Threshold-based forward guidance: hedging the zero bound *

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Abstract

We use a simple New Keynesian model to study forward guidance policies following a large recessionary shock that drives the policy rate to the zero lower bound (ZLB). In our model, forward guidance takes the form of a state-contingent commitment to hold the policy rate at the ZLB until macroeconomic variables breach particular 'thresholds'. In common with other policies, threshold-based forward guidance can stimulate spending at the ZLB via a commitment to hold the policy rate 'lower for longer'. But threshold-based guidance also acts as a hedge against the asymmetric effects of shocks at the ZLB via endogenous adjustment of liftoff. As a consequence, the incentive to renege is lower compared to lower for longer policies based purely on calendar time. Crucially, we show that the existence of a unique equilibrium requires the policymaker to specify how the thresholds should be interpreted, as well as what those thresholds are. The optimal design of the threshold conditions is model specific and depends on the relative importance of those shocks that induce a trade-off between stabilising output and inflation and those that do not.

Key words: New Keynesian model; monetary policy; zero lower bound; forward guidance; thresholds

JEL Classification: E17; E31; E52

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1 Introduction

The financial crisis of 2007/08 generated a severe and prolonged global contraction in output: the 'Great Recession'. In response, central banks around the world cut their policy rates towards the zero lower bound and implemented a range of unconventional monetary policy measures, including an increased use of 'forward guidance' about the future path of the policy rate.

One motivation for forward guidance is as the communication of a promise to hold the policy rate at the zero bound for long enough to reduce long-term real interest rates and provide near-term stimulus (Woodford, 2012). This type of behaviour resembles optimal commitment policy at the zero bound in New Keynesian models as first argued by Krugman (1998) and subsequently demonstrated by Eggertsson and Woodford (2003).[1] However, policymakers have tended to distance themselves from this interpretation, in part because they seem skeptical about their ability to commit credibly to behaviour that is well known to be time inconsistent.[2]

In this paper we study a form of 'threshold-based' forward guidance (TBFG), in which the policymaker commits to hold the policy rate at the zero bound until certain macroeconomic variables breach pre-specified 'thresholds'. We investigate whether this form of forward guidance can be used as a temporary policy measure at the zero bound to improve outcomes, while limiting the extent to which the policymaker promises to behave in a time inconsistent manner.

Our analysis is motivated by policies implemented by the FOMC and the Bank of England's MPC, both of which stated that policy rates would not be increased at least until (among other conditions) the unemployment rate fell below particular threshold values.[3]

The framework for our analysis is a simple New Keynesian model that is the workhorse for several other studies of monetary policy at the zero bound (for example, Adam and Billi (2006) and Bodenstein et al. (2012)). The model consists of log-linearised equations describing aggregate demand (the 'IS' curve) and the pricing decisions of firms (the New Keynesian Phillips curve). The IS curve contains a stochastic 'demand shock' and the Phillips curve contains a stochastic 'cost push shock'.

The monetary policymaker sets the short-term nominal interest rate to minimise the expected discounted value of a loss function derived from a second order approximation to household's utility, subject to the zero lower bound constraint. Our baseline assumption is that the policymaker acts with 'discretion', taking the behaviour of future policymakers as given. Under these assumptions, policy is time consistent. We solve the model using global methods to account for the nonlinearity introduced by the zero bound and by the form of the threshold-based policies that we consider.

As is common in the literature on monetary policy at the zero lower bound, we examine what happens when a large negative demand shock causes the zero bound to become a binding constraint. With our baseline assumption of time-consistent monetary policy, we observe a deep recession. Because of the zero bound, the short-term nominal interest rate cannot be cut enough to reduce the *real* interest rate sufficiently to stabilise aggregate demand. This motivates our experiments in which the policymaker attempts to improve outcomes by temporarily deviating from time-consistent policy. Specifically, under threshold-based forward guidance the policymaker makes a state-contingent commitment to hold the policy rate at the zero bound for longer than agents were expecting under the time-consistent policy. Once the state of the

[1]There are several other policy prescriptions (like price level targeting or the Reifschneider and Williams (2000) rule) that can also deliver better outcomes at the zero bound via the same mechanism.

[2]For example, when describing the introduction of forward guidance by the Bank of England's Monetary Policy Committee, Bean (2013) argues that: "While such a time-inconsistent policy may be desirable in theory, in an individualistic committee like ours, with a regular turnover of members, it is not possible to implement a mechanism that would credibly bind future members in the manner required."

[3]While some central banks used forward guidance to inject stimulus, others emphasized their role in clarifying central bank behaviour rather than in providing stimulus which is not subject of this paper.

economy is such that the forward guidance regime has come to an end (i.e. once the economy has improved sufficiently), the policymaker reverts back to setting the optimal discretionary policy forever more.

One key contribution of our paper is to show that threshold-based forward guidance policy is incomplete in the absence of sufficient detail about how the policymaker will act when the thresholds are breached. Put differently, in order for the private sector to be able to understand the policy, it is not sufficient for the policymaker to announce a set of threshold conditions, it is also necessary for them to announce precisely what those conditions mean.[4] For example, the real-world policies of the FOMC and MPC drew a distinction between 'thresholds', a necessary but not sufficient condition for liftoff to occur, and 'triggers', a sufficient condition. In this paper we model exit probabilistically as an increasing function of the amount by which the thresholds have been breached. Our mapping function permits us to model triggers as a limiting case, as well as looser threshold conditions that are more in line with survey-based evidence of expectations following the FOMC's threshold-based guidance policy announcement, which suggested that market participants attached a non-negligible probability to the federal funds rate remaining unchanged after unemployment fell below its threshold.

Our baseline results compare the behaviour of the model under the time consistent policy and various forms of forward guidance with thresholds that approximate triggers on inflation and the output gap. We find that threshold-based forward guidance can substantially improve welfare relative to time consistent optimal policy. Part of the mechanism behind the result is straightforward. In line with the 'textbook' remedy to mitigating the zero bound constraint, threshold-based forward guidance can be used to stimulate activity and inflation today by promising higher inflation in the future. But, as well as improving economic outcomes in expectation, threshold-based guidance can also be used to manage the variance of the distribution of possible outcomes. Agents know that if further negative shocks arise, prolonging the recession, the policy rate will be held at the zero bound for longer. By contrast, if positive shocks arrive, so that the economy recovers more quickly from the recession than originally expected, then exit from the zero bound will occur sooner and the policy stimulus will be removed.

So threshold-based forward guidance can be viewed as a hedge against the asymmetric effects generated by the zero lower bound. The magnitude of the effect can be seen by comparing losses under threshold-based guidance with those under calendar-based forward guidance, in which the policymaker promises to hold the policy rate at the zero bound for a pre-specified length of time regardless of the state of the economy.[5] As in the case of threshold-based forward guidance, this can improve outcomes in expectation and eliminate the negative skew in outcomes induced by the zero lower bound. However, calendar-based guidance leads to worse outcomes for both positive and negative realisations of future demand shocks than appropriately calibrated threshold-based guidance because it provides too much stimulus in 'good' states and insufficient stimulus in 'bad' states.[6] As a result, the variances of the distributions of the output gap and inflation are substantially larger.

Because our policy experiments are based on a temporary deviation from time-consistent behaviour, they are (by definition) time inconsistent. As such, the experiments may be regarded as less than fully credible by agents in the model. We investigate this by computing a measure of the extent to which the policymaker could achieve better outcomes by reneging on the

[4]In a previous version of this paper we analysed policies in which the set of feasible equilibria are those in which the threshold conditions are *not* breached in any state of the world in which the forward guidance regime remains in effect with a unique equilibrium selected from that set by choosing the one that maximised the expected duration of the forward guidance regime.

[5]Early incarnations of forward guidance by the FOMC and Bank of Canada had a calendar-based flavour, though also included (informal) threshold-based clauses.

[6]This result verifies the assertion of Campbell et al. (2012) that calendar-based guidance is likely to generate poor outcomes if the economy evolves differently to initial expectations.

threshold-based policy and reverting to the time-consistent policy. A corollary of the hedging property of threshold-based forward guidance is that the temptation to renege is much smaller than for calendar-based guidance. For realisations of shocks in which the economy recovers more quickly than originally expected, calendar-based guidance generates too much stimulus and the policymaker has a strong incentive to revert to the time-consistent policy. By contrast, under threshold-based guidance, for realisations of the shocks in which the economy recovers more quickly, the exit conditions are more likely to be met and policy automatically reverts to time-consistent behaviour.

One criterion to rank alternative threshold policies is the ex-ante loss. Using this criterion, optimised threshold-based policies can achieve ex-ante losses that are close to the optimal commitment benchmark. In order to deliver this result, the thresholds must be chosen carefully. A general requirement is that the thresholds must be set to generate an overshoot of goal variables from target. But above and beyond that requirement the optimised threshold values depend on the structure of the economy, the nature of the disturbances, and the interpretation of the threshold conditions. For example, in the baseline calibration in which demand shocks are dominant in driving the model's dynamics, optimised inflation and output gap threshold policies deliver similar losses. By contrast, an optimised inflation threshold performs better than an optimised output gap threshold in a version of the model in which cost-push shocks are more important.

To our knowledge, this is the first paper to analyse threshold-based policies similar to those actually implemented in response to the financial crisis in a fully stochastic setting. The closest paper to ours is Florez-Jimenez and Parra-Polania (2014), who also study threshold-based guidance in a small model. But their analysis is limited to a two-period model with a threshold defined in terms of an exogenous shock process. By contrast, we analyse threshold-based policies of indefinite duration and specify thresholds in terms of endogenous variables. Coenen and Warne (2013) consider a more realistic model and policy experiment, examining how a form of inflation forecast threshold can alter the performance of calendar-based forward guidance in the ECB's DSGE model. However, given the size of that model, they are restricted to perfect foresight approximations of expectations, whereas we compute a fully stochastic equilibrium.

The rest of the paper is organised as follows. Section 2 details the model and the baseline description of policy. Section 3 describes the policy experiments and the assumptions underpinning them. Section 4 defines equilibrium for both threshold-based and calendar-based forward guidance policies. Section 5 describes the methods we use to solve for equilibrium. Section 6 outlines the parameterisation of the model and the calibration of the state of the economy prior to the implementation of forward guidance. Section 7 describes the simulation results, including comparisons of threshold-based guidance to calendar-based guidance and optimal commitment policy. Secion 8 examines the sensitivity of the results to alternative calibrations of the model and interpretations of the threshold conditions. Section 9 concludes.

2 The model

The model is identical to that used by Adam and Billi (2006, 2007) and Bodenstein et al. (2012) to study monetary policy at the zero bound under optimal commitment, optimal discretion and 'loose commitment' respectively.[7] It is a prototypical New Keynesian model in which a representative household supplies labour to firms and consumes a bundle of goods to maximize expected lifetime utility, and in which monopolistically competitive firms maximize the discounted sum of expected future profits subject to Calvo (1983) pricing rigidities. The

[7]Under the loose commitment framework there is an exogenous, constant probability that the policymaker will renege on past commitments and re-optimise.

first-order conditions for the household and firms, together with standard market clearing and aggregation conditions give rise to an Euler equation for output and an optimal pricing decision.[8] Following previous studies of monetary policy at the zero bound (e.g. Adam and Billi (2006), Adam and Billi (2007), Nakov (2008) and Bodenstein et al. (2012)), we use a partially log-linearized version of the model where the only nonlinearity is due to the zero bound and the optimality conditions are log-linearised around the non-stochastic steady state.[9]

Throughout our analysis, our baseline assumption is that the monetary policymaker sets policy under optimal discretion. Specifically, we assume that the policymaker chooses the policy instrument each period to minimise a loss function derived from a quadratic approximation to the representative agent's utility function,[10] taking agents' expectations as given. As in Adam and Billi (2007), the policymaker solves the following constrained minimisation problem:

$$\min_{\{y_t,\pi_t,r_t\}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2) \tag{1}$$

$$s.t \quad r_t \geq 1 - \frac{1}{\beta} \tag{2}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + u_t \tag{3}$$

$$y_t = \mathbb{E}_t y_{t+1} - \sigma (r_t - \mathbb{E}_t \pi_{t+1}) + g_t \tag{4}$$

$$u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_t^u \tag{5}$$

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_t^g \tag{6}$$

$$\mathbb{E}_t \{y_{t+i}, \pi_{t+i}, r_{t+i}\}_{i=1}^{\infty} \text{ given}$$

$$\{u_t, g_t\} \text{ given}$$

where: π is inflation, y is the output gap, and r is the policy rate (all expressed in deviations from steady state); $\beta < 1$ is the discount factor; $\kappa = \frac{(1-\xi)(1-\xi\beta)}{\xi} \frac{\sigma^{-1}+\omega}{1+\omega\theta}$ is the slope of the Phillips curve, where ξ is the probability that a firm cannot adjust its price, ω is the elasticity of a firm's real marginal cost with respect to its own output level and θ is the price elasticity of demand for the goods supplied by the monopolistic firms; σ is the intertemporal elasticity of substitution; $\lambda = \kappa/\theta$ is the relative weight on output in the loss function; u and g are exogenous disturbances to inflation and demand, often called cost push and demand shocks[11], both of which are assumed to follow AR(1) processes with $\varepsilon_t^u \sim iid$ N(0,1), $\varepsilon_t^g \sim iid$ N(0,1), ρ_u and ρ_g the persistence parameters, and σ_u and σ_g the standard deviations.

In any period where the zero bound is not binding, the solution to this problem is the well-known targeting rule (e.g. Gertler et al. (1999)):

$$y_t = -\frac{\kappa}{\lambda}\pi_t \tag{7}$$

In the presence of an occasionally-binding zero bound, the targeting criterion may not always be achievable (Adam and Billi (2007)). In particular, the policymaker is unable to perfectly stabilise the economy (delivering equation with $\pi_t = y_t = 0$) in the face of negative demand shocks if the policy rate is constrained by the zero bound. This requires us to use numerical methods to solve for the model's equilibrium, as described in Section 5.

[8] See Woodford (2003) for a detailed derivation and discussion.

[9] This is not an innocuous assumption. For example, Fernández-Villaverde et al. (2012) and Braun et al. (2013) have shown that non-linearities in the competitive equilibrium conditions can play an important role in the dynamics of New Keynesian models in the presence of an occasionally-binding zero bound.

[10] See Woodford (2003) for a derivation and discussion.

[11] The natural rate is related to the stochastic process, g, as follows: $g_t = \sigma r_t^*$, where r_t^* is the natural rate. The microfoundation of this shock is typically as a stochastic process for government spending (along with an assumption that government spending is entirely wasteful) or household's rate of time preference.

3 The nature of the policy experiments

The policy experiments are ones in which a policymaker temporarily deviates from setting policy optimally but in the absence of a commitment device (optimal discretion). The temporary deviation is a one-off and fully credible forward guidance policy with the objective of achieving better outcomes, given an economic environment in which the policy rate has become constrained by the zero bound. As detailed in Section 4, the forward guidance policies can be characterised as a commitment by the policymaker to hold the policy rate at the zero bound in certain states of the world, in the case of threshold-based forward guidance, or for a particular number of periods, in the case of calendar-based forward guidance.

The precise sequence of events in all of our policy experiments is as follows. In some arbitrary period, $t = 0$, a negative demand shock arrives that is sufficiently large that the policy rate consistent with optimal discretionary policy is negative and hence constrained by the zero bound. Having observed this shock and the subsequent outcomes, the policymaker announces a forward guidance policy that becomes effective in period $t = 1$ and remains in effect until the regime termination conditions have been met. Once the regime has ended, the policymaker reverts to setting policy by optimal discretion forever more.

