Should bank capital requirements be less risk-sensitive?

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Abstract

Firms' borrowing constraints imply that optimal capital requirements are less risk-sensitive than purely risk-based ones as they trade off the efficient allocation of credit against the social costs of bank failures. We demonstrate this in a simple model and show that firms' productivity differences amplify the effect. However, when matched to US corporate loan data, the model suggests that adjusting the Basel/IRBA risk weights would only have second-order welfare benefits. These benefits are increasing in the correlation between firm productivity and risk, the bank equity premium, and decreasing in the social cost of bank failures.

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1 Introduction

After the Global Financial Crisis of 2007-2009, banks' equity capital requirements have been considerably increased in order to reduce the likelihood of future crises. The requirements take the form of a minimum amount of equity required per risk-weighted assets of a bank, where the risk-weighting scheme has been largely kept unchanged. Both the optimal level of capital requirements as well as optimal risk weights are still much debated in the literature. The question of the optimal level typically centers around the trade-off between reducing the likelihood of banking crises and possibly sacrificing short-term economic growth, as higher equity requirements may increase banks' funding costs and hence reduce bank lending in the economy.[1] However, this trade-off may also be affected by how credit is allocated across sectors and investment projects, and therefore the question of optimal risk weights also arises.

In this paper we study the optimal risk-weighted capital requirements when the potential trade-off between banks' stability and production by credit-constrained firms is taken into account.[2] Both sides of the trade-off are affected by credit allocation, which in turn is affected by the risk weights used to set banks' capital requirements. We find that the optimal risk weights may be flatter than those which are only set to buffer against banks' failures and their social costs. Hence, we provide an additional rationale for less risk-sensitive capital requirements.

[1] This is consistent with the macro-prudential view of bank capital regulation (Hanson et al., 2011). See e.g. Firestone et al. (2017); Elenev et al. (2017); Dagher et al. (2016); Mendicino et al. (2017); Thakor (1996, 2014), and Van den Heuvel (2008) and the literature covered therein.

[2] From hereon by capital requirement we refer to the effective amount of capital a bank has to hold against the loan given both the level of capital requirements and the risk weight associated with the loan.

The current capital regulation considers risk weights purely from the viewpoint of a bank's solvency or, more broadly, financial stability. In the current system, the risk weight on a corporate loan is determined by the loan's contribution to the bank's overall loan portfolio risk.[3] However, after the crisis, the current risk-weighting system has been criticized. Subsequently, it has been argued that risk weights should be determined in a more robust manner, or even be replaced by a non-risk-weighted (but sufficiently stringent) leverage restriction (see e.g. Admati and Hellwig 2013; Acharya et al. 2014). This would imply a flatter, if not an entirely flat, risk-weighting structure.[4] Among the concerns raised is that risk is hard to measure and that model-based risk weights are prone to manipulation by banks (see e.g. Beltratti and Paladino 2016; Mariathasan and Merrouche 2014; Plosser and Santos 2014 and Berg et al. 2015) and that risk-weighting may induce excessive pro-cyclicality (Kashyap and Stein, 2004; Goodhart et al., 2004; Gordy and Howells, 2006; Pennacchi, 2005 and Repullo and Suarez, 2013). Furthermore, Martinez-Miera and Repullo (2017) show that purely risk-based capital requirements may lead the riskiest borrowers to obtain credit from the shadow banking sector where monitoring is inefficiently low. Moreover, Admati and Hellwig (2013) argue that the current risk weights may create a bias against traditional business loans, which would typically obtain a relatively high risk weight in the current system. This suggests that the current risk-weighting system, together with the increase in the level of capital require-

[3]This is the case when a bank is allowed to use, subject to supervisory approval, the so called Internal Ratings Based Approach of the Basel rules for capital requirements. As a default option, typically smaller banks use a simpler risk-weighting system.

