocks, while the second two as financial-sector shocks.

All shocks have an AR(1) structure (in logs) with mean-zero and unit-variance iid innovations. So if X_t is the shock, we have

$$X_t = \rho_X X_{t-1} + \sigma_X \varepsilon_t \tag{2.28}$$

where ρ_X is the autoregressive coefficient and σ_X is the standard deviation of shock X_t.

[19] In our model variables appear only with timings $t + 1$, t and $t - 1$.

Firms and aggregate equations are standard and relegated to Appendix A

3 Calibration

We calibrate our model taking parameter values from the literature and adjusting them as necessary to ensure that the deterministic steady state and the standard deviation of key variables falls within empirical ranges for the US.[20] We make an exception for the interest rate rule that we impose in the calibration. For this we estimate a simple rule as described in Appendix B. We use the estimated rule as a reference for the policy rule coefficients used during calibration. That said, in the calibration of the shocks we include the standard deviation of the policy shock too, so to better match all the targeted moments. The normative analysis is only marginally affected by this procedure, as the policy shock does not matter for those results. The complete list of parameter values is displayed in Table 1. We estimate the standard deviations of our shocks targeting the median value of the standard deviation of six variables of interest – year-on-year inflation (we use the GDP deflator in the data), the policy rate (the three-month treasury rate), the consumption to GDP ratio, the spread (BBB corporate yield minus deposit rate)[21], the spread on government debt (10-year Treasury yield minus deposit rate),[22] and government spending.[23] Table 2 shows the inter-quartile range of the standard deviation in the data and the standard deviation in the model under our calibration. Key variables like inflation, spreads and consumption ratio are well captured by

[20]All data are retrieved from FRED, Federal Reserve Bank of St. Louis.

[21]The FRED code for these variables are BAMLC0A4CBBBEY and IR3TED01USQ156N – i.e. 90 days money market deposit rate – respectively.

[22]The FRED code for the 10-year yield is IRLTLT01USM156N.

[23]Government spending measured as total expenditures: FRED code W068RCQ027SBEA.

our model. Less so the interest rate. Given that our aim is to provide a ball-park quantitative comparison of optimal monetary policy and optimal balance-sheet policy, we deem these results satisfactory.

4 Effectiveness of balance-sheet interventions in normal times

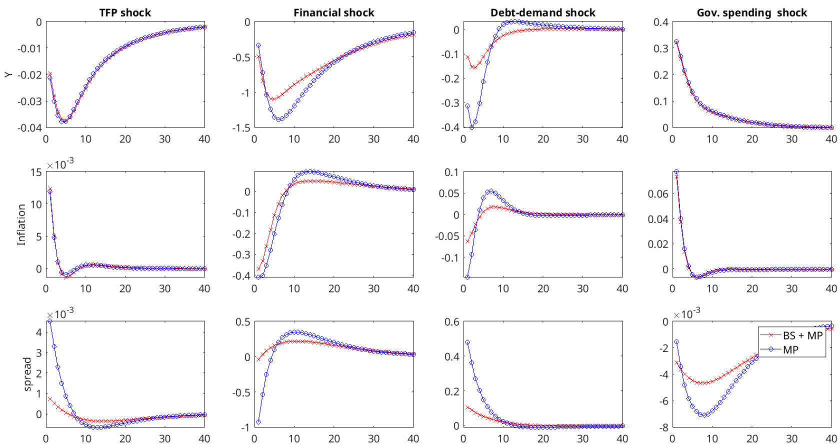
As mentioned in the Introduction, the literature has pointed out that, at least theoretically, balance-sheet policies can be effective also away from the ELB. Provided that typically asset purchases imply expansions of the supply of reserves, this result can be interpreted as a statement on the availability of two independent policy instruments in normal times: the supply of reserves and the policy rate. Below we argue that despite this theoretical proposition, the quantitative benefits from adjusting the central bank balance sheet counter-cyclical are modest. To clarify that this proposition is not about the ineffectiveness of the reserve tool during normal times, but a statement about optimal policy, Figure 1 compares the response of the economy to four shocks when the central bank adopt ad-hoc rules: The estimated interest rate rule as monetary policy and a simple feed-back rule for reserves, i.e.

$$B_{F,t} - B_{F,ss} = 0.96\,(B_{F,t} - B_{F,ss}) + 40\,(spread_t - spread_{ss}) \tag{4.1}$$

where $_{ss}$ denotes the deterministic steady state value.