There are two overarching assumptions governing the nature of our experiments. First, the forward guidance policy is assumed to be transitory or 'one off': before implementation, the policy is entirely unanticipated by agents in the model and, once the regime has ended, agents attach no probability to the policy being implemented again in the future. This assumption is common to several other papers in the literature that study temporary deviations of policy from a rule governing the timeless behaviour of the policymaker (e.g. del Negro et al. (2012), Coenen and Warne (2013), Haberis et al. (2014)). The assumption implies that such policy experiments are not conducted under fully rational expectations and so are subject to the issues discussed by Cooley et al. (1984) among others. Specifically, one may obtain misleading results from implementing a temporary policy regime change under the assumption that agents attach a zero *ex ante* probability to that regime change. In the context of our experiments with the policies implemented by some central banks in the wake of the financial crisis in mind, it is arguably reasonable to believe that the forward guidance policy may not have been anticipated, but is perhaps less reasonable to believe that agents would not expect policymakers to adopt a similar policy in the future, should the zero bound become a binding constraint on policy again. Such an anticipation would be expected to affect agents' decisions via expectations of how monetary policy will respond to future shocks. The results of our policy experiments are likely to be sensitive to this assumption.

Our second overarching assumption is that the forward guidance policy is fully credible. This assumption is seemingly at odds with a baseline description of policy being conducted in a fully time-consistent manner. Indeed, the mechanism by which the forward guidance policies we study are effective is through the manipulation of agents' expectations. In the absence of at least some credibility, the policymaker would be unable to affect agents' expectations and forward guidance of this sort would have no effect. Given the importance of this assumption, we pay particular attention to its likely validity by computing a measure of the incentive that the policymaker has to renege on the announced forward guidance policy. As argued by Nakata (2014), the assumption of full credibility may be reasonable if reneging on a policy has reputational costs for the policymaker. In that setting, the likelihood of the policymaker sticking to their policy plan (and hence the credibility of the announcement) depends on the costs and benefits of reneging: other things equal, a policy with a smaller incentive to renege is more likely to be viewed as credible than one with a larger incentive to renege.

4 Equilibrium in the forward guidance regime

The section defines equilibrium for threshold-based and calendar-based forward guidance policy given the environment described in Section 3 and the model described in Section 2.

4.1 Threshold-based guidance equilibrium

A key aspect of our approach is the assumption that exit from the forward guidance policy is probabilistic. That is, in the event that the threshold conditions are breached, exit from the forward guidance policy will occur with some probability strictly less than unity. There are two motivations for this approach.

The first motivation is to ensure that there is a unique equilibrium under threshold-based guidance. Using a simple deterministic example, we show in Appendix A that merely announcing threshold-based conditions for exit from the forward guidance policy may be insufficient for either existence or uniqueness of equilibrium. For example, we show that if exit is assumed to occur in the period that the threshold conditions are breached, then this may be inconsistent with agents expecting the policy to remain in place for long enough to generate a threshold breach that triggers exit. This type of result is a feature of the 'overshooting' generated by policy stimulus at the zero bound. In such cases, the equilibrium associated with a promise to exit with certainty once the threshold conditions have been breached may not be unique or may not exist at all. We also demonstrate that our probabilistic exit assumption alleviates this problem by making the expected exit date (a key determinant of the stimulus imparted by the policy) a continuous random variable.

The second motivation is that our probabilistic approach is consistent with the actual policies enacted by policymakers at the Federal Reserve and Bank of England, which clearly specified that the policy rate would remain at the zero bound "at least until" thresholds were breached.[12]

Implementing our approach requires two ingredients. First, we assume that the probability of exit is increasing in the distance between the threshold variable (for example, the output gap, y) and the threshold value, $\bar{y}$, according to an exponential distribution function:

$$f\left(y-\bar{y}\right)=\begin{cases}0 & \text{if } \ y\leq\bar{y}\\ 1-\exp\left(-\alpha_y^{-1}\left(y-\bar{y}\right)\right) & \text{if } \ y>\bar{y}\end{cases} \tag{8}$$

where the parameter $\alpha_y > 0$. The rationale for our choice of this function and its parameterisation are discussed in more detail in Section 6. At this point, we highlight that equation (8) embodies the assumption that the threshold is fully credible, in the sense that the probability that the policymaker exits is zero if the threshold is not breached ($y \leq \bar{y}$).

The second ingredient is a within-period timing assumption. Specifically, we assume that the sequence of events in each period t is as follows. First, shocks $\{\epsilon_t^u, \epsilon_t^g\}$ are realised and observed by all agents. Next, the private sector chooses $\{y_t, \pi_t\}$. Finally, the policymaker chooses the policy instrument r_t consistent with threshold-based guidance. For example, in the case of an output gap threshold, $\bar{y}$, the policymaker sets $r_t = 1-\beta^{-1}$ if $y_t \leq \bar{y}$, otherwise they will set r_t according to the optimal discretionary policy with probability $f\left(y_t-\bar{y}\right)$ and they set $r_t = 1-\beta^{-1}$ with probability $1-f\left(y_t-\bar{y}\right)$. That is, the policymaker exits the forward guidance regime with probability $f\left(y_t-\bar{y}\right)$.

Our timing assumption simplifies the computation of equilibrium because the policymaker's decision (whether or not to exit the forward guidance policy) is based on a variables that have already been determined by the decisions of private agents earlier in the period. By an appropriate choice of (8), it also allows us to approximate the situation in which exit

[12] For example, see the extract from the FOMC statement of December 2012 in Appendix B.

from the forward guidance policy occurs almost instantaneously when the threshold has been breached without introducing simultaneity between private sector and policymaker decisions.[13] Combined with a suitable calibration of the exit probability function (8), this allows us to examine threshold-based policies that approximate 'trigger' policies.

To define the equilibrium under the threshold-based guidance regime, we introduce some notation. The vector of endogenous variables is $x \equiv [\pi, y]$ and the state vector is $s \equiv [u, g]$. We drop time subscripts for period t variables and denote values in the following period using primes (e.g. s' is the state vector in the following period). We use the following notation for conditional expectations:

$$\mathbb{E}_{s'|s} h\left(x'\right) = \int h\left(x\left(s'\right)\right) \mathrm{d}W\left(s'|s\right) \tag{9}$$

for any function h, where $W\left(s'|s\right)$ is the distribution function of s' given s.

Formally, equilibrium in a threshold-based policy regime with inflation threshold, $\bar{\pi}$, and output gap threshold, $\bar{y}$ (so that $\bar{x} \equiv [\bar{\pi}, \bar{y}]$), is defined by policy functions, $\pi^{FG}(s)$ and $y^{FG}(s)$, that satisfy:

1. The competitive equilibrium conditions:

$$\begin{aligned} y^{FG}(s) &= p(s)\, \mathbb{E}_{s'|s} y^{OD}\left(s'\right) + \left(1 - p(s)\right) \mathbb{E}_{s'|s} y^{FG}\left(s'\right) \\ &\quad - \sigma \left\{ \begin{array}{l} p(s)\, r^{OD}(s) + \left(1 - p(s)\right)\left(1 - \beta^{-1}\right) \\ - \left[p(s)\, \mathbb{E}_{s'|s} \pi^{OD}\left(s'\right) + \left(1 - p(s)\right) \mathbb{E}_{s'|s} \pi^{FG}\left(s'\right)\right] \end{array} \right\} + g \qquad (10) \\ \pi^{FG}(s) &= \kappa y(s) + \beta \left[p(s)\, \mathbb{E}_{s'|s} \pi^{OD}\left(s'\right) + \left(1 - p(s)\right) \mathbb{E}_{s'|s} \pi^{FG}\left(s'\right)\right] + u \qquad (11) \end{aligned}$$

2. The probabilities of exiting (to optimal discretion) are given by the mapping:

$$p(s) = f\left(x^{FG}(s) - \bar{x}\right)$$

There are three features of this definition that are worth noting. First, expectations are defined as the probability weighted integral over all possible realisations of the shocks, accounting for the two different policy regimes: the case in which the forward guidance regime is still in effect (superscript ^{FG}), and the case in which policy has reverted back to optimal discretion (superscript ^{OD}). So the transmission of forward guidance policies in this model is via agents' expectations and the macroeconomic effect of the policy depends on the precise exit conditions that the policymaker specifies. It follows that threshold-based forward guidance can only affect outcomes to the extent that there are some states of the world in which the forward guidance regime still applies *and* those are states of the world in which the policy rate would be set to a positive value under optimal discretion. In this framework, threshold-based guidance is a state-contingent form of 'lower-for-longer' policy. Second, the 'one-off' nature of the policy is embodied in the equilibrium definition because state-contingent outcomes under optimal discretion are taken as given (and are not a function of outcomes in the forward guidance regime). Third, the function mapping threshold breaches to exit of the forward guidance policy is a crucial part of the equilibrium definition because it determines the probability of exit.

[13] For similar reasons to those discussed in Appendix A, simultaneous decision making may lead to situations in which an equilibrium does not exist. For example, conditional on the policymaker staying at the zero bound in the current period the private sector may choose an output gap or inflation level that breaches the threshold, but conditional on the policymaker exiting in the current period optimal private sector decisions may not breach the thresholds.

4.2 Calendar-based forward guidance equilibrium

The calendar-based forward guidance policy is characterised as a scalar number of time periods, K, for which the policymaker commits to hold rates at the ZLB regardless of the state of the economy. Equilibrium is defined by a *set* of policy functions, $\{\pi_t^{FG}(s), \ y_t^{FG}(s)\}_{t=1}^{K}$, that satisfy:

1. The competitive equilibrium conditions:
$y_t^{FG}(s) = \mathbb{E}_t^{FG}(s)\, y_{t+1} - \sigma\left(1 - \frac{1}{\beta} - \mathbb{E}_t^{FG}(s)\,\pi_{t+1}\right) + g$
$\pi_t^{FG}(s) = \beta\mathbb{E}_t^{FG}(s)\,\pi_{t+1} + \kappa y_t^{FG}(s) + u$, where:
$\mathbb{E}_t^{FG}(s)\, y_{t+1} = \mathbb{E}_{s'|s}\left[\mathbb{I}_{t+1}^{EXIT} y^{OD}(s') + \left(1 - \mathbb{I}_{t+1}^{EXIT}\right) y_{t+1}^{FG}(s')\right]$
$\mathbb{E}_t^{FG}(s)\,\pi_{t+1} = \mathbb{E}_{s'|s}\left[\mathbb{I}_{t+1}^{EXIT} \pi^{OD}(s') + \left(1 - \mathbb{I}_{t+1}^{EXIT}\right) \pi_{t+1}^{FG}(s')\right]$
$u' = \rho_u u + \epsilon^{u'}$
$g' = \rho_g g + \epsilon^{g'}$
$\epsilon^{u'} \sim \mathbb{N}(0, \sigma_u)$
$\epsilon^{g'} \sim \mathbb{N}(0, \sigma_g)$.

2. The criterion for exit:
$\mathbb{I}_t^{EXIT} = 0 \ \forall \ t <= K$ and $\mathbb{I}_{K+1}^{EXIT} = 1$.

As in the case of threshold-based guidance, it is evident from the equilibrium definition that calendar-based guidance affects economic outcomes in this setting via the manipulation of agents' expectations. The key distinction between the two policies is that regime exit is determined as a function of time under calendar-based guidance, while regime exit is determined as a function of the state of the economy under threshold-based guidance.

5 Solution method

5.1 Optimal discretion with a zero lower bound

To solve the model described in Section 2 we find time-invariant policies for inflation, $\pi^{OD}(s)$, and the output gap, $y^{OD}(s)$, that satisfy the equilibrium conditions (i.e. the Phillips and IS curves) and that solve the policymaker's optimal discretion problem, subject to the zero bound constraint and the stochastic cost-push and demand processes.

The zero bound means that there is no analytical solution to this problem, so it is necessary to use numerical methods to approximate the solution. In doing so, we follow the approach described in Adam and Billi (2007). The approach is a time iteration implementation of policy function approximation using linear interpolation and quadrature to approximate expectations. The algorithm is initialised with a guess for the solution defined on a pre-specified grid of values for the state variables (cost-push and demand process outturns). For our initial guess, we use the solution to a version of the model in which the zero bound is ignored (which can be solved analytically). The algorithm is then comprised of an outer layer and an inner layer. In the outer layer, the output of each successive time iteration is a new guess at the solution on the state grid, using the previous guess to approximate agents' expectations for inflation and the output gap at each node in the state grid (which represents a particular combination of cost-push and demand process outturns). In the inner layer, outcomes for the endogenous variables are solved analytically as a sequence of independent static problems (for each node in the state grid) conditional on the approximation of expectations.[14] The time iteration is terminated when the difference between the latest guess for the solution (the output of the time iteration) and the

[14] First, solve for outcomes on the assumption that the zero bound is not binding in the following way: (i) use the first-order condition for the policymaker in equation (2) to substitute the output gap out of the Phillips

previous guess (the input of the time iteration used to approximate expectations) is sufficiently small.

We implement the algorithm using a $20,000$ state grid formed of the tensor product of 100 and 200 node uni-dimensional grids of values for the cost-push and demand states respectively. These nodes are uniformly spaced between lower and upper bounds for each state, set to ensure that the policy experiment simulations are unlikely to require us to extrapolate the policy functions. This means that the lower and upper bounds for both states in the grid are functions of the particular parameterisation of the model we use. In the case of the baseline parametrisation outlined in Section 6, the bounds for the cost-push and demand state are set to ± 0.66 and ± 22 respectively (reflecting that the demand process is more persistent and has a higher variance than the cost-push process). In approximating expectations at each node in the state grid, we use a 25 node quadrature scheme formed of the tensor product of two separate 5 node Gauss-Hermite schemes for the cost-push and demand shocks. We terminate the time iteration when the largest absolute difference between the latest and previous guesses for the policy functions is less than $1e^{-6}$.[15]

5.2 Threshold-based guidance experiments

The objective is to find policy functions for inflation, $\pi^{FG}(s)$, and the output gap, $y^{FG}(s)$, that satisfy the equilibrium conditions (i.e. the Phillips and IS curves) and are consistent with the exit probabilities of the regime, as defined in Section 4.1. We use a time iteration approach, solving for policy functions conditional on a guess for exit probabilities and updating exit probabilities according to the function f in an iterative fashion.

The structure of the algorithm is as follows, where the subscript $\langle i \rangle$ denotes iteration i:

0. Initialise policy functions $y^{FG}(s)$, $\pi^{FG}(s)$ and the probabilities $p(s)$. Policy functions are initialised using the policy functions under optimal discretion:

$$x^{FG}_{\langle 0 \rangle}(s) \equiv \begin{bmatrix} y^{FG}_{\langle 0 \rangle}(s) \\ \pi^{FG}_{\langle 0 \rangle}(s) \end{bmatrix} = \begin{bmatrix} y^{OD}(s) \\ \pi^{OD}(s) \end{bmatrix}$$

   The probabilities are initialised as

$$p_{\langle 0 \rangle}(s) = \delta_0 f\left(x^{FG}_{\langle 0 \rangle}(s) - \bar{x}\right) + (1 - \delta_0)\mathbf{1}$$

   where $\delta_0 \in (0, 1]$ is a damping factor and $\mathbf{1}$ is the unit vector (i.e., exit with certainty for every state s).