[4]In actuality, risk weights on corporate loans were flat in the first Basel Accord from 1998 (Basel I). The shift to model-based risk weights was introduced in the second Basel agreement in 2004 (Basel II), implemented e.g. in the EU in 2007. A relatively modest capital to assets (leverage ratio) requirement without any risk-weighting has been supplemented in Basel III.

ments agreed in Basel III, may not be optimal from the viewpoint of economic growth if it disproportionately constrains lending to the riskiest sectors. This echoes concerns from both academic and banking circles.[5]

As suggestive evidence, consider the case of how banks have responded to calls for increased requirements culminating in Basel III. In the years following the Global Financial Crisis, Cohen and Scatigna (2016) and Andrle et al. (2017) find that although most of the adjustment to higher regulatory capital ratios was accomplished via the accumulation of retained earnings, there was also a decrease in the average ratio of risk-weighted to total assets. This stylized fact is supported by micro-level evidence documented elsewhere. See for instance Cortes et al. (2018); Gropp et al. (2016); Juelsrud and Wold (2017); Celerier et al. (2017); Auer and Ongena (2016) and De Jonghe et al. (2016) for evidence of a shift in lending away from riskier borrowers in response to higher bank funding costs and in particular increases in required capital.

To study the welfare implications of this credit allocation effect of risk-weighted capital requirements, we build a simple model of banking where financial risks and economic rewards pose a trade-off. Entrepreneurs become borrowers and may differ in terms of the productivity of their sector. Importantly, they are collateral-constrained and can only borrow up to a fraction of the value of their investment project.[6] This is a key imperfection in credit markets which

[5] "The current structure of the regulations may actually introduce biases against making (business) loans" (Admati and Hellwig, 2013, p. 222). Further, representatives of the banking industry especially in Europe have also raised concerns that the increased capital requirements (together with the current risk-weighting system) may jeopardize sufficient lending to small and medium-sized enterprises, which are often seen as crucial to European economies. See Christian Clausen, president of the European Banking Federation, in Financial Times, 16 November 2014.

[6] This follows the tradition of e.g. Kiyotaki and Moore (1997); Holmstrom and Tirole (1997); Bernanke and Gertler (1989) and Bernanke et al. (1999).

drives our central result, and a notable difference with respect to the portfolio theoretic model underlying the current regulation which implicitly assumes frictionless markets. Further, there is a continuum of banks where each specializes in lending to entrepreneurs from a given sector and face a sector-specific risk in their loan portfolios. Hence banks are subject to a failure risk themselves. Consistent with the view that banks play a special role in facilitating economic activity, bank failures in our model generate pecuniary externalities which provide the rationale for capital regulation. Due to un-modeled wedges such as tax distortions or agency costs, bank equity capital is a more costly source of financing for banks than deposits.[7] Banks are competitive and hence pass on the cost of capital requirements to their borrowers. Loan demand responds accordingly such that capital requirements play a significant role in the allocation of bank credit.

Banks do not internalize the social costs of their own failure which are modeled as a pecuniary externality. Banks are subject to a deposit insurance scheme which is otherwise actuarially fair but does not cover pecuniary externalities. Productivity of entrepreneurial investment projects does not factor into banks' loan pricing problem because perfect competition among banks implies that they do not profit more from lending to more productive sectors. Entrepreneurs lever up to their collateral constraint such that being more productive does not generate a cushion against negative shocks that can lead to their defaulting on their loans. Higher capital requirements can help *price* loans appropriately. In particular, capital requirements reduce (i) bank leverage which in turn reduces both

[7]See e.g. Modigliani and Miller (1958); Myers and Majluf (1984); Jensen (1986), and Gorton and Pennachi (1990) on how tax distortions, agency costs, information asymmetries, and liquidity premia can generate a spread between the cost of issuing bank equity and deposits. See also Admati et al. (2013) on why this spread may itself decline as bank leverage falls and for a discussion on the social cost of bank equity.

the frequency and size of bank failures and (ii) borrower leverage by raising the cost of borrowing which tightens the collateral constraint. The optimal capital requirements trade off these two leverage effects - productive investment and bank failure risk.