The choice of parameter values is arbitrary and meant only to prove the point that the two instruments have both traction on the economy. This simple experiment should help interpreting the optimal-policy results discussed below. Specifically, keeping the balance sheet constant (MP) implies responses of the three key variables displayed that are different from the response when reserves are adjusted counter-cyclically (i.e. increase reserves when spreads increase). Interestingly, not all variables are equally sensitive to the activation of both instruments across all four shocks. The distinction runs between real (demand and supply) shocks and financial-sector shocks (the "financial shock" and "debt-demand shock"). For example, for the former group, GDP and inflation respond virtually identically, whether the central bank balance sheet is adjusted or not. This is not the case for the shocks impacting directly financial markets. In this case inflation and GDP responds in markedly different ways depending on the policy strategy. The spread, in contrast, responds to any of the shocks in ways that strongly depend on the policy instrument mix.[24]

Figure 1: Effectiveness of reserves supply in normal times



5 Optimal monetary and balance sheet policies

Banks in our model choose their portfolios taking into account asset-price fluctuations from holding state-contingent private loans and the sensitivity

[24]A similar point is made by Karadi and Nakov (2021) .

to inflation of their balance sheet that arise from nominal contracts. Moreover, since long-term debt is more sensitive to inflation (or variations in interest rates, aka duration risk) than reserves, more of the former would further increase the sensitivity of a bank to inflation.

As a consequence the central bank should optimally set both balance sheet and interest rate policy in light of these considerations.

We consider three regimes. The first consists of a jointly optimal monetary and balance-sheet policy. The second and third consists of an optimal monetary policy without the counter-cyclical use of the balance sheet. The difference between these two regimes is the size of the balance sheet. In one case we assume that the central bank operates with a balance sheet of the same size as that under the fully optimal policy. In the other case we assume that the size of the balance sheet is only a small fraction of that under the fully optimal policy. In the first subsection we discuss the long-run properties of the economy under these regimes, while in the second subsection we conduct an impulse-response analysis under specific shocks under each of the regimes.

5.1 The role of the optimal long run balance sheet under different policy regimes

We have seen that counter-cyclical balance-sheet interventions can have real differential effect even when combined with conventional moentary policy in normal times. It is therefore legitimate to expect that a fully optimal policy strategy would activate both instruments to steer the economy in response to shocks. The question that we seek to answer is which is the relative role of size of the long-run balance sheet relative to optimal balance-sheet adjustments in response to shock. In other words: Should the central bank be concerned only with the long-run size of its balance sheet, or should it carry out asset-purchases/sales along the business cycle?

To address this question, we must first determine the optimal long-run size of the central bank's balance sheet. The first result of our analysis is that when the central bank faces no additional costs related to the size and adjustment of its balance sheet, it would optimally choose a corner solution: issue zero reserves, if $\kappa_B \leq \kappa_B^* \leq \kappa_F$; or hold all sovereign debt otherwise. A key determinant of the threshold κ_B^* is the average maturity of government debt δ_p.

These results cannot be shown analytically under either of the optimal policy regimes. We thus illustrate it graphically as follows. We start by abstracting from both the monetary and the nominal-rigidity frictions, including the fact that deposits, bonds and reserves are nominal.[25] In this way we focus exclusively on how the central bank balance sheet can affect the degree of financial friction, and thus welfare. Moreover, we switch off the government-debt demand shock to highlight the possible symmetry between debt and reserves.[26]

Under these assumptions, there is only one friction in the economy (the financial friction) and the central bank has only one instrument: the size and composition of its balance sheet.

For given values of κ_F, κ_B and δ_p, we plot the conditional mean welfare (in percent of permanent consumption units) as well as the steady state welfare under the BSP regime, without any central bank's preference cost.[27]

We start by considering the effect of switching around the relative rank-

[25] We thus impose that inflation is zero and constant.

[26] Including the government debt shock would not change the results qualitatively (and obviously not at all for the deterministic steady state), but would affect the relative welfare effects of exchanging debt for reserves.

[27] In the deterministic steady state there is a continuum of values for the policy instruments that satisfies the decentralized equilibrium conditions. In general though only a subset of values of the policy instruments satisfies the FOCs of the policy problem. In the case we are discussing, the FOCs are satisfied for all the range of reserve supply that we are entertaining. We thus look at welfare directly to infer the optimal steady state value of reserves.

ing of κ_F and κ_B (minimum value 0.12; maximum value 0.15) keeping the same maturities for debt and reserves ($\delta_p = 0$). Figure 2 shows the result. The left vertical axis reports the conditional welfare (in percent of permanent consumption units) while the right vertical axis reports the deterministic steady state of welfare (in natural "utils" units). In both cases the distance from the minimum value is displayed. The horizontal axis reports the share of central bank's holding of government debt. The minimum amounts to zero reserves, while the maximum consists of the central bank holding all available debt. Both perspectives (utils or permanent consumption units) provide the same results: Welfare is maximized when banks hold the least of the asset with the highest ICC cost. In particular, in the empirically most relevant case ($\kappa_B > \kappa_F$) the central bank should hold all the debt in circulation and replace it with reserves.