   Then for each iteration, $i = 1, \ldots$:

curve (equation (3)) and rearrange to compute inflation as a function of expected inflation and the cost-push state; (ii) compute the output gap using the policymaker's first-order condition; (iii) rearrange the IS curve (equation (4)) to compute the interest rate as a function of the output gap, the expected output gap, expected inflation and the demand state. If the interest rate is greater than or equal to the zero bound $(1 - \beta^{-1})$, then the solution (conditional on expectations) has been found and stop. If the interest rate violates the zero bound constraint then: (i) set the interest rate equal to $1 - \beta^{-1}$; (ii) compute the output gap conditional on the interest rate, expectations and the demand state using the IS curve; (iii) compute inflation conditional on the output gap, expectations and the cost-push state using the Phillips curve.

[15] The algorithm takes 151 iterations to converge in 67 seconds in 64-bit MATLAB 2012b using a single Intel i7 CPU @ 2.90GHz. Key to that performance is the pre-computation of the state index numbers and weights for linear interpolaton in the approximation of expectations (noting that all the state variables are exogenous and so each possible realisation of next period's state given the quadrature scheme and this period's state is known in advance and does not vary across the iterations).

1. Taking $p(s)$ as given, solve equations (10) and (11) using time iteration. This is done in an analogous fashion to optimal discretion.[16] Denoting policy functions based on time iteration j conditional on $p_{\langle i-1\rangle}(s)$ as $x^{FG}_{\langle j|i-1\rangle}$, iteration proceeds until $\left\|x^{FG}_{\langle j|i-1\rangle} - x^{FG}_{\langle j-1|i-1\rangle}\right\|_{\infty} < \tau$. The resulting policy functions are denoted $x^{FG}_{\langle *|i-1\rangle}$.

2. The policy functions from step 1 are used to update the exit probabilies $p_{\langle i\rangle}(s)$. We first compute the probabilities consistent with the latest estimate of the policy functions using the mapping (12):
$$\widetilde{p}(s) = f\left(x^{FG}_{\langle *|i-1\rangle}(s) - \bar{x}\right) \tag{12}$$
The probabilities are then updated for the next iteration i according to
$$p_{\langle i\rangle}(s) = (1-\delta)\, p_{\langle i-1\rangle}(s) + \delta\widetilde{p}(s) \tag{13}$$
where $\delta \in (0,1]$ controls damping.

3. Check for convergence.
   (a) Convergence is achieved if:
      (i.) The policy functions have converged $\left\|x^{FG}_{\langle *|i\rangle} - x^{FG}_{\langle *|i-1\rangle}\right\|_{\infty} < \varepsilon_x$; *and*
      (ii.) The exit probabilities have converged $\left\|\widetilde{p}_{\langle i\rangle}(s) - \widetilde{p}_{\langle i-1\rangle}(s)\right\|_{\infty} < \varepsilon_p$.
   (b) If convergence is not achieved return to step 1.

We implement this algorithm using the same 20,000 node state grid and linear interpolation scheme described in Section 5.1 and the same quadrature nodes for cost-push and demand shocks. The overall tolerance applied to policy function convergence is the same as for the optimal discretion solution (i.e., $\varepsilon_x = 1 \times 10^{-6}$) and the convergence for exit probabilities is set to $\varepsilon_p = 1 \times 10^{-4}$.[17] The 'within iteration' convergence criterion for the time iteration step 1 was set to a relative loose value, $\tau = 1 \times 10^{-1}$, in conjunction with substantial damping of the exit probability updates: $\delta = 0.025$ in most cases.[18]

This design reflects the very strong feedback from the exit probabilities p to the policy functions x, which motivates substantial damping in the updating of exit probabilities in Step 2. Given this damping, the algorithm takes longer to converge (around 10 times longer than the optimal discretion solution). For this reason, the tolerance τ is set loosely to avoid over-refining policy function estimates conditional on exit probabilities that are far away from the equilibrium probabilities. As the policy functions converge, updating of the policy functions and exit probabilities becomes sequential, speeding convergence.[19]

[16] For example, in each iteration we solve the IS curve (10) for $y^{FG}(s)$ for each s by computing the terms on the right hand side as follows. The terms $p(s)$, $\left(1-\beta^{-1}\right)$, $g(s)$ and $r^{OD}(s)$ are known. The expressions $\mathbb{E}_{s'|s}y^{OD}(s')$ and $\mathbb{E}_{s'|s}\pi^{OD}(s')$ are also known (we pre-compute them using the OD policy functions). The expectations $\mathbb{E}_{s'|s}y^{FG}(s')$ and $\mathbb{E}_{s'|s}\pi^{FG}(s')$ can be computed by numerically approximating the integral using the previous guess for the FG policy functions (just as we do in each iteration of the solution for the OD policy functions). The Phillips curve is solved in an analogous manner.

[17] This was rarely the binding constraint on convergence. The exit probabilities had typically converged to within 1×10^{-5} or less by the time that the policy functions had converged.

[18] More moderate damping was used for the initialisation, $\delta_0 = 0.05$. For cases in which the threshold values $\bar{x}$ were further from zero, even stronger damping was required and $\delta = 0.005$ was used.

[19] That is Step 1 converges to the required tolerance, τ in a single iteration.

5.3 Calendar-based guidance experiments

Solving for the approximate policy functions that characterise a one-off calendar-based forward guidance policy is relatively straightforward via backward induction. In period K, the final period of the regime, the policy functions can be computed under the assumptions that the policy rate is pegged at the zero bound regardless of the state and that expectations are determined by outcomes in the optimal discretion regime. With the period K policy functions in hand, it is straightforward to work backwards from period $K-1$ to period 1 imposing that the policy rate is pegged at the zero bound and using the policy functions already computed for the period ahead to approximate expectations. We use the same state grid, linear interpolation and quadrature schemes as detailed above.

5.4 Optimal commitment

In Section 7 we compare outcomes generated by threshold-based forward guidance with other policies. A natural benchmark is the optimal commitment policy as it delivers the best achievable outcomes. In the case of optimal commitment, the policymaker is able to commit to an interest rate plan that minimises the entire discounted sum of future losses subject to the zero lower bound constraint on interest rates and the equilibrium conditions:

$$
\begin{aligned}
&\min_{\{y_t,\pi_t,r_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t(\pi_t^2 + \lambda y_t^2) \\
&s.t \;\; r_t \geq 1 - \frac{1}{\beta} \\
&\quad\;\; \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + u_t \\
&\quad\;\; y_t = \mathbb{E}_t y_{t+1} - \sigma\left(r_t - \mathbb{E}_t \pi_{t+1}\right) + g_t \\
&\quad\;\; u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_t^u \\
&\quad\;\; g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_t^g \\
&\quad\;\; \{u_0, g_0\} \;\; \text{given}
\end{aligned}
$$

As in the case of optimal discretion, in the presence of an occasionally-binding zero bound constraint, it is not possible to solve for the equilibrium of the economy analytically.[20] Furthermore, unlike in the case of optimal discretion, the optimal targeting rule that would apply in the absence of the zero bound is invalid even if the zero bound is not binding in the current period, provided that it has bound at some point in the past. This is a direct consequence of the history dependence of policy which must be accounted for when the model is solved. We solve the model using the approach of Adam and Billi (2006).

6 Parameterisation and experiment scenario

For the baseline, which we use to conduct the majority of the analysis in Section 7, we parameterise the model in exactly the same way as Adam and Billi (2006), Adam and Billi (2007) and Bodenstein et al. (2012).[21] The baseline parameter values we use are outlined in Table 1

[20] There are analytical expressions that characterize the solution – see Adam and Billi (2006) – but they also include Lagrange multipliers from the first-order conditions to the Lagrangian representation of the constrained minimization problem.

[21] The parameters κ, σ and λ originate from Woodford (2003). The parameters of the stochastic processes and the discount factor were estimated by Adam and Billi (2006) on US data using the approach of Rotemberg and Woodford (1998).

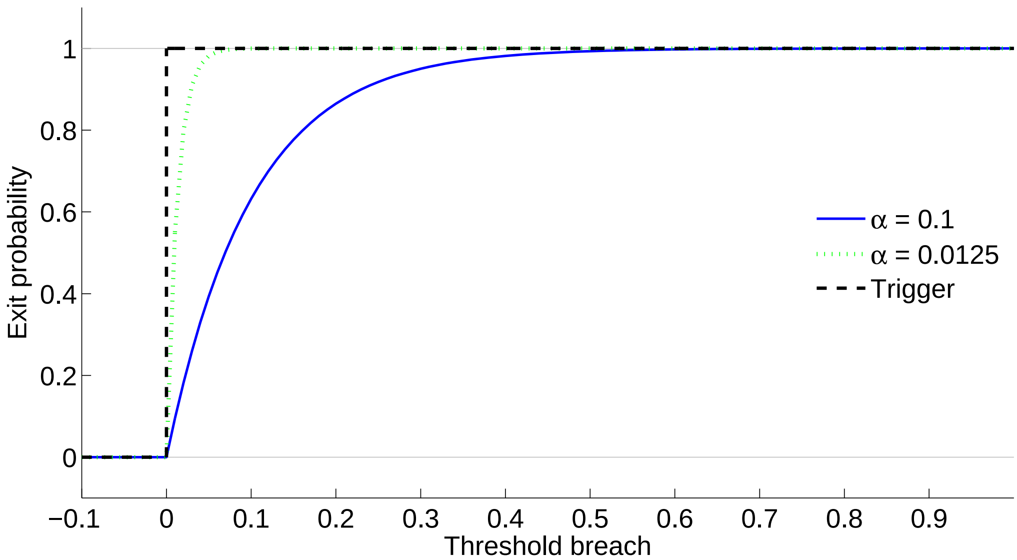
(where the model is interpreted as a quarterly model). Sensitivity of our policy experiments to an alternative parameter values is discussed in Section 8.

Table 1: Baseline model calibration

Parameter	Description	Value
ξ	Calvo parameter	0.6600
β	Discount factor	0.9913
σ	Intertemporal elasticity of substitution	6.2500
θ	Price elasticity of demand	7.6600
ω	Elasticity of marginal cost	0.4700
ρ_u	Persistence of cost-push process	0.0000
σ_u	Standard deviation of cost-push shocks	0.1540
ρ_g	Persistence of demand process	0.8000
σ_g	Standard deviation of demand shocks	1.5240
κ	Slope of the Phillips curve	0.0240
λ	Weight on output in loss function	0.0031

A key element of our approach is the function that determines exit probabilities p from the extent to which the threshold $\bar{x}$ is breached. As noted in Section 4.1, we choose an exponential distribution function (8). This distribution is attractive for two reasons. First, it has just one parameter, which minimises the number of degrees of freedom when specifying the distribution. The second, related reason is that the α parameter acts as an index of the extent to which the function approaches a 'trigger': as $\alpha \to 0$, the density function gets steeper (see Figure 1).

Figure 1: Alternative calibrations of the mapping $f(x - \bar{x}) = 1 - \exp(-\alpha^{-1}(x - \bar{x}))$



We use different baseline values of α_y and α_π (for output gap thresholds and inflation thresholds respectively) because the slope of the Phillips curve implies that the size of output gap and inflation responses to shocks are markedly different. In Appendix B we use survey evidence to find baseline values of these parameters that imply f functions for output gap and inflation thresholds that are similarly 'steep'. This delivers baseline values of $\alpha_y = 0.1$ and $\alpha_\pi = 0.0125$.

As explained in Section 4.1, allowing for the possibility that exit from the forward guidance regime will not occur with certainty once the threshold has been breached is important to deliver an equilibrium. However, we choose our baseline parameterisations of α_y and α_π to be close to a 'trigger'. This decision is motivated by a desire to ensure that the results are not excessively influenced by the choice of the f function. In other words, we wish to minimise the extent to which the results rely on the assumed link between threshold breaches and exit probabilities. Although the approximation to a trigger is a useful benchmark, Appendix B presents evidence that these thresholds are tighter than implied by survey evidence of financial market participants when the FOMC announced threshold-based guidance in December 2012. Reflecting that evidence, we examine sensitivity of our results to a looser calibration for α_y and α_π in Section 8.1.

We also examine the behaviour of dual thresholds, applied to both the output gap and inflation. Such policies are arguably better approximations to real-world policies. Indeed, the threshold-based forward guidance policies implemented by the FOMC and the Bank of England's MPC applied conditions to more than one macroeconomic variable.[22] We consider two simple approximations to a 'dual threshold' applying to both the output gap and inflation. In the 'OR' specification, the probability of exit from the forward guidance regime is zero if neither variable satisfies the threshold condition, but is positive if at least one variable breaches its threshold. In the 'AND' specification, the probability of exit is only positive if both variables breach the thresholds. We use the same exponential functions to compute exit probabilities as in the baseline case of single variable thresholds. The way they are combined for the 'OR' and 'AND' variants are given in equations (14) and (15) respectively:

$$f(x-\bar{x}) = \begin{cases} 0 & \text{if } \ y \leq \bar{y}, \pi \leq \bar{\pi} \\ 1-\exp\left(-\alpha_y^{-1}(y-\bar{y})\right) & \text{if } \ y > \bar{y}, \pi \leq \bar{\pi} \\ 1-\exp\left(-\alpha_\pi^{-1}(\pi-\bar{\pi})\right) & \text{if } \ y \leq \bar{y}, \pi > \bar{\pi} \\ \left[1-\exp\left(-\alpha_y^{-1}(y-\bar{y})\right)\right]\left[1-\exp\left(-\alpha_\pi^{-1}(\pi-\bar{\pi})\right)\right] & \text{if } \ y > \bar{y}, \pi > \bar{\pi} \end{cases} \tag{14}$$

$$f(x-\bar{x}) = \begin{cases} \left[1-\exp\left(-\alpha_y^{-1}(y-\bar{y})\right)\right]\left[1-\exp\left(-\alpha_\pi^{-1}(\pi-\bar{\pi})\right)\right] & \text{if } \ y > \bar{y}, \pi > \bar{\pi} \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

As described in Section 3, the policy experiments are ones in which a large negative demand shock drives the policy rate to the zero bound, prompting the policymaker to implement a one-off forward guidance policy. We calibrate the size of the demand shock to deliver a modal fall in the output gap of 7.5pp in period one of our simulations for a policymaker who continues to follow optimal discretion.[23] This is approximately equal to the amount by which quarterly GDP fell in the United States during the Great Depression. Section 8.3 examines sensitivity to an alternative inital condition calibrated to match the fall in output in the United States during the Great Recession.

7 Results

In Section 5 we showed how to solve for equilibrium under threshold-based forward guidance for particular values of the thresholds $\bar{\pi}$ and $\bar{y}$. Here we consider results for cases in which the values of these thresholds have been optimised to deliver the minimum welfare loss (as measured by equation (1)). To do this we solve for the policy functions for pairs $\{\bar{y}, \bar{\pi}\}$ on a grid. For each pair we use the policy function to construct 100,000 simulations of length 24

[22]In both cases, a threshold was applied to the unemployment rate but the policy was also contingent on inflation expectations remaining well anchored. In the case of the Bank of England, the conditions applied to inflation expectations (and also financial stability) were termed 'knockouts' indicating a lexicographic dominance over the unemployment threshold: see Monetary Policy Committee (2013) for a comprehensive discussion.

[23]In Section 7, we use modal simulations: those in which no further shocks arrive in periods $t = 1, \ldots$ as one way of comparing responses under different policies.

periods (starting from the initial condition $g_0 = -9.4$) and compute the mean discounted loss across those simulation paths.[24] Results for threshold values that deliver the minimum average loss are reported here.