We mimic the current risk-based capital requirements (i.e. Basel Internal Ratings-Based Approach or IRBA) by imposing the constraint that capital requirements are set such that all banks must have the same probability of failure. When comparing these to the unconstrained optimal requirements we find the latter to be flatter or less risk-sensitive, even if there are no productivity differences across sectors.

The "flattening" result can be understood as follows: because of collateral constraints, the risk weights that are needed to make banks which lend to high-risk sectors as safe as other banks will be so high that they reduce production in high-risk sectors too much. Hence, it is better to tolerate a higher probability of bank failures in high-risk sectors than in low-risk sectors and have a more even distribution of production across sectors. The flattening of risk weights is further amplified if risk and productivity across sectors are positively correlated.

To evaluate the quantitative importance of our results, we match key features of the model with US corporate loan data to assess the relative importance of the mis-allocation of credit induced by a purely risk-based risk weighting system. The purely risk-based approach is designed to mimic the Basel II risk-weighting scheme which has largely been unchanged in Basel III. We find that welfare losses from adopting this purely risk-based regulation tend to be small and roughly equivalent to the loss from a policy with the right risk weights but the average level of capital requirements too high by about three quarters of a percent. These welfare losses are increasing in the correlation between firm productivity and risk, the bank equity premium, and decreasing in the social cost of bank failures.

Our model is closely related to Mendicino et al. (2017) who study the optimal level of dynamic bank capital requirements on two sectors representing corporate loans and household mortgages. Our focus on the risk-weighting aspect of regulation is shared with Martinez-Miera and Repullo (2017) who study the effects of risk-weighted capital requirements on the propensity of borrowers to obtain market-based, bank-based, or shadow bank-based credit. Our focus is on borrowing constraints as well as productivity and risk differences across various corporate borrower risk classes and their effect on the optimal risk-weighted capital requirements. Our results formalize a new argument for flatter risk weights, based on the trade-off between bank risk and the efficient allocation of credit when borrowers are collateral-constrained and possibly differ in productivity. Our emphasis on the importance of the composition of credit, due to differences in productivity and credit constraints, and the role that capital requirements play is related to Harris et al. (2017). In this sense, we are also related to the empirical literature on the impact of financial frictions on capital and credit mis-allocation and consequently output and productivity.[8] For instance, Gilchrist et al. (2013); Hassan et al. (2017) and Gopinath et al. (2017) show that differences in credit constraints, and credit allocation by banks, has led to lower aggregate productivity in the United States and in Southern Europe.

The rest of the paper is structured as follows. Section 2 presents the model and section 3 analyzes optimal capital requirements. Section 4 covers results from a quantitative evaluation of the model predictions and finally, Section 5

[8]See Restuccia and Rogerson (2013, 2017) for a review of the literature.

concludes. Proofs and some important extensions are provided in appendices.

2 Model

The key contribution of our analysis concerns the effect of risk-weighted capital requirements on the cross-sectional allocation of bank financing. To capture the trade-off between borrowing frictions on the one hand and societal pecuniary costs of bank failures on the other, we consider bank credit in a two-period model where borrower risk and productivity varies across sectors. There is a continuum of competitive retail banks who each specialize in lending to one sector and face sector-specific loan portfolio risk. Agency frictions motivate collateralized borrowing which is limited by banks' valuations of collateral.[9]

For simplicity, we assume an actuarially fair deposit insurance scheme, which implies that banks have no excessive risk taking incentives arising from deposit insurance. However, banks prefer high leverage because bank equity is scarce and is hence assumed to bear a premium with respect to deposit financing. Moreover, if banks fail, society suffers pecuniary costs. This motivates bank capital requirements because banks do not internalize these costs (see also Gale and Ozgur, 2005 on why pecuniary externalities arising from bank failures may be appropriate as a motivation for capital requirements).