To complement this result, Figure 3 shows the (unconditional) mean of the spread between the return on private (state contingent) loans and the deposit rate, capturing the degree of financial friction. Consistently with Figure 2, the financial friction falls when the central bank swaps the most ICC-costly asset with the cheaper one.

While this result is straightforward, given the assumptions of our model, it illustrates quite clearly the incentives of the central bank.

By increasing the duration of public debt to 20 quarters ($\delta_p = 0.95$) affects the relative premium on sovereign debt and reserves, now comprising a term premium (but no inflation-risk premium, as inflation is constant). The implication, shown in Figures 4 and 5, is that any given level of mean welfare (and spread) is achieved with a different mix of reserves and public debt in the central bank's balance sheet, compared to the $\delta_p = 0$ case. The deterministic steady state relationship remains unchanged. Also in this case, though, the optimal size would be achieved at either of the corners, depending on the same relative ranking of κ_B and κ_F as before.

Figure 2: Welfare and central bank debt holding: $\delta_p = 0$

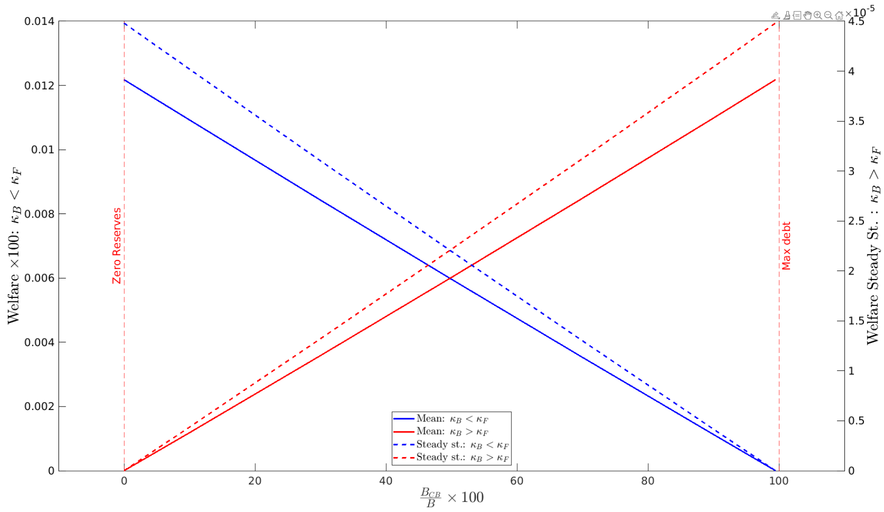


Figure 3: Spread and central bank debt holding: $\delta_p = 0$

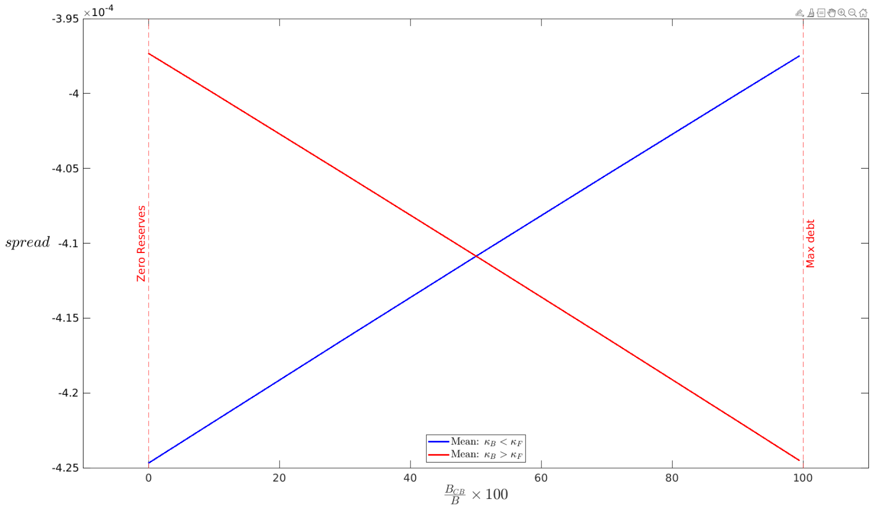


Figure 4: Welfare and central bank debt holding: $\delta_p = 0.95$

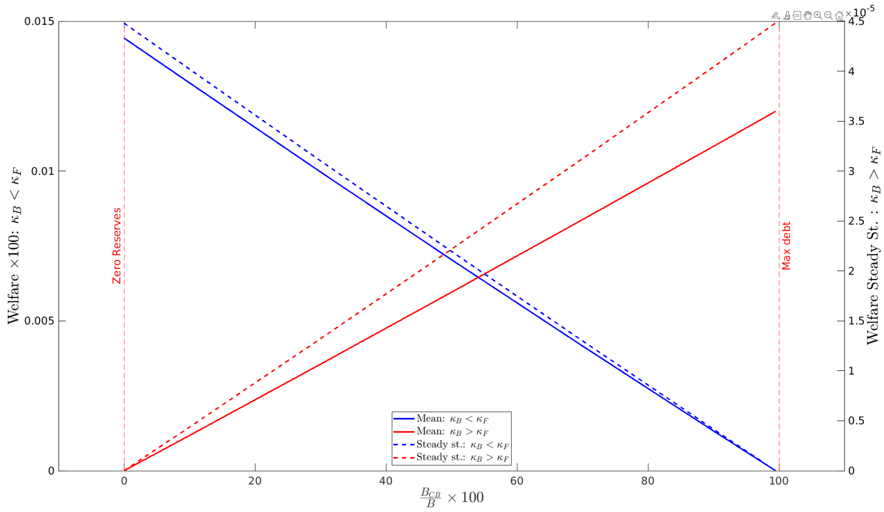
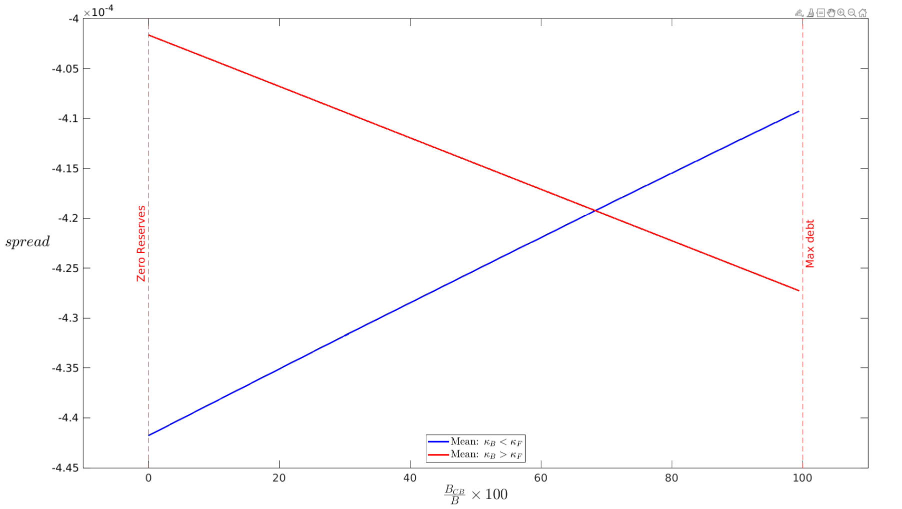


Figure 5: Spread and central bank debt holding: $\delta_p = 0.95$



Allowing for variable inflation (on top of the longer maturity) adds an inflation risk premium to the relative return on public debt. This further increases the asymmetric welfare effect of reserves and (net) debt supply (not shown). Overall though the principle remains the same. To the extent that public debt and reserves are identical (same κ_s, same maturity, same premia etc.) the central bank has no incentives to change their relative (net) supply. Once either of these conditions changes, making the two public liabilities different, the central bank's incentives change too.