7.1 Headline results

We first consider the performance of alternative specifications of threshold-based guidance, with reference to the baseline policy of optimal discretion. Specifically, we consider policies based on a single inflation threshold ($\bar{\pi}$), a single output gap threshold ($\bar{y}$) and 'dual' threshold specifications. The dual threshold specifications are such that the probability of exit from threshold-based guidance depends on both the output gap and inflation relative to their threshold values. The dual threshold cases are specified as 'OR' and 'AND' variants, as described by equations (14) and (15) respectively, according to whether one or both of the thresholds must be breached for there to be a non-zero probability of exiting the threshold-based guidance policy.

Table 2: Results for baseline calibration of model

Threshold type	$\bar{\pi}^*$	$\bar{y}^*$	Loss	$\frac{\text{Loss}}{\text{Loss(OD)}}$
Inflation threshold	0.15	–	0.377	0.444
Output gap threshold	–	2	0.334	0.394
Dual OR threshold	0.3	2.25	0.333	0.392
Dual AND threshold	-0.05	1.75	0.332	0.391

Table 2 records the optimised threshold values for a variety of threshold specifications and the average losses achieved. All threshold specifications considered reduce losses relative to the baseline optimal discretion policy by more than 50% (final column).

Although the expected loss associated with the optimised inflation and output gap thresholds is similar, the alternative policies do not deliver the same outcomes in all circumstances. Under an inflation threshold, exit from forward guidance can be triggered by either a demand or cost-push shock. By contrast, exit is less dependent on cost-push shocks for an output gap threshold-based policy. This reflects that cost-push shocks do not affect output directly in the baseline parameterisation of the model in which they are assumed to be *iid*.[25]

This logic also explains why an output gap threshold can deliver comparable results to an inflation threshold despite the relatively small weight on output in the loss function (Table 1). The benefit of an inflation threshold is that it can directly mitigate losses arising from high inflation. The cost is that exit from the forward guidance regime can be triggered by transitory cost-push shocks in states of world where underlying inflationary pressure is weak (because demand is weak). Unsurprisingly, this result is overturned when cost-push shocks are autocorrelated, as shown in Section 8.2. In that case, the benefit associated with avoiding high inflationary losses exceeds the cost of cost-push driven exit in weak demand states of the world and an optimised inflation threshold performs better than an optimised output gap threshold.

We also observe that the optimised threshold values depend on the way that dual threshold policies are specified. For the 'AND' variant, the optimised values for the inflation and output gap thresholds ($\bar{\pi}^*$ and $\bar{y}^*$) are both larger than the optimised values for individual inflation and output gap thresholds. In contrast, for the 'OR' variant, the optimised threshold values

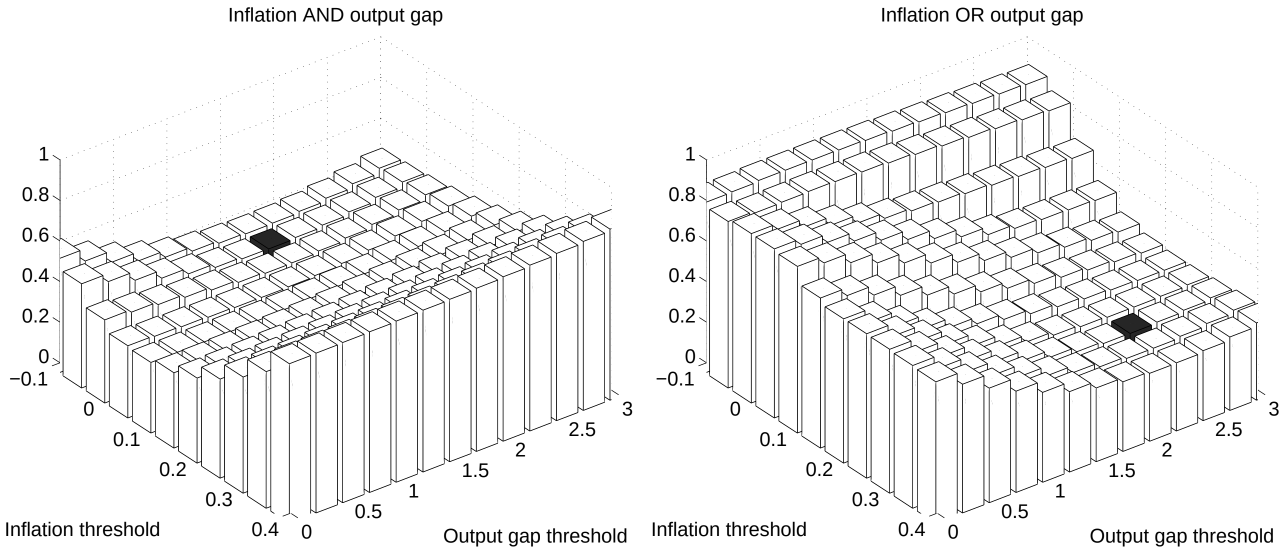
[24] The grid for $\bar{\pi}$ runs from -0.4 to 0.4 in increments of 0.05 and the grid for $\bar{y}$ runs from -1 to 6 in increments of 0.25.

[25] The presence of cost-push shocks in the model does affect output via the optimal behaviour of the policymaker under optimal discretion, but that reflects an active response by the policymaker to act on the inflationary consequences of cost-push shocks, which is not present when rates are held at the zero bound.

are smaller than the optimised values for individual inflation and output gap thresholds. The *ex ante* losses associated with these policies are almost identical.

This result reflects the role of cost-push shocks (u) in generating a tradeoff between inflation and output stabilisation, an issue that we will discuss further in Section []. The intuition for the result is relatively straightforward. Starting first with the 'OR' specification, we note that, for given values of $\bar{y}$ and $\bar{\pi}$, there are more states of the world in which at least one of the two thresholds is breached than in which either threshold is breached individually.[26] As a result, an 'OR' dual threshold specification for given $\bar{y}$ and $\bar{\pi}$ values will impart less stimulus than individual policies based on the same threshold values. So delivering the optimal state contingent stimulus with an 'OR' specification requires higher threshold values for $\bar{y}$ and $\bar{\pi}$.

Figure 2: Losses delivered under alternative dual threshold specifications



The intuition for the 'AND' specification is the converse. For given values of $\bar{y}$ and $\bar{\pi}$, there are fewer states of the world in which both of the two thresholds are simultaneously breached than in which either threshold is breached individually. As a result, the optimal state contingent stimulus can be achieved with lower threshold values because a positive exit probability requires both thresholds to be simultaneously satisfied. The fact that the inflation threshold is below the target implies that there are some states of the world in which an output gap based threshold would not permit the policymaker to exit the forward guidance regime even when a positive cost push shock arrives. The 'AND' specification allows the policymaker to exit in response to such shocks, if the state of demand is sufficiently strong (that is, if the output gap threshold is also breached). Figure 2 illustrates this logic by showing the shape of the loss surface for the set of $\{\bar{y}, \bar{\pi}\}$ grid points considered, with the loss-minimising case highlighted for the 'OR' and 'AND' specifications.

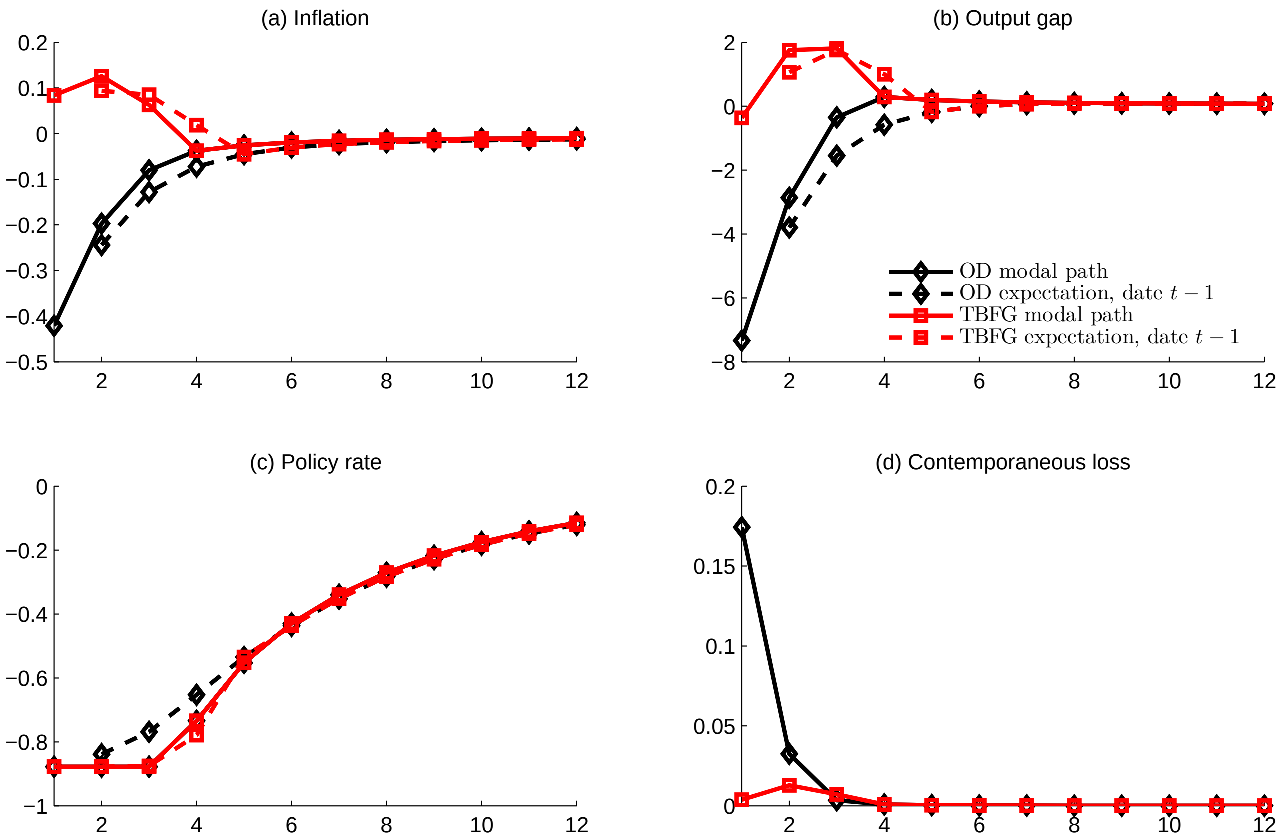
7.2 Inspecting the mechanism

To inspect the mechanism at work, we examine the most likely (that is, modal) outcomes from the optimised dual threshold-based policy[27] and compare them to the modal outcomes under the baseline optimal discretion policy. These paths are constructed by initialising the simulation using $g_0 = -9.4, u_0 = 0$ and assuming that the shocks for period 1 onwards are equal to their modal value of zero: $\epsilon_t^g = \epsilon_t^u = 0, t = 1, \ldots$.

[26] This follows directly from inspection of the probability mapping function (14).
[27] The 'AND' formulation with $\{\bar{y}^*, \bar{\pi}^*\} = \{1.75, -0.05\}$: see Table 2.

Panels (a)–(c) of Figure 3 plot the modal responses of the endogenous variables (measured in quarterly deviations from steady state) given the alternative policy strategies. Panel (d) shows the loss in each period associated with the per-period outcomes for the output gap and inflation generated by the two policies.

Figure 3: Modal responses under optimised dual threshold policy and optimal discretion



Notes: The baseline policy assumption of optimal discretion (OD) is represented by the black lines with diamond markers. The loss-minimising threshold-based forward guidance (TBFG) policy is shown in the red lines with square markers. This corresponds to a dual threshold 'AND' specification with $\{\bar{y}^*, \bar{\pi}^*\} = \{1.75, -0.05\}$. The responses are computed under the assumption that no shocks arrive after the initial period 0 and from the initial condition $g_0 = -9.4, u_0 = 0$. Inflation, the output gap and the policy rate are plotted as quarterly percentage point deviations from steady state.

Under the discretionary policy, inflation and the output gap are negative while the policy rate is at the zero bound because the policymaker cannot cut rates below zero and cannot commit to any policy plans that would be inconsistent with loss minimization in the future. Other things equal, this reduces expectations of future inflation and activity, which in turn reduces current spending and inflation. This effect can be seen in the dashed lines (with diamond markers) which plots the expected outcomes at date t, conditional on information at the end of period $t-1$. Under optimal discretion the modal paths for inflation and the output gap lie above the sequences of outcomes expected at the end of the preceding period. This reflects the well-known negative skew in the distribution of inflation and outcome under discretionary policy when the zero bound is binding. The negative skews arise because in states of the world where demand is low, the policymaker has no ability to stimulate the economy by reducing the policy rate or by manipulating expectations. Indeed, the expected outcomes for the policy rate lie above the modal outcomes reflecting the fact that the policymaker cannot commit to holding the policy rate below the rate consistent with period-by-period optimisation.

In contrast, outcomes under threshold-based forward guidance embody a temporary commitment mechanism such that the expectations for the policy rate in periods 1–3 are below the modal path. These expectations are consistent with the thresholds not being breached in period 1 or expected to be breached in period 2.[28] The lower expected path for the policy rate, relative to optimal discretion, increases the expected amount of policy stimulus thus reducing the skew in the distributions for inflation and the output gap in periods 1 and 2.

Note also that inflation and the output gap deviate from the optimal discretion equilibrium in period 4 even though the policymaker exits the threshold-based policy (and lifts off from the zero bound) in this period. This reflects our within-period timing assumption: output and inflation are chosen before the policymaker chooses the policy rate. These choices are influenced by the expected policy rate in period 4, which lies slightly below the modal path.[29]

7.3 Threshold-based guidance versus other policies

In this section we compare the optimised dual threshold policy to two alternatives: a calendar-based forward guidance policy and the optimal commitment policy. For each alternative policy, we compute the equilibrium policy functions as described in Sections 5.3 and 5.4. These policy functions are used to construct modal paths and to generate simulations from the initial condition $g_0 = -9.4, u_0 = 0$.[30] These simulated paths are used to compute the average loss. In the following, we focus on the loss-minimising calendar-based forward guidance policy which involves holding the policy rate at the zero bound for 4 periods ($K = 4$).

Figure 4 shows that modal paths for the optimised threshold-based forward guidance policies are close to the optimal commitment benchmark.[31] As documented in Adam and Billi (2006) and Nakov (2008), the optimal commitment policy stabilizes the economy by promising inflation above target and positive output gaps in the future. An (appropriately calibrated) calendar based guidance has similar effects. A calendar-based guidance policy that holds the policy rate at the zero bound for an additional period (that is, setting $K = 4$) stimulates inflation and activity today via the effect of a commitment to looser policy in the future. This mechanism is not unique to the policies we consider here. A common theme of related work is that history dependent policies such as optimal commitment, price level targeting or the Reifschneider-Williams rule can substantially improve outcomes at the zero bound by manipulating inflation expectations to reduce the ex-ante real interest rate as a substitute for cutting the policy rate.[32]

The modal paths under optimal commitment, calendar-based forward guidance and threshold-based guidance in panels (a)–(c) of Figure 4 are relatively similar. Although, relative to the other policies, calendar-based guidance fails to stabilise the output gap as effectively in period 1, the overshoot in periods 2 and 3 is smaller. As a result, inflation is better stabilised in periods 2 to 4 and per-period losses are smaller in these periods. Despite these differences, all three alternative policies appear to substantially outperform the baseline assumption of optimal discretion. However, a proper assessment of the policies requires consideration of the distribution

[28] To be precise, the expected values for the output gap and inflation do not breach the thresholds. There is, of course, a positive probability that the thresholds will be breached in period 2 as there will be states in which the cost push or demand shocks are sufficiently large to generate a breach.