[9]Our setting is in the same spirit as Gale and Hellwig (1985) and the literature that follows. Agency costs also imply that a standard debt contract is the optimal mode of external financing. See as well Townsend (1979) and Kiyotaki and Moore (1997).

2.1 Entrepreneurs

Consider a two-period economy populated by two sets of risk-neutral agents, entrepreneurs and bankers, who maximize old-age consumption of a numeraire good. First, entrepreneurs with a unit mass and indexed by i belong to a unit mass of sectors indexed by j. They are born with an endowment of the good e (equity) which, along with potential borrowing, they can invest into projects.

An entrepreneur's investment opportunity allows her to convert one unit of the numeraire today into A_j units of a specialized good tomorrow. After production she may then sell the product for price $\epsilon_{i,j}$ of the numeraire. This price is a random variable realized in period two and is log-normally distributed with a mean of one and a variance which may differ across sectors.

Assumption 1. *Independent and identically distributed entrepreneur shocks*

$$log(\epsilon_{i,j}) \sim i.i.d. \mathcal{N}(-\sigma_j^2/2, \sigma_j^2) \tag{1}$$

The independence assumption is a useful abstraction to simplify the analysis. In section 2.2 describing the banking sector, we introduce a sector-specific (or aggregate) shock which a bank may not diversify. This is a reduced-form way of incorporating correlated risk between entrepreneurs. In Appendix C we also provide a version of the model were correlated shocks are explicitly incorporated.

In the case that the good is transferred to a banker, as collateral, the banker can convert it into $\theta_j < 1$ units of the numeraire. To prevent entrepreneurs from strategically defaulting and running away with the borrowed funds, banks limit lending up to their valuation of the specialized good which serves as a constraint on borrowing (cf. Gale and Hellwig, 1985; Kiyotaki and Moore, 1997).

Entrepreneurs solve the following program,

$$\begin{aligned}\max_{B_{i,j}} & \quad \mathbb{E}\left[\epsilon_{i,j}K_{i,j} - R_j^b B_{i,j}\right]^+ \\ \text{s.t.} & \\ K_{i,j} &= A_j(B_{i,j}+e) \\ R_j^b B_{i,j} &\leq \theta_j K_{i,j}\end{aligned}$$

where R_j^b is the loan rate set for loans in sector j. We consider the interesting case where all entrepreneurs desire borrowing and are borrowing-constrained. This amounts to the following assumptions.

Assumption 2. *Productivity and borrowing constraints:*

- *Entrepreneurs are sufficiently productive so as to desire borrowing and entrepreneurs are borrowing constrained*

$$A_j > R_j^b \quad \forall\, j \tag{2}$$

$$R_j^b > \theta_j A_j \quad \forall\, j \tag{3}$$

The program above yields the following optimal size of borrowing and output,

$$B_j^* = \frac{\theta_j A_j}{R_j^b - \theta_j A_j} \quad \text{and} \quad K_j^* = \frac{A_j}{R_j^b - \theta_j A_j} R_j^b \tag{4}$$

where we have that the borrowing constraint is binding $R_j^b B_j^* = \theta_j K_j^*$ and we have normalized the initial net worth e of entrepreneurs to one. In turn, expected consumption is given by

$$\mathbb{E}\left[\epsilon_{i,j} K_j^* - R_j^b B_j^*\right]^+ = (1 - \Phi_j)\left(\bar{\epsilon}_j^s - \theta_j\right) K_j^* \tag{5}$$

where $\Phi_j = Pr(\epsilon_{i,j} < \theta_j)$ is the probability of default and $\bar{\epsilon}_j^s = E[\epsilon_{i,j} | \epsilon_{i,j} \geq \theta_j]$ is the mean of the price shock conditional on not defaulting which reflects the gains from limited liability.