Further increasing the number of frictions (price stickiness and monetary transaction costs) blurs the intuition. In the more general case, the welfare implications of inflation, of the composition and size of banks' balance sheet and of the amount of money in circulation, are not mutually independent. But it remains true that the size and composition of the central bank's balance sheet is a key determinant of allocations and thus welfare. In the next section we further explore this role by disentangling the benefits of a long-run size and composition of the central bank's balance sheet from the benefits of its cyclical adjustment.

5.2 Cyclical vs long-run balance sheet

The second central result that emerges from our model is that the most important factor affecting the response of the economy to shocks is the long-run structure of banks' balance sheet and not asset-purchases/sales along the business cycle. To wit, the valuation effect of the assets and liabilities held by banks, upon a shock, can have a more material effects on allocations than the portfolio adjustment induced by the central bank. We demonstrate this result by comparing MP with the joint regime MP+BSP under two scenarios and under all the three frictions entertained in our model: monetary, financial and nominal. The first scenario we consider consists of the MP regime operating under the same balance sheet size of the MP+BSP. The second consists of the MP regime operating with a BS that is 10% of that under MP+BSP.

To gain more control of the size and volatility of the central bank's balance sheet, and in light of the possibility of corner solutions discussed above, we introduce a parametrized "preference cost" for the optimizing central bank. The cost depends on deviations of central bank's holding from a given target. Without this cost, in our exercise, a number of variables (in particular debt holding) have unreasonable volatility or the solution is explosive. We interpret this result as a dynamic reflection of the the same feature that leads to the corner solutions discussed a above. In particular, when the long-run central bank holding of debt is at either corner, stochastic fluctuations would breach the bounds. An alternative would be to impose occasionally non-negativity constraints. Yet this solution would be computationally more involved (we evaluate our model at second order of accuracy) and, we believe, it would not add to the main result we are conveying with our analysis.