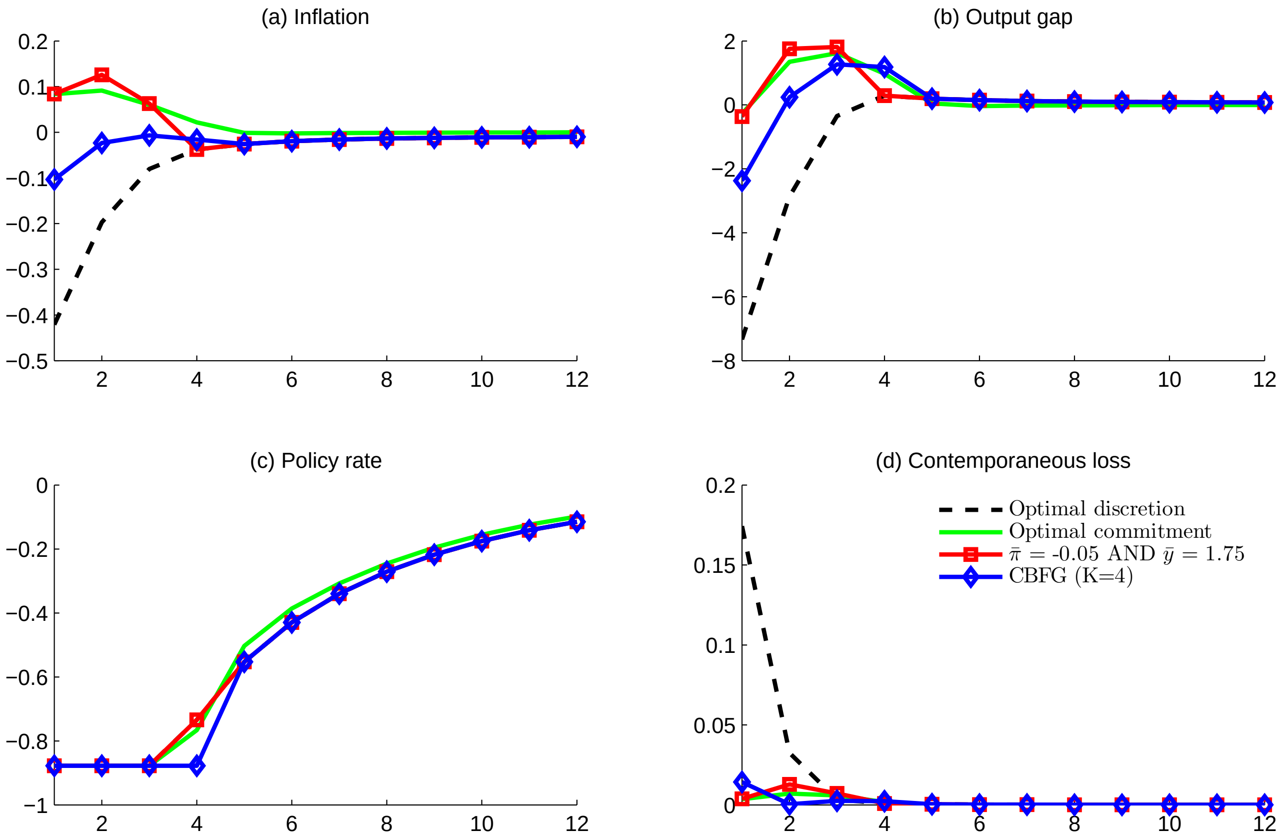
[29] The modal path is selected by choosing the exit date with the highest *ex ante* probability, computed using the outcomes generated under the modal sequence of shocks $\epsilon_t^g = \epsilon_t^u = 0, t = 1, \ldots$. The *ex ante* probability of exit occurring in period 4 is around 0.43.

[30] We use the same set of 100,000 simulated shock trajectories for all policy specifications.

[31] The finding that the optimal commitment policy does not keep the policy rate at the ZLB longer than the optimal discretionary policy if the state evolves in line with expectations is just a coincidence in our particular experiment. If the initial condition for the demand state is set to -10 instead of -9.4, the modal ZLB duration under the optimal commitment policy is one period longer than under optimal discretion. This is a consequence of the discrete-time setting used here. Werning (2011) uses a continuous-time setting to show that optimal commitment always involves setting the policy rate at the zero bound for longer than under optimal discretion.

[32] See also Adam and Billi (2006), Adam and Billi (2007), Nakov (2008), Hills and Nakata (2014), Bundick (2014) and Chattopadhyay and Daniel (2014).

Figure 4: Modal responses under optimised threshold-based policies, optimal commitment and discretion



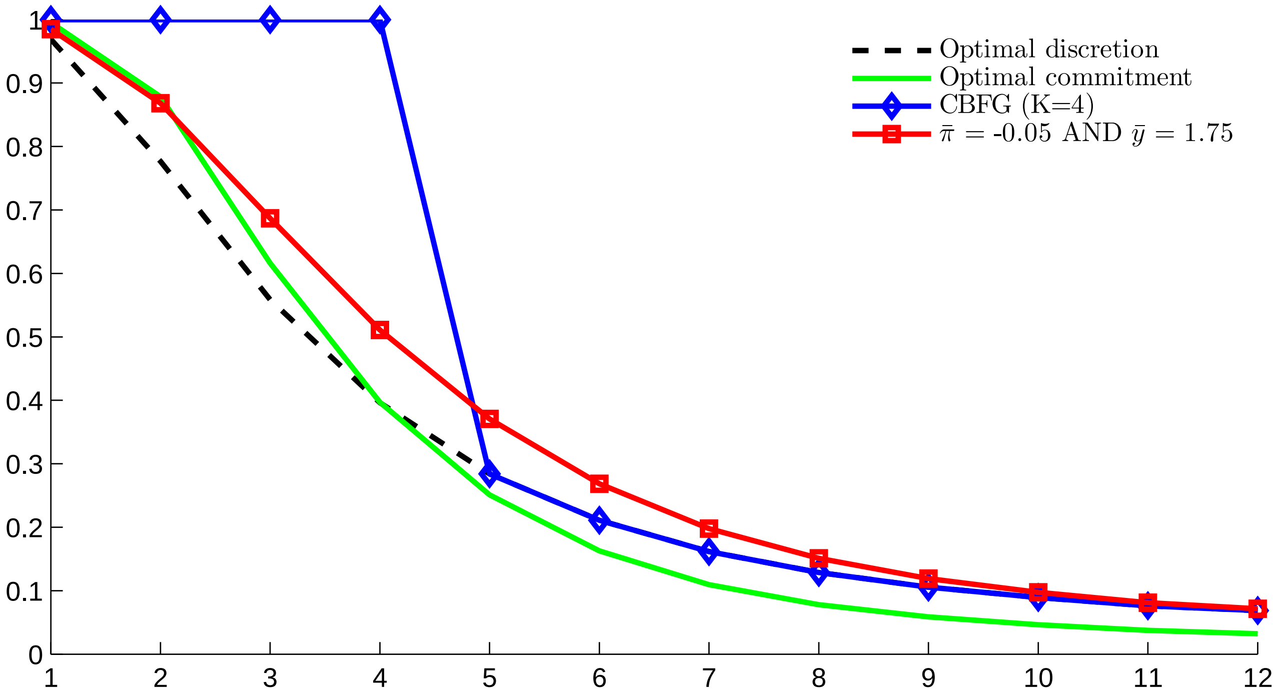
Notes: The baseline policy assumption of optimal discretion (OD) is represented by the dashed black lines (no markers). The solid green lines (no markers) correspond to optimal commitment. The blue lines with diamond markers show the responses under a calendar-based forward guidance (CBFG) in which the policymaker commits to hold the policy rate at the zero bound for $K = 4$ periods. The loss-minimising threshold-based forward guidance (TBFG) policy is shown in the red lines with square markers. This corresponds to a dual threshold 'AND' specification with $\{\bar{y}^*, \bar{\pi}^*\} = \{1.75, -0.05\}$. The responses are computed under the assumption that no shocks arrive after the initial period 0 and from the initial condition $g_0 = -9.4, u_0 = 0$. Inflation, the output gap and the policy rate are plotted as quarterly percentage point deviations from steady state.

of outcomes, rather than the outcomes corresponding to a particular realisation of the shocks.

Figure 5 plots the frequency of simulations in which the policy rate is at the zero bound for the alternative policies considered. Under threshold-based forward guidance, the probability of remaining at the zero bound is persistently higher than the baseline assumption of optimal discretion (dashed black lines). In contrast, the calendar-based forward guidance holds the policy rate at the zero bound with certainty for four periods (thereafter setting policy according to optimal discretion). This observation suggests that the overall performance of calendar-based guidance policy may be compromised by the lack of state contingency in the policy setting for the first four periods. In the first two periods, optimal commitment (solid green lines) generates a slightly higher probability of rates at the zero bound compared to threshold-based guidance. Thereafter, the probability that rates are at the zero bound is lower, gradually falling below the probability generated by optimal discretion. This latter result reflects the fact that optimal commitment policy operates through the policymaker's ability to manage expectations. Since the private sector internalises these effects when forming expectations of future outcomes, the probability of hitting the zero bound in future periods can be reduced by current promises.

Figure 6 plots the distributions of outcomes under the baseline policy of optimal discretion, optimised calendar-based guidance, optimal commitment and optimised threshold-based

Figure 5: Probability that the policy rate is at the zero bound under alternative policies



Notes: Computed from a stochastic simulation of 100,000 draws over 24 periods from the initial condition for the state of $g_0 = -9.4$ and $u_0 = 0$. The baseline policy assumption is optimal discretion (dashed lines). The results from optimal commitment are shown in the solid green line (no markers). CBFG ($K = 4$) refers to a calendar-based forward guidance policy in which rates are held at the zero bound for 4 periods (blue lines with diamond markers). Results from the loss-minimising threshold-based forward guidance (TBFG) policy is shown in the red lines with square markers. This corresponds to a dual threshold 'AND' specification with $\{\bar{y}^*, \bar{\pi}^*\} = \{1.75, -0.05\}$.

guidance. As discussed in Section 7.2, the distributions of inflation and the output gap are negatively skewed under optimal discretion (first column) because in states of the world where demand is low, the policymaker has no ability to stimulate the economy by reducing the policy rate or by manipulating expectations.[33]

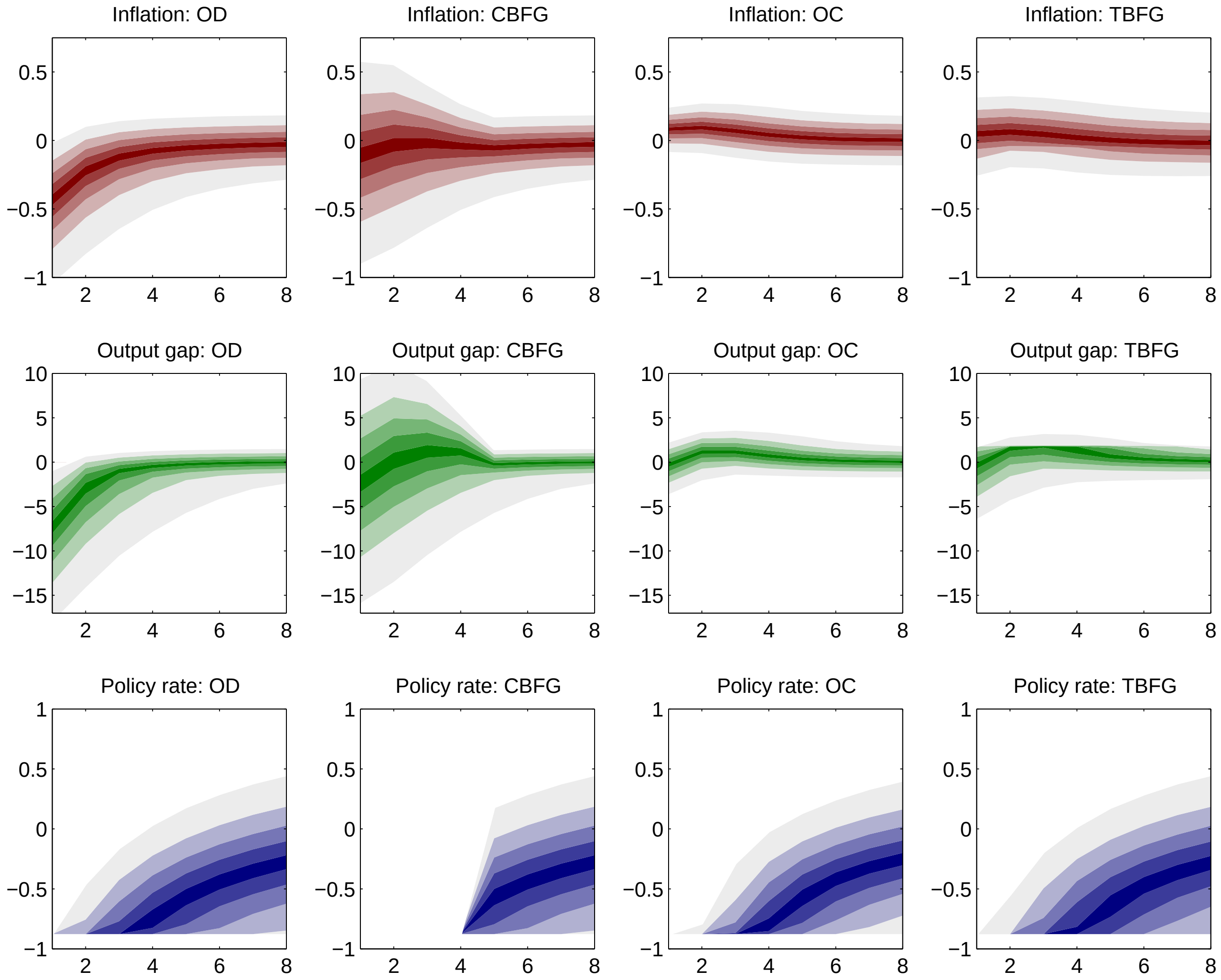
The contrast with calendar-based guidance (second column) is stark. Calendar-based guidance imparts stimulus regardless of the state of the economy. While it reduces the negative skew because it raises expectations sufficiently to reduce the impact of the zero bound constraint (Figure 3), it leads to worse outcomes in both good and bad states than appropriately-calibrated threshold-based policies. That is because calendar-based guidance provides too much stimulus in good states and insufficient stimulus in bad states. As a result, the variance of the distributions of inflation and the output gap increases substantially.

Threshold-based guidance (fourth column) cannot stabilise the distributions of inflation and the output gap as well as the optimal commitment policy (third column). Nevertheless, the improvement over the distribution generated by the baseline policy of optimal discretion (first column) is substantial. The state-contingent promise to hold rates at the zero bound until the thresholds are breached provides additional stimulus in bad states of the world, hedging against some of negative skew associated with the zero bound.