Key to our environment is that entrepreneurs (borrowers) benefit from external financing and agency problems generate the need for borrowers to collateralize debt with future output. Since investment generates a specialized good which is ex ante of greater value to entrepreneurs than to bankers, the size of borrowing is limited to a fraction of potential output. Banks' lower valuation of the collateral reflects the degree of specialization or tangibility of the sector's output. Consequently, it is possible that investments in the most productive projects are severely limited due to financing constraints while other, less productive projects, may get external financing more easily.

2.2 Banks

The second set of agents are bankers. A continuum of retail bankers indexed by j fund their lending activities with bank equity e_j^b and deposits d_j:

$$B_j \leq e_j^b + d_j \tag{6}$$

Each retail banker serves sector j and faces the threat of competitive entry for their entire loan portfolio. There is a single wholesale banker who holds all of bank equity and is willing to rent it at an expected return ρ to retail bankers. Similarly, a perfectly elastic supply of deposit funds is available to all retail bankers at a required return given by R^d. Further, we assume that the required expected return on bank equity is more costly than deposit financing:[10]

Assumption 3. *Bank equity is more costly than deposit funding.*

$$\rho > R^d \tag{7}$$

Deposit funds taken out by bankers are subject to a deposit insurance scheme such that R^d may be interpreted as the risk-free rate of return in our economy. The deposit insurance scheme guarantees the repayment of deposits in the case of a bank failure and charges an actuarially fair premium on surviving banks (in the spirit of Merton, 1977; Ronn and Verma, 1986).[11] The expected return on bank equity ρ is defined as net of this insurance premium. Finally, the share of lending financed with bank equity for each retail banker has to be greater than or equal to an exogenously set capital requirement κ_j:

$$e_j^b / B_j \geq \kappa_j \quad \forall\, j \tag{8}$$

Each retail bank in operation services the continuum of borrowers in her

[10]As typically assumed in the literature, see e.g. Repullo and Suarez (2004, 2013), deposit supply is perfectly elastic at the rate R^d. These deposits may be interpreted as savings from elsewhere in the economy, e.g. households. In addition to the standard corporate finance-related arguments, there are several reasons in the banking literature on why bank equity requires a higher expected return (see for instance Holmstrom and Tirole, 1997; Diamond and Rajan, 2000). Here we motivate it as primarily arising from the relative scarcity of bank equity in a similar vein to Repullo and Suarez (2004) and Mendicino et al. (2017).

[11]See Appendix B for details on our actuarially fair deposit insurance scheme and its relation to bank leverage and bank failure probabilities.

sector. Because the borrowers are hit with independent identically distributed idiosyncratic shocks (see Assumption 1), a constant fraction of the loan portfolio will default. Further, the binding collateral constraint implies that the proceeds from the fraction of defaulted borrowers will yield the same return to the bank as the non-defaulting borrowers. To incorporate risk, we include a loan portfolio shock ξ_j after borrower default has taken place as in Clerc et al. (2015) and Mendicino et al. (2017). This may be interpreted as a reduced-form way of incorporating correlated risk of default of the individual loans (as a result of a systematic risk factor; see e.g. Gordy, 2003).

Assumption 4. *The portfolio shock is log-normally distributed with unit mean and independently distributed across sectors.*

$$log(\xi_j) \sim N(-\eta_j^2/2, \eta_j^2) \tag{9}$$

One may interpret a positive portfolio shock ($\xi_j > 1$) as the realization of lower realized loan losses than the loan loss reserves for a given portfolio of loans.[12] When the shock is sufficiently large and negative, then the return on the portfolio is insufficient to cover the bank's liabilities and the bank itself goes into default. Denote a bank's failure probability as Ψ_j,

$$\Psi_j \quad = \quad Pr(\xi_j R_j^b B_j < R^d d_j) \tag{10}$$

For simplicity, in this section the variance of the portfolio shock does not de-

[12] See also the interpretation in Clerc et al. (2015); Mendicino et al. (2017) who have similar modeling approaches and where the Log-Normality assumption facilitates the solution to these medium-scale *DSGE* models.

pend on individual borrower risk characteristics and the shocks are independent across sectors. However, an extension of the model in Appendix C demonstrates how portfolio risk (η_j) may endogenously arise as an increasing function of entrepreneur risk (σ_j) and also allows for correlated portfolio shocks.