Table 3 shows that when the size (and composition) of the long-run BS is the same under the two policy regimes, the resulting equilibria are virtually indistinguishable (for shocks calibrated to imply commensurable magnitudes with observed moments), except for the standard deviation of spreads (higher under the joint regime) and of interest rates (lower under the joined regime, albeit marginally). Needless to say, the standard deviation of the share of debt held by the central bank varies more under the joined regime, as it is an instrument of policy.

Quite different is the case displayed in Table 4. In this case the MP regime operates with a long-run balance sheet of the central bank that is 10% of the balance sheet underlying the MP+BSP regime (in the latter the optimal long-run balance sheet must be imposed).

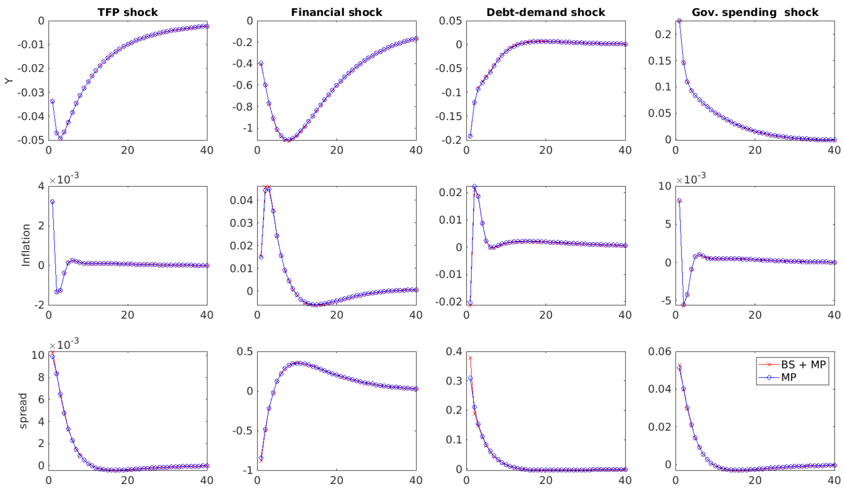
5.3 Optimal policy in response to specific shocks

We compare two policy scenarios in response to a set of interesting shocks: TFP, net-worth (financial) shock, banks' demand for debt shock and government spending shock. The first scenario is the standard optimal monetary policy, while the balance sheet is maintained constant. The second is the jointly optimal monetary and balance sheet policies. For the two policy regimes and each of the shocks we consider the two cases entertained above: i) MP operates with a constant supply of reserves of the same amount supplied by the BSP+MP regime in the long run (deterministic steady state); ii) MP operates only with 10% of the supply or reserves as in the previous case.

Figure 6 shows the response of three variables – output, private contingent-loan spread and inflation – to the four shocks when the supply of reserves in the deterministic steady state is identical across regimes. Clearly, the two regimes imply identical responses of the three variables to the three shocks, except for spreads under the debt-demand shock, at least initially. In this regard our model suggests that indeed, sudden changes in the relative demand for liquid assets will be associated with spikes in spreads. These can be minimized if the central bank is able and willing to accommodate changes in market preferences quickly and amply enough. That said, the model also suggest that short-lived "excess" spikes in spreads should not be a source of major concern for the policymaker, as the real and inflationary consequences of such an event are minimal. Indeed, the experience during September 2019 in the United States and during the gilt crisis in 2022 in the United Kingdom suggest that the central banks could react to these spikes in a targeted manner avoiding real consequences. This statement is clearly subject to the the premise that the central bank can conduct an optimal policy under either regime.

A key feature of either policy regimes is that the policymaker brings

Figure 6: Response to shocks when the (deterministic) long run supply of reserves is the same under MP and BSP+MP regimes



about an "impact" response of inflation (and to a lesser extent of output) that is markedly different from the subsequent paths. While the policy is fully predictable (being under commitment), the shock is not. On impact, nominal contract (including debt) are pre-set and the shock engenders a redistribution of income. On impact, the central bank leverages this margin which is not available thereafter. For example, upon a negative TFP shock, the central bank engineers a one-period spike in inflation followed by a smooth deflation for several quarters. Output does not fall as much on impact, and then converges smoothly to the steady state.

Figure 7 shows the second scenario: MP operates with a constant supply of reserves that is only 10% of that in the previous case. Under this assumption the two regimes are markedly different (the BSP+MP IRFs are the same as above). Starting with the TFP shock (first column), output behaves in a similar way across regimes (except for a small gap on impact), unlike inflation and spread. Under MP, inflation remains always above its steady state value, after an initial spike not dissimilar from the BSP+MP case. As for the private-sector credit spread, it increases more substantially under MP than under the alternative regime.