That threshold-based forward guidance comes close to replicating outcomes under optimal commitment at the zero bound is certainly of relevance to policymakers. Threshold-based guidance has some practical advantages over optimal commitment. In particular, it may be

[33]This has implications for policy even if the zero bound does not bind. As discussed in e.g. Nakov (2008), the optimal discretionary policy features a "deflationary bias", whereby the average rate of inflation falls short of its target. Accordingly, the output gap is above target on average: in the presence of an occasionally binding zero bound, demand shocks induce a policy trade-off (see, for example, Adam and Billi, 2006; Nakov, 2008).

Figure 6: Distributions of outcomes under alternative policies



Notes: Distributions of endogenous variables when policy is set according to: optimal discretion (OD); calendar-based forward guidance (CBFG); optimal commitment (OC) and threshold-based forward guidance (TBFG). The calendar-based forward guidance policy is one in which the policy rate is held at the zero bound for $K = 4$ periods TBFG is the loss-minimising threshold-based forward guidance policy: a dual threshold 'AND' specification with $\{\bar{y}^*, \bar{\pi}^*\} = \{1.75, -0.05\}$. Distributions for all cases are computed from a stochastic simulation of $100,000$ draws over 24 periods from the initial condition for the state of $g_0 = -9.4$ and $u_0 = 0$.

much easier for the public to understand than a fully state-contingent optimal commitment policy, particularly as thresholds can be specified directly on goal variables that are used to frame a lot of central bank communications. In this way, threshold-based forward guidance could be thought of as an approximate implementation of optimal commitment policy at the zero bound.

The results above demonstrate that an appropriately calibrated threshold-based policy can achieve substantially better outcomes at the zero bound than the optimal discretionary policy. But engineering an overshoot of inflation and/or the output gap is time inconsistent because once inflation and the output gap exceed their targets, the policymaker can improve welfare by reneging on the policy and reverting to discretion (with an increase in the policy rate). A measure of the size of the policymaker's incentive to renege in any given period can be computed as the probability-weighted integral of the welfare gains from reneging on the forward guidance policy and reverting to the time-consistent policy (ignoring states in which welfare is higher if policy remains in the forward guidance regime). More formally, denote the measure of time inconsistency of a particular policy, P, in period t, as $\mathbb{T}_t^P$:

$$\mathbb{T}_t^P = \int_u \int_g \psi_t^P(u,g)\left(\mathbb{L}_t^P(u,g) - \mathbb{L}_t^{OD}(u,g)\right)\mathbb{I}\left(\mathbb{L}_t^P(u,g) - \mathbb{L}_t^{OD}(u,g) > 0\right) \mathrm{d}g\,\mathrm{d}u \tag{16}$$

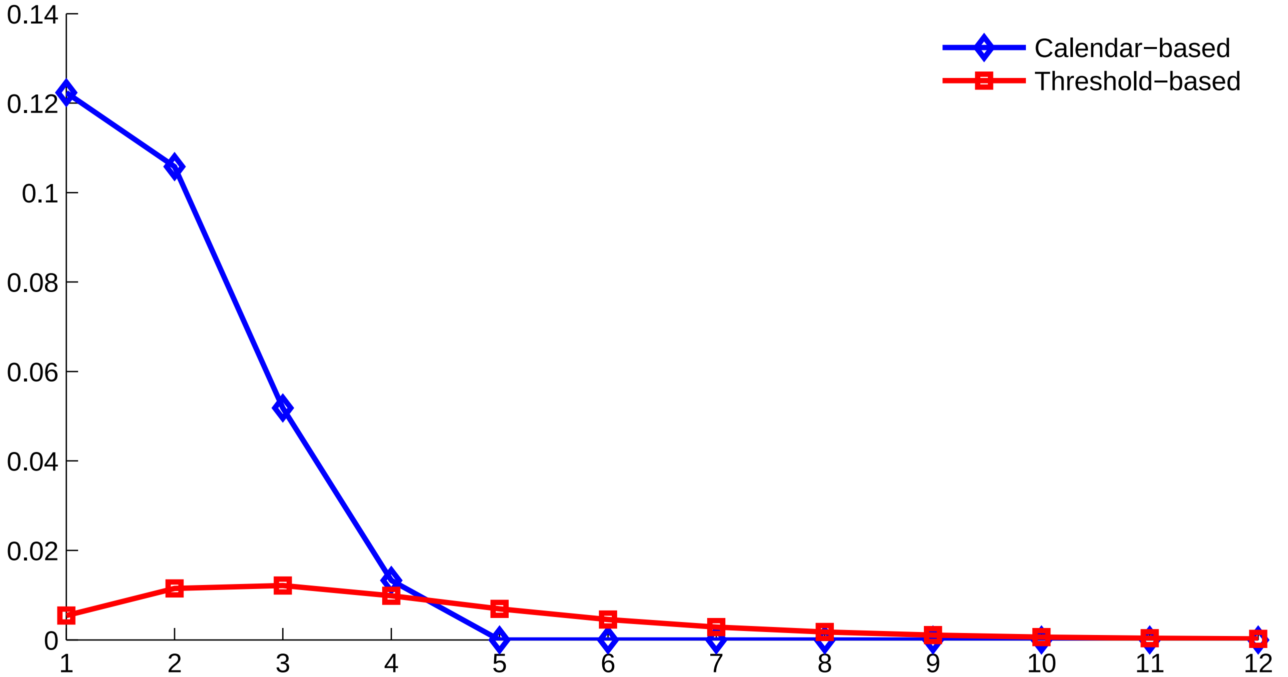
where $\psi_t^P(u,g)$ is a measure of the likelihood that policy P is in effect in period t, $\mathbb{I}(.)$ is an indicator function taking a value of 1 if the loss associated with following the policy concerned exceeds that associated with optimal discretion and 0 otherwise and:

$$\mathbb{L}_t^J(u,g) = \sum_{s=t}^{\infty} \beta^{s-t}\mathbb{E}_t(u,g)\left(\pi_s^J(u,g)^2 + \lambda y_s^J(u,g)^2\right) \tag{17}$$

is the welfare loss associated with policy $J \in \{P, OD\}$ given the state, $\{u, g\}$.

Figure 7 illustrates that the incentive to renege on threshold-based forward guidance is small relative to the incentive to renege on calendar-based guidance. Even though the threshold-based forward guidance policy generates higher modal inflation and the output gap than the calendar-based policy (see Figure 3), the incentive to renege on the threshold-based guidance is smaller than for calendar-based guidance (until the calendar-based policy reverts to the optimal discretionary policy from period 5 onwards). This demonstrates that threshold-based guidance can be less time inconsistent than calendar-based guidance, even when it imparts more stimulus in expectation.

Figure 7: Time-inconsistency measures for inflation threshold-based forward guidance and calendar-based forward guidance policies



Notes: Computed by replacing the density measure in equation (16) with a discrete approximation on the state grid from from the initial condition for the state of $g_0 = -9.4$ and $u_0 = 0$, which we use to replace the integral with a finite sum – see, for example, Chapter 5 of Heer and Maussner (2005).

Although there may be less temptation to renege from threshold-based guidance it is nevertheless time inconsistent to some degree. But that does not necessarily make such policies uninteresting from the perspective of a policymaker because there may exist alternative mechanisms to overcome the time-inconsistency problem. For example, Nakata (2014) demonstrates that policies of this sort can be made time consistent if the policymaker is concerned about their reputation and zero bound episodes are sufficiently frequent and persistent. In that context, threshold-based guidance is more likely to be supportable by a concern for reputation than calendar-based guidance because it embodies less time inconsistency in the absence of reputational mechanisms.[34]

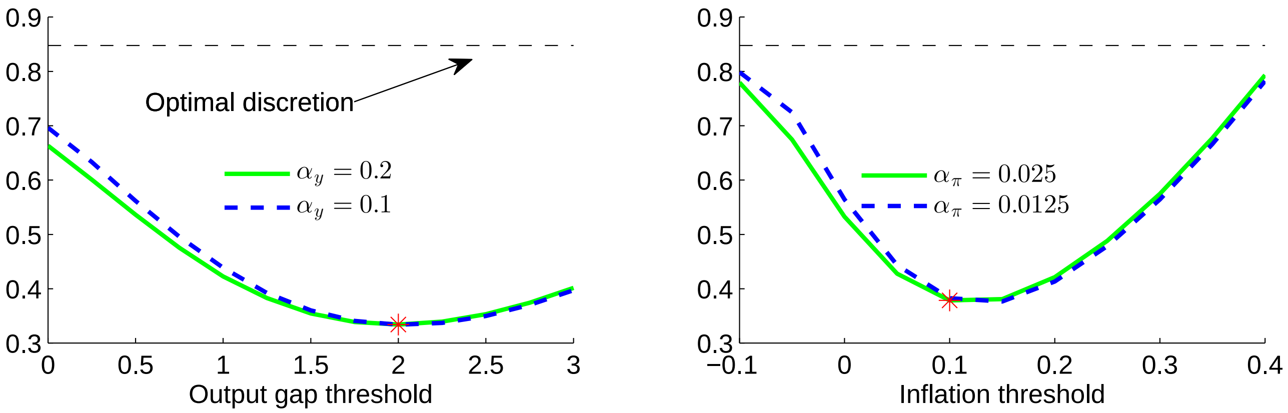
8 Sensitivity analysis

In this section, we consider the sensitivity of our results along several dimensions. In Section 8.1, we consider an alternative calibration of the mapping from threshold breaches to the probability of exiting the policy. In Section 8.2, we examine the sensitivity of our findings to variants of the model with alternative parameter values. Finally, Section 8.3 considers a recalibration of the experiment so that the recession is less severe than the baseline case.

8.1 Threshold specification

As discussed in Section 6, our baseline specification for the mapping from threshold breaches to the probability that the policymaker will exit from the forward guidance regime is intended to mimic a 'trigger' (see Figure 1). Survey evidence, presented in Appendix B, suggests that financial market participants may have viewed the FOMC's December 2012 threshold-based guidance as less strict than a trigger. That is, market participants seem to have assigned a non-negligible probability to exit from the forward guidance regime occurring only once macroeconomic variables had moved substantially beyond the stated threshold values. We mimic these types of beliefs by recomputing the threshold-based forward guidance experiments using a 'flatter' f function (8): we double the value of α.[35]

Figure 8: Optimal threshold values for alternative α calibrations



Notes: Losses are plotted for different threshold values for the output gap (left panel) and inflation (right panel). Dashed blue lines represent the results for the baseline calibration of the function (8) mapping threshold breaches to exit probabilities: $\alpha_y = 0.1, \alpha_\pi = 0.0125$. Solid green lines denote results from a 'flatter' calibration of the function: $\alpha_y = 0.2, \alpha_\pi = 0.025$. The red asterisk denotes the loss-minimising threshold for the 'flatter' calibration.

Figure 8 plots the losses associated with alternative threshold values using the higher calibrations for α (solid green lines) together with the results for the baseline calibration (dashed

[34] In principle, it could be possible to make threshold-based policies time consistent by allowing the central bank to issue option contracts where the buyer has the right (but is not obliged) to borrow at $\underline{r}$ and lend at: $\underline{r} + (r_t - \underline{r})(T_\pi - \pi_t)$, where $\underline{r}$ is the effective lower bound of the policy rate and T_π is an inflation threshold. The option expires when inflation exceeds its threshold for the first time. If the central bank honours its promise and keeps the policy rate at the zero bound until the threshold is reached, the option is out of the money. In contrast, if the central bank reneges on its promise and increases the policy rate before the threshold is met, then the option is in the money. See Tinsley (1999) for an early variant of this type of idea.

[35] This means that for inflation and output gap thresholds we use $\alpha_\pi = 0.025$ and $\alpha_y = 0.2$ respectively.

blue lines). As we would expect, the optimal threshold values under the higher α calibrations are (slightly) lower than for the baseline case.[36] This reflects the fact that a given threshold breach is associated with a lower exit probability for the higher α values. That is, a given output gap threshold policy could be approximated by an alternative policy with a lower value of $\bar{y}$ and a higher α_y. This logic is supported by Figure 8, which shows that the profiles of losses for the baseline calibration (blue dashed lines) lie to the right of the profiles with the higher values of α.

8.2 Model parameterisation

The precise results obtained in Section 7 will naturally depend on the parameter values used. To test the sensitivity of our results, we and consider three alternative parameterisations of the model, based on the sensitivity analysis presented in Adam and Billi (2007).

Table 3: Alternative model parameterisations

	Baseline	'RBC calibration'	'Lower IES'
σ	6.2500	1.0000	1.0000
ρ_u	0.0000	0.3600	0.0000
σ_u	0.1540	0.1710	0.1540
σ_g	1.5240	0.2940	0.2940
κ	0.0240	0.0569	0.0569
λ	0.0031	0.0074	0.0074
g_0	-9.4000	-2.8750	-3.2500
g_0/σ	-1.5040	-2.8750	-3.2500

Table 3 presents the alternative parameterisations of the model that we consider. The table documents the baseline values of the relevant parameters and the values used in the alternative variants. Since some of the parameterisations change the values of 'composite' parameters (namely κ and λ) the values of these parameters are also reported. The changes in model calibration also change the value of the initial demand state, g_0, that is consistent with an initial fall in output of 7.5% (in the 'modal' case). The values of g_0 used to generate this outcome (to ensure that the simulations across model variants are more easily comparable) is also reported. All other parameters remain at the baseline values shown in Table 1.

Following Adam and Billi (2007) we consider an 'RBC calibration' which, relative to the baseline model, features a smaller intertemporal elasticity of substitution ($\sigma = 1$), which increases the slope of the Phillips curve (κ) and the weight on the output gap in the loss function (λ). This variant of the model also features cost push shocks with greater persistence ($\rho_u = 0.36$) and standard deviation ($\sigma_u = 0.171$). The standard deviation of the demand shock ($\sigma_g = 0.294$) is smaller than the baseline value.[37] The RBC calibration involves changes in both the responsiveness of demand to real interest rates and the properties of the disturbances to the model, in particular the relative importance of cost push shocks (u) and demand shocks (g). To isolate the relative importance of these factors, we also consider a 'lower IES' calibration that reduces σ and σ_g in line with the RBC calibration, while holding ρ_u and σ_u at their baseline values.

Table 4 confirms that the optimal threshold values depend on the specific parameterisation of the model. However, some broad themes emerge from the results. First, threshold-based forward guidance to reduces losses relative to optimal discretion by at least 10% in all cases

[36] Optimal threshold values for the higher α (baseline) case are: 2 (2.25) for the output gap threshold and 0.1 (0.15) for the inflation threshold.

[37] This implies that the standard deviation of the disturbance to the IS curve measured in 'real interest rate units' (that is, $\sigma^{-1}\sigma_g$) is virtually unchanged from the baseline parameterisation.

Table 4: Results for alternative calibrations

Variant	Threshold type	$\bar{\pi}^*$	$\bar{y}^*$	$\frac{\text{Loss}}{\text{Loss(OD)}}$
Baseline calibration	Inflation threshold	0.15	–	0.444
	Output gap threshold	–	2	0.394
	Dual OR threshold	0.3	2.25	0.392
	Dual AND threshold	-0.05	1.75	0.391
'RBC' calibration	Inflation threshold	0.25	–	0.264
	Output gap threshold	–	1	0.284
	Dual OR threshold	0.35	1.5	0.26
	Dual AND threshold	0.15	0	0.254
Lower IES	Inflation threshold	0.25	–	0.215
	Output gap threshold	–	1	0.198
	Dual OR threshold	0.4	1	0.191
	Dual AND threshold	0	0.75	0.189

considered. The relative improvement over the baseline assumption of optimal discretion does depend on the parameterisation of the model, as discussed below. Second, there are regular patterns between the optimised values for single and dual threshold specifications. In all cases, a dual 'OR' specification requires values for the inflation and output gap thresholds that are both larger than the optimised values when thresholds are applied to each of the variables individually.[38] In the dual 'AND' specification, optimised threshold values are smaller than the optimised single variable thresholds. This implies that the same mechanism explained in Section 7.1 – that the dual 'AND' specification can detect large positive demand shocks with lower threshold values than the 'OR' specification – is in operation for all of the variants we consider.