Given the threat of entry, the retail bankers' problem may be written as minimizing the loan rate,

$$\min R_j^b \qquad \text{s.t.}$$
$$\rho \;\leq\; \left(R_j^b B_j - R^d d_j\right)/e_j^b$$

The above constraints along with equations (6) and (8) bind in equilibrium and yields the competitive loan rate,

$$R_j^b = R^d + \kappa_j(\rho - R^d) \tag{11}$$

Thus, we can rewrite the probability of bank failure as

$$\Psi_j \;=\; \Phi(\frac{\eta_j}{2} - \frac{\tilde{\kappa}_j}{\eta_j}) \tag{12}$$

where $\tilde{\kappa}_j \equiv log(1 + \frac{\kappa_j}{1-\kappa_j}\frac{\rho}{R^d})$ and $\Phi(\cdot)$ is the normal density.[13] This probability is completely determined by the riskiness of the bank's portfolio and leverage which in turn is determined by the bank's capital requirement. Given the competitive loan rate, we may rewrite Assumption 2 as the following parameter conditions which set a floor on the level of productivity (A_j) and a cap on the borrowing

[13]As noted in Repullo and Suarez (2004), the net interest margin also acts as an additional buffer against bank failure.

constraint parameter (θ_j).

$$\kappa_j < \frac{A_j - R^d}{\rho - R^d} \quad \forall j \tag{13}$$

$$R^d > \theta_j A_j \quad \forall j \tag{14}$$

The key friction motivating the need for capital requirements in our model is that bank failures generate pecuniary externalities. Similar to Repullo and Suarez (2004); Clerc et al. (2015) and Mendicino et al. (2017), we assume that when a bank fails society as a whole suffers a cost proportional to the size of the bank's balance sheet:

Assumption 5. *The societal cost of a bank's failure, in terms of the consumption good is linear in the size of the bank's balance sheet.*

$$w_j^{bf} = -\gamma R_j^b B_j \tag{15}$$

where γ is a scale parameter.

The linearity assumption is done to simplify the model. Consistent with a macro-prudential view of capital regulation as in Hanson et al. (2011), our optimal set of risk-weighted capital requirements is concerned with more than just the cost of bank failures. We define societal welfare as the sum of all expected consumption by entrepreneurs and payments by bankers (who make zero profits) less bank failure costs. A sector's contribution to societal welfare is given by,

$$\begin{aligned} \mathbb{E}[w_j] &= \mathbb{E}[c_j] + \left[\rho e_j^b + R^d d_j\right] + \Psi_j w_j^{bf} \\ &= \mathbb{E}[c_j] + \theta_j K_j - \gamma \Psi_j R_j^b B_j \end{aligned}$$

where

$$\begin{aligned} \mathbb{E}[c_j] &= (1-\Phi_j)(\bar{\epsilon}^s - \theta_j)K_j \\ &= \left[1 + \Phi_j(\theta_j - \bar{\epsilon}^d)\right] K_j - \theta_j K_j \\ &= \tilde{K}_j - \theta_j K_j \end{aligned}$$