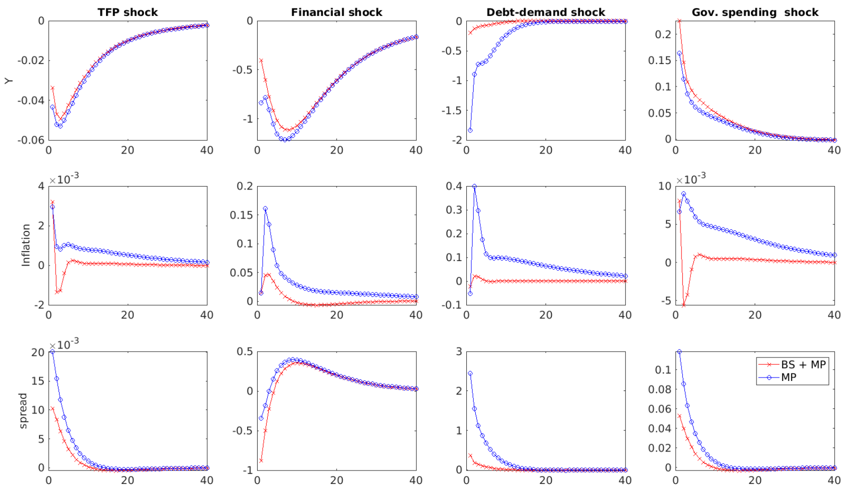
The financial friction shock (middle column) induces responses that are also markedly different across regimes. Output, in this case, differs more visibly on impact. The MP regime engenders a response in the "surprise" period that is considerably different from the subsequent one. After that the two regimes bring about a very similar adjustment of output. Inflation is also markedly different on impact relative to the following path across regimes. MP though engenders a much larger and persistent inflation response. Finally, spreads fall in the first few periods before increasing persistently. By adjusting the supply of reserves, the BSP+MP regime is able to make the initial spread contraction much larger while allowing output and inflation to move by less.

The debt demand shock generates the most significant difference across regimes. In this case it is helpful to start from the spread, which, under MP, increases more than five times than under the other policy regime. This financial tightening engenders a large contraction of output (again very severe in the first period and only milder but protracted thereafter). Financial tightening means also a higher cost of production, with the consequence that inflation remains above the long-run equilibrium persistently, except for the "surprise" period.

Finally, the government spending (demand) shock shares some similarities with the supply (TFP) shock, with an even more pronounced discrepancy in the response of inflation across policy regimes.

In summary, the impulse response function analysis confirms that the average size of the central bank balance sheet is significantly more important than the ability to adjust the supply of reserves in response to economic conditions. Comparing these impulse responses with those under ad-hoc policies (discused in Section 4) we can conclude that the small role that balance-sheet adjustment play under optimal policies must be attributed exactly to the optimality of interventions. Only when conventional mon-

Figure 7: Response to shocks when the (deterministic) long run supply of reserves under MP is 10% of that under the BSP+MP



etary policy is fully optimal, the cyclical adjustment of the balance sheet loses importance. Yet, even in this case, the size of the balance sheet remains crucial.

It is important to restate here that we are not studying the effects of balance-sheet policies when the interest rate is at its lower bound. An ample literature has already shed considerable light on the benefit of balance sheet policies in that case. Our results show instead that in the presence of financial and liquidity shocks, what matters the most is to have in place a resilient configuration of balance sheets even away from the effective lower bound, i.e. during "normal" times.

In this regard, the long-run balance sheet of the central bank can be seen also from a macroprudential perspective. For a given supply of public debt and preferences (habitat), banks can end up with a portfolio that is less resilient to aggregate shocks than what a policymaker could envisage. By supplying reserves and absorbing excessive government debt in circulation, the central bank can induce banks to move to a less vulnerable asset structure. Once this is achieved, monetary policy alone is able to steer the economy in a way very similar to what can be implemented by adjusting reserves in a counter-cyclical way.

In essence, our model, under the most empirically relevant parametrization, calls for the supply of ample reserves and thus a reduction of banks' exposure to longer-term debt.

6 Conclusion

Is there a role for balance sheet policies in the central bank policy toolbox away from the effective lower bounds on interest rates? A key lesson from the Global Financial Crisis and the Covid-19 crisis is that financial vulnerabilities confront policymakers with harsh trade-offs. These trade-offs often call for multiple policy tools. Balance sheet policies, operating at the core of financial frictions, are an obvious candidate to alleviate these trade-offs.