Turning to the differences from the baseline specification, we first consider the RBC calibration. Relative to the baseline specification, the optimised values for inflation thresholds are larger and the optimised values for the output gap thresholds are smaller. Compared with the baseline parameterisation, the benefits of threshold-based guidance are larger: optimally chosen thresholds achieve losses of less than 30% of the level delivered by optimal discretion (compared with around 40% for the baseline variant of the model).

We also observe that, for the RBC variant, the loss from the optimised inflation threshold is smaller than loss from the optimised output gap threshold. This finding is consistent with the increased role for cost push shocks under this calibration. Importantly, when cost push shocks are persistent, they affect the output gap at the zero bound through the effects on inflation expectations and hence the real interest rate. So the output gap becomes a less reliable indicator of the demand state (g) if a sizeable fraction of its variability is explained by cost push shocks and a threshold-based policy based solely on the output gap performs worse than a threshold-based policy based on the inflation rate.

This logic is confirmed from the results for the variant of the model with a lower intertemporal elasticity of substitution. That case is consistent with the baseline parameterisation: we observe smaller losses with an optimised output gap threshold than with an inflation threshold. This suggests that it is the increased role of cost push shocks in the RBC calibration that makes the output gap a less useful threshold variable relative to the baseline parameterisation.

For both the 'lower IES' and 'RBC' variants, the demand state g_0 consistent with an initial 7.5% fall in output implies a much longer average duration at the zero bound. This can be

[38]For example, in the RBC calibration, the optimised single inflation threshold is 0.25 whereas the optimised inflation threshold in the dual 'OR' specification is 0.35. The optimised single output gap threshold is 1 and the dual 'OR' specification is 1.5.

seen from the final row of Table 4 which reports g_0/σ, mapping the size of the initial demand state into interest rate units. The demand state needs to be substantially weaker for the 'lower IES' and 'RBC' variants, which generates a more prolonged period at the zero bound under the baseline policy of optimal discretion. This implies that the benefits from additional stimulus are substantial so that losses relative to the baseline case can be reduced by over 80% in some cases.

8.3 Scale of the recession

Our baseline specification of the recession scenario was chosen to deliver a modal fall in output in the first quarter of the simulation equal to around 7.5% when policy is set under optimal discretion.[39] The magnitude of the recession is therefore broadly in line with the Great Depression: Eggertsson (2011) asserts that US output fell by around 30% on an *annual* basis. In this section we consider a specification that is more in line with the Great Recession. Christiano et al. (2011) argue that the Great Recession generated a decline in annual output of around 7%. We therefore set the initial condition for the demand state to $g_0 = -6.925$ which generates a modal fall in quarterly output of around 1.75% in the first quarter of the simulation under optimal discretion.

Figure 9: Optimal threshold values for 'Great Recession' experiment

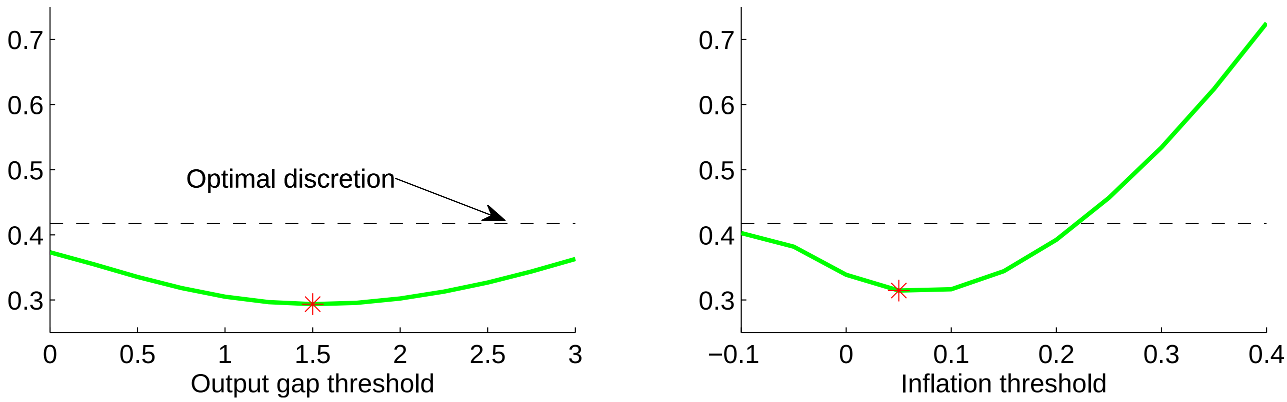


Figure 9 shows the optimal output gap and inflation thresholds for this case. The optimal threshold values are lower than the baseline scenario because the initial fall in output is smaller. A smaller recessionary shock means that the optimal amount of stimulus is also smaller. We also observe that the ability of threshold-based guidance to improve outcomes relative to optimal discretion is somewhat reduced: *ex ante* losses under optimal thresholds are around 0.3 relative to the loss of 0.4 under optimal discretion. This represents a 25% reduction in loss, whereas losses are more than halved in the baseline case. Again, this reflects the fact that the recession scenario is less severe in this instance, so that the zero bound on the policy rate has a smaller effect on the distributions of outcomes in this case.

9 Conclusions

Motivated by policies implemented by the FOMC and the Bank of England's Monetary Policy Committee, this paper has studied the efficacy of stylised 'threshold-based' forward guidance as a temporary policy tool to impart stimulus at the zero lower bound. We have shown that

[39] The modal response is recorded from a simulation in which shock realisations for $t = 1, 2, \ldots$ are set to zero. Results from such simulations are shown in Figure 3, for example.

threshold-based guidance can improve outcomes at the zero bound as a state-contingent form of 'lower-for-longer' policy: the policymaker commits to hold rates at the zero bound for longer than would have been the case (under optimal discretionary policy) in at least some states of the world. By doing so, the policymaker can gain leverage over inflation expectations, reduce the real interest rate and improve outcomes in the same way as first argued by Krugman (1998). But the state-contingency of the commitment also means that threshold-based guidance can act as a hedge against the asymmetric effects generated by the zero bound: the date at which the policymaker is expected to lift rates off the zero bound endogenously reacts to developments in the economy. This allows the policymaker to manage the variance of possible outcomes, as well as to improve outcomes in expectation. This intuition is borne out in a quantitative analysis, where we find that threshold-based policies are associated with lower mean losses and a lower incentive to renege when compared to forward guidance based purely on calendar time.

Crucially, in order for threshold-based guidance to be effective, it is necessary for the private sector to understand precisely how the policymaker intends to behave. We demonstrate that that requires the policymaker to specify how they intend to interpret the threshold conditions. For example, we adopt an interpretation that requires the policymaker to specify precisely how breaches of the thresholds relate to the probability of exiting the forward guidance commitment.

We also rank alternative threshold guidance policies based on their ex-ante losses. We show that threshold-based forward guidance improves outcomes relative to the time-consistent optimal policy as long as the threshold conditions are consistent with generating an expected overshoot of goal variables from target. We also show that threshold-based guidance can deliver losses that are reasonably close to the optimal commitment benchmark if they are optimally chosen. Both the optimal choice of threshold values and the efficacy of alternative types of threshold depend on the structure of the economy and the nature of the disturbances that drive economic fluctuations.

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A Equilibrium selection with threshold-based policies

In this appendix, we provide some intuition for why a threshold-based guidance policy may fail to deliver a unique equilibrium using a simple deterministic example. We also explain how introducing probabilistic exit from the forward guidance policy helps to resolve this problem.

A.1 Intuition using a deterministic example

Here we build intuition for our approach to threshold-based forward guidance described in Section 4.1 by using a deterministic example. The basic environment is the same as that outlined at the beginning of Section 3: the economy begins with a very low state of demand in period $t = 0$; the policymaker (who ordinarily sets policy on an optimal discretionary basis) announces a one-off, fully credible forward guidance policy which takes effect in period $t = 1$. However, in this case we assume that the environment is deterministic in the sense that the probability of future shocks arriving is understood to be zero by all agents. The deterministic setting is instructive because, by definition, there is a single state in each period that is perfectly forecastable by agents. This means that there is no uncertainty about when the policy rate will liftoff and so there is an equivalence between threshold-based and calendar-based forward guidance in which the policymaker commits to hold rates at the zero lower bound for a specific number of periods.

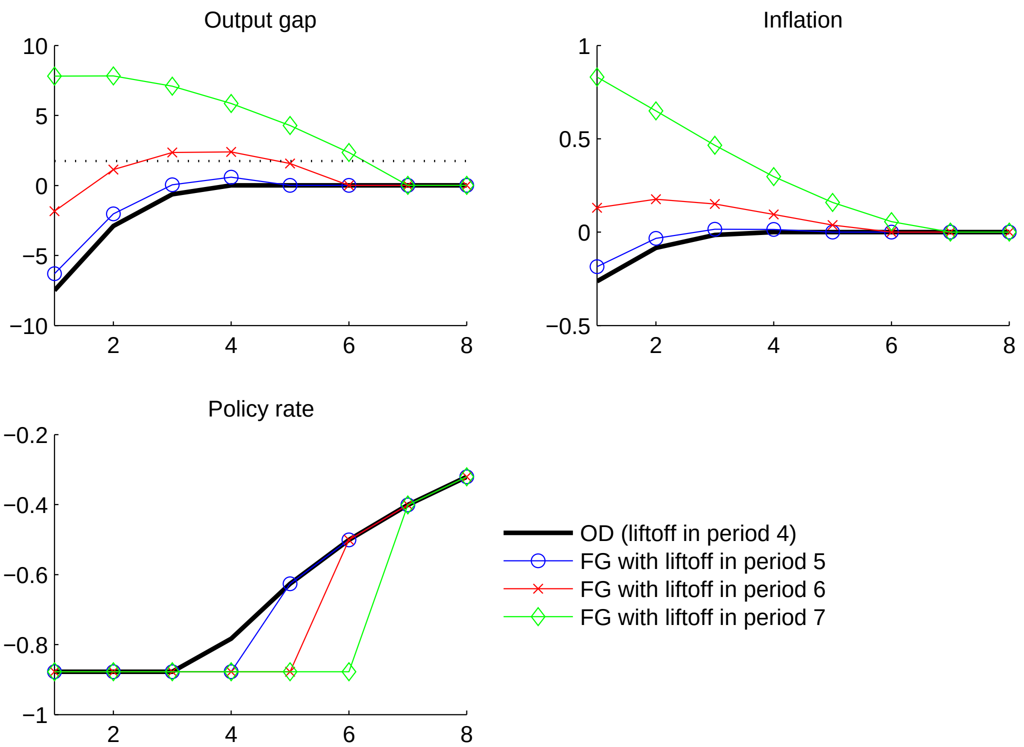
Figure 10 shows alternative paths for the output gap in a deterministic version of the model outlined in Section 2 given alternative policies implemented from period 1 and given an initial condition for the state of demand, $g_0 = -11.95$.[40] In the case in which the policymaker continues to set policy following optimal discretion (the black line), the policy rate 'lifts off' in period 4 (after which the output gap is closed and inflation is at target at all times by virtue of the deterministic setting). The figure also shows paths in cases where the policymaker credibly commits to hold rates at the zero bound for one, two and three additional periods in the blue line with circle markers, the red line with cross markers and the green line with diamond markers respectively. The differences in outcomes are large and are a non-linear function of the duration of the calendar-based guidance policy as has been documented by, for example, Carlstrom et al. (2012).

Suppose that instead of announcing a calendar-based policy, the policymaker announces a threshold-based policy with an output gap threshold of 1.75, as indicated by the horizontal dashed black line in Figure 10. Which of the paths shown in Figure 10 is the equilibrium for the threshold-based policy? In the absence of additional information, *any* of these paths could be an equilibrium. To see that, consider the policies with liftoff in periods 5 (the blue line with circles) and 6 (the red line with crosses), which are arguably the most plausible candidates. The policy with liftoff in period 6 would result in a path for the output gap along which the threshold was breached in *some periods* prior to exit, but by the smallest amount of all such policies. The policy with liftoff in period 5 delivers an output gap path that does not breach the threshold in any period, but which comes closest to doing so among all such policies.[41] But the policy with liftoff in period 7 could also be an equilibrium if the policymaker intended that

[40]The model has been re-solved for the deterministic case (with the standard deviations of the shocks set to 0). The initial condition for the state was set to deliver roughly the same fall in output in period 1 if the policymaker continues to set policy according to the optimal discretion prescription as the mean outcome for output in the stochastic version of the model used for the policy experiments in Section 7.

[41]It should be noted that New Keynesian models with endogenous state variables (e.g. indexation) exhibit 'sign-flipping' behavior, whereby outcomes for output and inflation are an increasing function of the duration for which rates are pegged at the zero bound until that duration crosses a certain threshold when the responses flip sign (see, e.g., Carlstrom et al. (2012)). In that context, the above statements should be interpreted as 'local' statements applying to zero bound durations that do not result in sign-flipping.

Figure 10: Outcomes under alternative policies in a deterministic setting



Notes: Computed from an initial condition of $g_0 = -11.95$ and $u_0 = 0$. No shocks arrive or are expected to arrive thereafter. Otherwise, the model is identical to that described in Section 2 with the baseline calibration outlined in Section 6.

the threshold be breached in *every* period (but by the smallest amount among all such policies) prior to liftoff.

This discussion demonstrates that even in a simple deterministic setting, the announcement of a threshold is not sufficient to pin down the equilibrium outcome, it is also necessary for the policymaker to specify precisely how they will determine regime exit. As an example of the necessity for precision in the policy announcement, suppose that the policymaker announces the output gap threshold along with a statement that the threshold should not be breached at any point prior to regime exit. This policy announcement would rule out the policies with liftoff in periods 6 and 7 as equilibria, but would leave open policies with liftoff in any period up to 5 because none of these would result in the threshold being breached in any period.[42]

The deterministic example considered here also reveals that equilibrium selection concerns the entire path of outcomes. It is not sufficient to determine exit on a period-by-period basis because the entire expected path for rates matters for outcomes in the preceding periods. To see that, suppose that the economy is on the path determined by the policy with liftoff in period 5 (the blue line with circles) along which the threshold is not breached in any period. Notice that, on arrival in period 5, it would be possible for the policymaker to extend the period for which rates are held at the zero bound by 1 period without the threshold condition being breached. However, note that if agents had known that the policymaker would behave in this way prior to period 5, then the equilibrium path would be governed by the policy with liftoff in period 6 along which the threshold is violated in periods 3 and 4.

Conversely, suppose that the policymaker announces that exit will occur in the period immediately after the threshold has been breached. The path corresponding to liftoff in period

[42] In a previous version of this paper, we selected a unique equilibrium by maximising the expected duration of the policy regime subject to the condition that the threshold is *not* breached in any state of the world (which would select the blue line with circles in the deterministic example of Figure 10).

6 cannot be supported as an equilibrium in this case. To see this, note that this path first breaches the threshold in period 3, implying that exit would occur in period 4. However, if agents know that exit will occur with certainty in period 4 their expectations would be consistent with outcomes under the baseline policy (optimal discretion, OD) and the resulting equilibrium would be as depicted in the black lines.