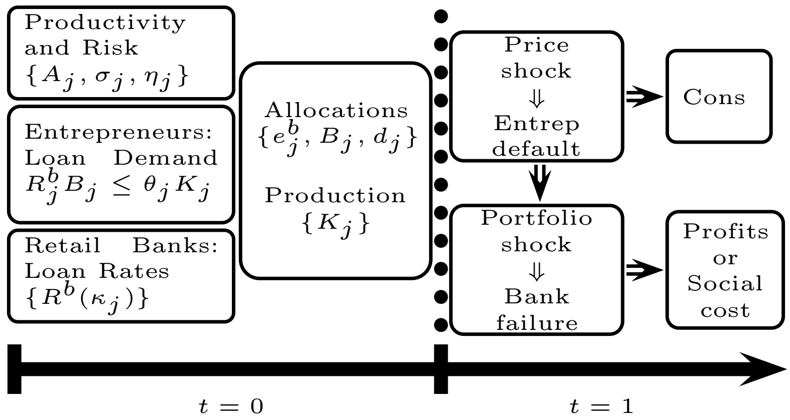
and where $\bar{\epsilon}_j^d = E[\epsilon_{i,j}|\epsilon_{i,j} < \theta_j]$ is the mean of the price shock conditional on defaulting. Aggregate welfare is simply the sum across sectors of expected entrepreneurial output (in terms of the numeraire) net of expected bank failure costs,

$$W = \int \mathbb{E}[w_j] = \int \tilde{K}_j - \gamma \Psi_j R_j^b B_j$$

2.3 Timing

In the first period, each retail banker meets the set of entrepreneurs in her sector and makes a loan offer by posting a loan rate. Given loan rates, the solution to the entrepreneurs' problem yields loan demand. The retail banker then presents her loan portfolio to the wholesale banker and asks for bank equity as per capital requirements. Once loans are made the first period ends and production takes place. In the second period, the entrepreneurs' price shock $\epsilon_{i,j}$ is realized and some entrepreneurs repay while others default. In the latter case, retail banks appropriate the collateral which they then sell for θ_j. Finally, the bank portfolio shock ξ_j is realized and some banks fail. The sequence of choices and shock realizations are illustrated in Figure 1.

Figure 1: Timing



2.4 Equilibrium

Given the set of parameters $\{A_j, \sigma_j, \theta_j, \eta_j, \kappa_j\}$ for all sectors and aggregate parameters $\{\rho, R^d\}$, equilibrium is defined as the set of choices $\{B_j, R_j^b, e_j^b, d_j\}$ for all entrepreneurs and sectors such that equations (4) and (11) hold given that the constraints in equations (6) and (8) along with the entrepreneurs' borrowing constraint are binding.

3 Capital requirements

The equilibrium allocation depends on the set of capital requirements. In this section, we first derive and characterize the set of optimal capital requirements, and compare these with simple rules which approximate the spirit of current regulation. Note that in practice, the capital requirement for a bank is determined as the minimum percentage of (equity) capital of the bank's risk-weighted assets. However, in our simple model we work directly with the capital requirement per loan in a given sector without having to specify the minimum percentage of capital and sector-specific risk-weights separately. For example, two banks holding a portfolio of loans granted to the same sector have the same capital requirement in our model, but two banks lending to different sectors have different capital requirements.

To simplify the analysis, from hereon and unless otherwise specified, we assume that all parameters other than bank portfolio risk (η_j), entrepreneur productivity (A_j), and capital requirements (κ_j) are the same across sectors.

Assumption 6. *Sectors differ only in productivity (A_j) and risk (η_j)*

$$\theta_j = \theta \quad \forall\, j \tag{16}$$

$$\sigma_j = \sigma \quad \forall\, j \tag{17}$$

3.1 Optimal capital requirements

Suppose a constrained social planner wants to maximize aggregate welfare by choosing a set of capital requirements for each sector. The optimal capital requirements chosen by the planner solves,

$$\begin{aligned} \max_{\{\kappa_j\}} \quad & \int \tilde{K}_j - \gamma \Psi_j R_j^b B_j \\ \text{s.t.} \quad & 0 \leq \kappa_j \leq 1 \quad \forall\, j \end{aligned}$$

where K_j,B_j, and R_j^b are given by equations (4) to (11). An interior solution to the problem yields the following first-order condition for each sector,

$$\overbrace{(1+\Phi(\theta-\bar{\epsilon}^d))\frac{\partial K}{\partial \kappa}}^{\text{Entrep collateral constraint}} = \gamma\theta \overbrace{\left(K \underbrace{\frac{\partial \Psi}{\partial \kappa}}_{\text{Frequency}} + \Psi \underbrace{\frac{\partial K}{\partial \kappa}}_{\text{Size}} \right)}^{\text{Bank failure}} \tag{18}$$