We show that, even away from the ELB constraint of monetary policy the balance sheet of the central bank plays a crucial role. In particular, under the empirically plausible parametric assumptions entertained in our analysis, central banks supplying ample reserves can improve on allocations even by simply conducting optimal conventional monetary policy, i.e. even keeping the supply of reserves constant. Key in this proposition is the optimality of central bank decisions. Our analysis shows that, when the central bank does not operate optimally, activating two instruments can imply very different responses of the economy to shocks. We show this in a canonical DSGE model with financial intermediaries constrained by agency problems. A key contribution of our analytical set-up is to spell out more thoroughly the balance sheet of the central bank. Doing so we force the central bank to face explicitly the demand for its liabilities (money and reserves) and to face the competing demand for public debt coming from the financial sector.

Importantly, our analysis focuses on "conventional times", i.e. times in which the policy rate can freely move up or down (i.e. monetary policy can operate in a conventional manner). A large literature, also using models similar to ours, has convincingly argued that balance sheet interventions, during period of policy rates at their effective lower bonds can be effective. Our results thus should be seen as completing the emerging positive picture of the central bank's balance sheet as an additional policy tool. In particular, the average balance sheet as a policy tool has a strong macroprudential flavor. The average volume of reserves supplied by the central bank can affect the composition of depository institution's balance sheet. Obviously, there are many more instruments within the macroprudential trove that can have similar effects. We leave for future research the comparison between balance-sheet policies and classic macroprudential tools.

Relatedly, our model falls short of fully capturing the complexity of the broader policy problem. For the sake of clarity, we have decided to abstract from optimal fiscal policy. Fiscal decisions, especially concerning public debt issuance and maturity do play an important role in our model and deserve to be further investigated. Future work should relax our assumption of exogenous supply of public debt and bring this analysis to the next level: the joint optimal design of fiscal and monetary policy interventions.

Table 1: Parameters

Description	Parameter	Value
Households		
Discount factor	β	0.99
Labor share	α	0.3
Labor utility weight	χ	0.5
Risk-aversion	σ	4
Inverse Frisch elasticity	ψ	1.5
Cash demand	A_m, B_m	$0.0111, 0.0752$
Firms		
Capital adjustment cost	η	1.0
Depreciation	δ	0.025
Demand elasticity	σ_p	6
Calvo probability	ς	0.75
Banks		
Survival probability	θ	0.94
Start-up transfer	δ_T	0.008
Risk-weight coefficients		
Lending	κ_K	0.48
Long-term bonds	κ_B	0.15
Reserves	κ_F	0.08
Government		
Spending (as share of SS output)	G/Y	0.16
Perpetuity expiry probability	δ_p	0.95
Perpetuity fixed return	$\bar{r}_p$	0.01
Central Bank		
Inflation objective (quarterly)	π	$2\% p.a.$
Central bank transfer	γ_{CB}	0.9
Bond adjustment cost (deviation of SS)	$\lambda_{\mathcal{B}_{CB}}$	0.01
Bond adjustment cost (change)	$\lambda_{\Delta\mathcal{B}_{CB}}$	0.0001
Long-run bond-holdings	$\tilde{\mathcal{B}}_{CB}/\mathcal{B}$	0.17
Shocks		
All shock persistence	ρ_{shocks}	0.9
Productivity shock Stdev	σ_A	0.0006
Policy shock size	σ_R	0.0004
Net-worth shock Stdev	σ_N	0.0273
Bond-demand shock Stdev	σ_B	0.5859
Government spending shock Stdev	σ_B	0.0234

Table 2: Standard deviation in the data and in the model

	Stdev	Data (Quartiles) 25%	50%	75%	Model
Targeted	π_{yoy}	0.141	0.170	0.201	0.163
	R_d	0.310	0.415	0.439	0.133
	C/Y	0.764	0.855	1.33	1.19
	$spread$	0.281	0.364	0.432	0.392
	$spread_B$	0.301	0.319	0.34	0.273
	G/Y	7.54	8.43	10.5	4.88
Not-targeted	ΔY	0.509	0.549	0.687	0.513
	I/Y	6.18	8.61	11.7	5.73
	Δvel	1.11	1.15	1.44	0.217
	B_f/Y	44.4	66.6	191	1.01
	$\Delta B_{cb}/Y$	1.02	7.79	8.15	0.577

Note: Percentages. Sample 1987Q1-2019Q4. Interquartile range of 40 quarters rolling standard deviations.

Table 3: Mean and standard deviation under the two alternative policy regimes: Equal steady-state BS size

	MP+BSP			MP		
Variable	Det. ss	Mean	Stdev[§]	Det. ss	Mean	Stdev[§]
π	1	0.9994	0.0927	1	0.9995	0.0909
Y	1.913	1.958	4.312	1.913	1.958	4.3
$spread$	1.008	1.006	1.679	1.008	1.006	1.641
$P_B B_{cb}$	2.295	2.165	4.03	2.295	2.317	2.685
P_B	0.9816	0.9473	5.172	0.9816	0.9478	5.151
N_{cb}	1	1.008	2.018	1	1.009	2.019
R_d	1.009	1.007	1.66	1.009	1.007	1.649

[†] The steady-state balance sheet of the central bank under MP is 100% of that under MP+BSP.