It is evident that one issue that complicates the selection of an equilibrium such as the red line in Figure 10 is the dynamic responses of the macroeconomic variables in response to delayed liftoff. That is, in order to generate near term stimulus the policymaker must promise a subsequent overshoot of the output gap (or indeed inflation). But such an overshoot implies that the desired trajectory for the output gap approaches steady state equilibrium from above. The fact that the output gap is falling in the period prior to exit from the forward guidance regime complicates the use of a threshold-based policy that ties exit to a condition that the output gap must *exceed* a particular threshold.

A.2 Probabilistic exit

We now extend our deterministic example to consider the case in which a breach of the threshold conditions is associated with *probabilistic* exit from the forward guidance regime.

To examine the outcomes in this case, we use the algorithm developed by Haberis et al. (2014). A full explanation and derivation of the algorithm is provided by Haberis, Harrison, and Waldron (henceforth HHW), so here we summarise the key elements as they apply to our specific model and policy experiment.

The HHW algorithm is applied to linear, deterministic models with the following timing structure.

- At the start of period t households and firms choose output and inflation $\{y_t^*, \pi_t^*\}$. They do so on the basis of the expected path of the policy rate set by the policymaker, *including the rate that will be set in period* t.

- With probability p_t the policymaker exits from the zero bound and sets the policy rate according to the usual policy rule (that is, optimal discretion). With probability $1 - p_t$ the policymaker continues to hold the policy rate at the zero bound.

- If the policymaker choses to revert to optimal discretion at date t then it continues to set policy under optimal discretion forever more. So the forward guidance policy is a 'one-off' policy and reversion to optimal discretion is an absorbing state.

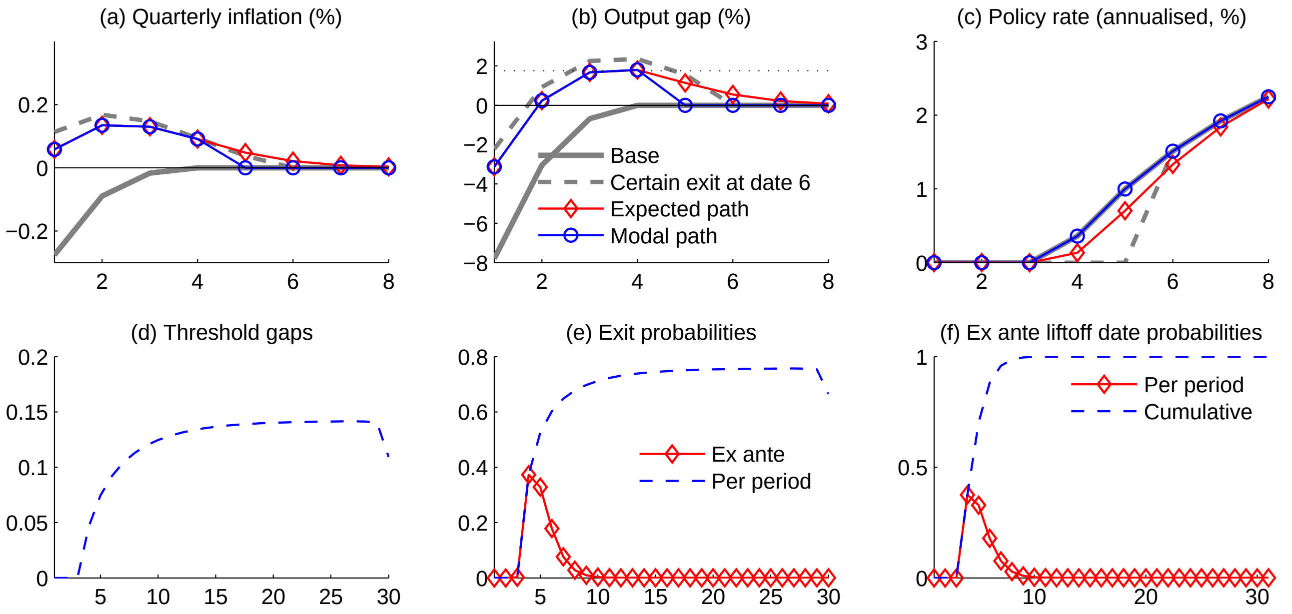
- If the policymaker chooses to continue to set the policy rate at the zero bound in date t then we enter date $t+1$ with households and firms choosing $\left\{y_{t+1}^*, \pi_{t+1}^*\right\}$ as described above.

- If we reach the end of a particular period K and the policymaker has not reverted to setting policy according to optimal discretion in any of the periods $t = 1, \ldots, K$, then the policymaker reverts to optimal discretion for sure in period $K+1$. At the start of the policy ($t = 0$) the date K is known to all agents.

HHW show that there are $K+1$ possible paths for the model, corresponding to reversion to optimal discretion in periods $t = 1, \ldots, K+1$. They also show how to solve simultaneously for these paths, conditional on a set of exit probabilities $\{p_t\}_{t=1}^K$. The solution procedure is a 'stacked time' approach in which the $K+1$ possible paths are stacked together, weighted according to the probabilities of each path occurring. Those probabilities are straightforward to compute given the fact that reversion to optimal discretionary policy is an 'absorbing state'.

Solving for the case of threshold-dependent exit probabilities requires a straightforward iterative approach. Conditional on a guess for the exit probabilities $\{p_t\}_{t=1}^{K}$, the equilibrium paths can be computed using the algorithm described above. These paths can then be used to compute a new guess for the exit probabilities, using a pre-specified mapping from threshold breaches to exit probabilities. Iteration between updating equilibrium paths and exit probabilities continues in this way until a fixed point is reached.

Figure 11: Threshold-based probabilistic exit



Notes: Computed from an initial condition of $g_0 = -11.95$ and $u_0 = 0$. No shocks arrive or are expected to arrive thereafter. The model is identical to that described in Section 2 with the baseline calibration outlined in Section 6. Exit from the zero bound is determined by a probabilistic exit criterion: the probability of exit in period t is $1-\exp\left(-\alpha^{-1}\left(y_t - \bar{y}\right)\right)$ if the output gap y_t exceeds the threshold $\bar{y}$ and 0 otherwise. The parameters are $\alpha = 0.1$ and $\bar{y} = 1.75$ (the dotted horizontal line in panel (b)).

We apply the HHW algorithm to our deterministic example, using the baseline calibration for the mapping between (output gap) threshold breaches and exit probabilities from the main text. We set K, the maximum number of periods of the forward guidance policy, to 30. The results are shown in Figure 11. Panels (a)–(c) show the responses of inflation, the output gap and the policy rate. The solid grey lines are the baseline case of optimal discretion. The dashed grey lines correspond to a calendar-based forward guidance policy with exit *for certain* in period 6. The dotted horizontal line in the output gap chart is at 1.75% and we note that the dashed grey line (exit in period 6 with certainty) is the trajectory that is 'close' to the threshold but cannot be supported as an equilibrium in which exit triggered with certainty when the threshold is breached (as discussed in Appendix A.1 above).

Still focusing on the top row, the red lines with diamond markers are the *expected* paths from the simulation in which exit from the zero bound is probabilistic, where the mapping from threshold breach to exit probability is given by (8) with $\alpha = 0.1$ and $\bar{y} = 1.75$. The blue lines are the *modal* paths, corresponding to exit at $t = 4$.

The fact that the modal path is for exit in period 4 is confirmed by the second row of Figure 11. In panel (e), we plot the probability that the policymaker exits the forward guidance policy and reverts to the optimal discretion policy. The red line with diamond markers corresponds to the *ex ante* probabilities of reversion. These are computed using information at the beginning of the experiment (ie the start of period 1). The modal exit date is date 4, which has the highest *ex ante* probability of occurring. The blue dashed line in panel (e) shows the probabilities of reverting in each period, *conditional on the FG policy still being in effect at that date*. These probabilities rise over time because, as the economy recovers back to steady state, the stimulus associated with continuing to hold the policy rate at the zero bound increases. Indeed, panel (d) plots the 'threshold gaps', conditional on the policymaker having not already exited the forward guidance policy. The threshold gaps increase over time consistent with an increasing amount of stimulus associated with delayed exit from the zero bound.

Finally, panel (f) shows the *ex ante* distribution of liftoff dates. The blue dashed line shows the cumulated ex ante liftoff probability (ie the probability that liftoff occurs before a particular period). This shows that there is very small probability that the policy rate remains at the ZLB beyond period 10.

In terms of the economics underpinning these results, there are several points to note.

We first note that the modal path for the policy rate is identical to the baseline path corresponding to optimal discretionary policy. Here stimulus is being provided *in expectation* because there is a chance that the policymaker will exit later than the baseline case. A similar effect occurs in our the stochastic model studied in the main text because of the probability that additional stimulus (later exit) will be applied in future 'bad' states. But in the case considered here it is probabilistic exit alone that gives rise to the expected stimulus.

What matters for inflation and output is the *expected* path of the policy rate. Probabilistic exit affects the expected path for rates in a fairly smooth fashion (the expected path is a weighted average of paths in which exit occurs at a discrete date). We can see that the expected path for the policy rate in panel (c) lies below the baseline path for the policy rate, gradually converging to it. This reflects the fact that the sequence of *ex ante* exit probabilities imply a monotonically increasing probability of liftoff (dashed blue line in panel (f)) so that the *ex ante* probability that the policymaker has returned to setting policy according to optimal discretion approaches unity as the horizon increases.

This observation provides intuition for why probabilistic exit helps to pin down the equilibrium under a threshold-based policy. In a discrete time, deterministic model exit must occur at a particular date. The example in Appendix A.1 showed that there may be cases where applying a threshold-based exit criterion *with certainty* is incompatible with exit at date T because that implies insufficient stimulus for the threshold to be breached at that date, but also incompatible with exit at date $T+1$ because that would imply sufficient stimulus to breach the threshold at an earlier date. Probabilistic exit implies that the expected exit date is a continuous random variable. In the simulation presented in Figure 11, the *ex ante* expected number of periods at the zero bound is 5.09. This provides a substantial amount of stimulus, even though the most likely outcome is for exit to occur in period 4 (as in the baseline case in which policy is set using optimal discretion). The expected duration of the probabilistic forward guidance policy lies in between the dates $T(=5)$ and $T+1(=6)$ that are incompatible with equilibrium if the threshold-based exit criterion is applied with certainty.

A monotonic and convex mapping from the threshold breach to the exit probability creates a feedback between threshold breach and exit probability that supports an equilibrium. Other things equal, a set of beliefs about exit probabilities (that is, $\{p_t\}_{t=1}^{K}$) that imply a longer duration at the zero bound will impart more stimulus and increase the extent to which the threshold variable is expected to breach the threshold. A monotonic and convex mapping to exit probabilities will tend to map a large expected stimulus into higher probabilities of exit, which will reduce the expected stimulus.

Finally, we note that it does not appear that our results are sensitive to the assumption that policy reverts to optimal discretion with certainty by period 31, as the probabilities attached to late exit are extremely low.[43] There is a kink in the profiles for the 'per period' threshold

[43] Recomputing the equilibrium using $K = 15$ generates results that are almost identical.

gaps and exit probabilities (panels (d) and (e)) in the final period of the simulation. These are conditional on reversion to optimal discretion not having occurred already. Because reversion to optimal discretion happens with certainty in period 31, the stimulus of setting policy at the ZLB in period 30 is much reduced relative to stimulus in previous periods (because outcomes in previous periods incorporate some probability of remaining at the ZLB to period 30).

B Calibrating the exit probability function

As noted in the main text, the behaviour of the exponential distribution approximates a 'trigger' (exit occurs for sure when the threshold is breached) as $\alpha \to 0$. Our goal is to find positive values for α that allow us to approximate a trigger, while retaining the ability to compute an equilibrium in light of the discussion in Section 4.1. This reflects our desire to choose a conservative variant of the exit probability function, f.

To inform our calibration, we use information from the Primary Dealer Survey conducted by the Federal Reserve Bank of New York following the introduction of threshold-based forward guidance by the FOMC in December 2012.[44] Our main focus is the survey conducted in January 2013, following the FOMC's announcement in their statement on 12 December 2012 that they would pursue threshold-based guidance:

> the Committee decided to keep the target range for the federal funds rate at 0 to $\frac{1}{4}$ percent and currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above $6\frac{1}{2}$ percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

The January 2013 Primary Dealer Survey investigated respondents' views about the conditions under which the FOMC would lift off from the zero bound, in light of the forward guidance issued in December 2012.

Question 12a of the survey asked respondents for their *modal* estimates of the joint outcomes for the unemployment rate and headline 12-month PCE inflation rate at the date of the first increase in the Federal funds rate. For the unemployment rate, the median and 75th percentile responses were 6.5% indicating that the majority of respondents viewed the threshold as a trigger. However, the 25th percentile response was 6.25% so that 25% of respondents thought it most likely that liftoff would occur with the unemployment rate 0.25 percentage points or more below the threshold.

Using a simple Okun's law relationship, this implies that 25% of respondents thought it most likely that liftoff would occur when the output gap was more than 0.5 percentage points above the threshold value.[45] Given our calibration of $\alpha = 0.1$, the probability of the policymaker exiting the forward guidance regime when the output gap is 0.5 percentage points above the threshold value is 0.67%. Although it is impossible to directly map from a survey distribution of heterogeneous beliefs about the modal rate of unemployment consistent with liftoff to our f function, our calibration implies that outcomes which a substantial proportion of market participants thought to be the *most* likely are, under our calibration, very unlikely. This suggests that our calibration is closer to a trigger than real world guidance was perceived to be by market participants.

[44] The survey results are available here: https://www.newyorkfed.org/markets/primarydealer_survey_questions.html.

[45] Okun's law is a widely used rule of thumb that a one percentage point unemployment gap corresponds to a two per cent output gap.

In terms of beliefs about inflation at liftoff, question 12c concerns the conditions under which exit would occur for alternative assumptions about unemployment and the inflation projection. Specifically, respondents were asked at what maximal unemployment rate they thought the FOMC would lift off from the zero bound conditional on alternative assumptions about the outlook for inflation.

Table 5: Maximum unemployment rates at liftoff conditional on inflation forecast

Inflation forecast :	2.50%	2.75%	3.00%
Percentile 25	6.50%	6.50%	6.70%
Percentile 50	6.50%	7.00%	7.25%
Percentile 75	6.50%	7.00%	7.80%

Table 5 contains the responses to this question. Most respondents regard the 2.5% inflation forecast threshold as a trigger: liftoff would occur at unemployment rates higher than 6.5% for inflation forecasts higher than 2.5%. However, 25% of respondents thought that an inflation forecast of 2.75% would be compatible with the unemployment threshold (that is liftoff would occur at an unemployment rate equal to or below the threshold of 6.5%).

Mapping to our model is difficult, not least because in the model the threshold relates to the current inflation rate whereas the relevant variable in the FOMC's guidance was expected inflation. However, we can say that a sizeable fraction of respondents (one quarter of them) thought that exceeding the inflation forecast threshold by 0.25 percentage points would be possible while remaining within the FOMC's threshold-based guidance regime. Mapping 0.25 percentages points to a quarterly inflation rate gives around 0.06 percentage points. Setting $\alpha = 0.0125$ in our f function implies that the probability of exceeding the threshold by 0.06 percentage points without triggering exit is 0.8%, which is comparable to the 'tightness' we applied to the output gap threshold in the calibration above.