[§] In percent.

Table 4: Mean and standard deviation under the two alternative policy regimes: Small BS under MP

	MP+BSP			MP[†]		
Variable	Det. ss	Mean	Stdev[§]	Det. ss	Mean	Stdev[§]
π	1	0.9994	0.0927	0.9998	0.994	0.7147
Y	1.913	1.958	4.312	1.913	2.029	5.314
$spread$	1.008	1.006	1.679	1.008	1.009	3.679
$P_B B_{cb}$	2.295	2.165	4.03	0.657	0.7952	35.21
P_B	0.9816	0.9473	5.172	0.9861	0.7237	10.93
N_{cb}	1	1.008	2.018	1	1.002	2.821
R_d	1.009	1.007	1.66	1.008	1.001	7.236

[†] The steady-state balance sheet of the central bank under MP is 10% of that under MP+BSP.

[§] In percent.

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Appendices

A Firms

The representative firm produces using a Cobb-Douglas production technology:

$$Y_t = A_t K_{t-1}^{\alpha} H_t^{1-\alpha}. \tag{A.1}$$

subject to the accumulation law for capital

$$K_t = I_t + K_{t-1}(1-\delta). \tag{A.2}$$

Price dispersion measures:

$$\Delta_t = \varsigma\, \Delta_{t-1} \left(\frac{\pi_t}{\pi}\right)^{\sigma_p} + (1-\varsigma)\ (\Pi_t^*)^{(-\sigma_p)} \tag{A.3}$$

Optimal price set by price-changing firms:

$$\Pi_t^* = \frac{F_t}{G_t} \tag{A.4}$$

where

$$F_t = Y_t\, MC_t + \varsigma\, \Lambda_{t+1|t} \left(\frac{\pi_{t+1}}{\pi}\right)^{\sigma_p} F_{t+1}, \tag{A.5}$$

and

$$G_t = \frac{Y_t\, P_t}{\frac{\sigma_p}{\sigma_p - 1}} + \varsigma\, \Lambda_{t+1|t} \left(\frac{\pi_{t+1}}{\pi}\right)^{\sigma_p - 1} G_{t+1}, \tag{A.6}$$

and MC_t is the marginal cost.
Relative producer price index dynamics are:

$$1 = \varsigma \left(\frac{\pi_t}{\pi}\right)^{\sigma_p - 1} + (1-\varsigma)(\Pi_t^*)^{1-\sigma_p} \tag{A.7}$$

The labor market clearing condition is:

$$W_t = MC_t(1-\alpha)\frac{Y_t}{H_t}, \tag{A.8}$$

and gross return on capital is given by:

$$R_{K,t} Q_{t-1} = \alpha MC_t \frac{Y_t}{K_{t-1}} + (1-\delta) Q_t. \tag{A.9}$$

and the investment Euler equation defines Tobin's Q:

$$Q_t = 1 + \frac{\eta}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \eta\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} - \eta \Lambda_{t+1|t}\left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right). \tag{A.10}$$

Finally, the goods market clearing condition is:

$$Y_t/\Delta_t = (1 + s(v_{D,t}) + s(v_{M,t}))C_t + I_t + \frac{\eta}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \eta\left(\frac{I_t}{I_{t-1}} - 1\right) + G. \tag{A.11}$$

B Estimation of the Taylor rule

To calibrate the model we assume that the central bank follows a Taylor-type rule for the deposit rate (the short-term market rate in our model).

We derive the coefficient of the policy rule by estimating a linear regression of the three-month treasury yield on one-period lag of the same and one lag of yoy inflation rate (measured using the GDP deflator).[28]. We dropped the growth rate of real GDP as it turned out to be insignificant. The regression results are shown in Table 5.

Table 5: Policy rule estimation results

	Dependent variable:
	3-month Treasury yield$_t$
3-month Treasury yield$_{t-1}$	0.961***
	(0.015)
Inflation$_{t-1}$ (yoy)	0.133***
	(0.030)
Constant	-0.0005^{**}
	(0.0002)
Observations	145
R^2	0.972
Adjusted R^2	0.972
Residual Std. Error	0.001 (df = 142)
F Statistic	2,507.247*** (df = 2; 142)
Note:	*p<0.1; **p<0.05; ***p<0.01

[28]All data retrieved from FRED, Federal Reserve Bank of St. Louis. Our sample consists of quarterly data from 1987Q2 to 2023Q2.