The optimal capital requirement is determined by the following trade off: on the one hand, capital requirement affects the collateral constraint and thereby productive investment (left-hand side); on the other hand, it affects the bank failure externality (right-hand side).

A useful benchmark is the equilibrium under no capital regulation but with deposit insurance still in place. This leaves bank leverage unconstrained and in this case banks maximize the return on equity by holding as little of it as possible. In effect, as they compete to provide as low a lending rate as they can, banks maximize their own failure rate. Allocations are completely determined by relative productivity without regard for the bank failure externality. Given deposit insurance, this would be optimal only if bank failure costs are negligible yielding a corner solution for all sectors in the planner's problem.

When bank failures are socially costly, imposing capital requirements will bind and improve welfare. As intended, capital requirements deliver lower bank leverage, which directly reduces failure probabilities. However, doing so will raise the cost of borrowing by requiring loans to be partly financed by scarce (and thus more costly) bank equity (cf. Mendicino et al., 2017). In our model, loan demand is elastic. Borrower leverage is reduced with more costly borrowing. This also implies that the sector with a higher capital requirement is also relatively smaller in terms of the aggregate banking portfolio. All of these aspects are featured in equation (18).

3.2 Features of optimal capital requirements

We now characterize some features of the optimal capital requirements in terms of differences in borrower productivity and bank portfolio risk. First, note that all else equal banks with riskier portfolios are subject to higher capital requirements.

Lemma 1 (Capital requirements for risky and safe bank portfolios)**.** *For any two sectors with identical productivity but sector k exhibiting higher portfolio risk than sector j ($\eta_k^2 > \eta_j^2$), the optimal risk weight for bank k is higher than bank j whenever the resulting bank failure probabilities take reasonable values.*[14]

$$\eta_k^2 > \eta_j^2 \Rightarrow \kappa_k^* > \kappa_j^*$$

Second, when all sectors have the same portfolio risk, lending to more productive borrowers merit lower capital requirements.

Lemma 2 (Capital requirements by productivity)**.** *For any two otherwise identical sectors but with sector j more productive than sector k ($A_j > A_k$), the optimal risk weight for bank j is lower than bank k with a corresponding higher*

[14] The lemma holds for sufficiently low values of portfolio risk such that the resulting bank failure probabilities are not too large (e.g. when $\max(\Psi_j) < \Phi(-1) = 15.87\%$). We assume this to be the case. Details and proof for this lemma, as well as the subsequent lemmas and propositions, are in Appendix A.

frequency of bank failure

$$A_j > A_k \Rightarrow \kappa_j^* < \kappa_k^* \ , \ \Psi(\kappa_j^*) > \Psi(\kappa_k^*)$$

Combining both differential risk and productivity leads us to the following characterization of optimal capital requirements:

Proposition 1 (Capital requirements summary). *Let sector 0 with characteristics $\{A_0, \eta_0\}$ have an optimal capital requirement given by κ_0^* and j be another sector which is otherwise identical to sector 0,*

- *Whenever $A_j > A_0$ and $\eta_0^2 > \eta_j^2$ then $\kappa_j^* < \kappa_0^*$*
- *In general, whenever $A_j > \underline{A}(\eta_j; \kappa_0^*)$ then $\kappa_j^* < \kappa_0^*$*
- *Conversely, whenever $\eta_j < \bar{\eta}(A_j; \kappa_0^*)$ then $\kappa_j^* < \kappa_0^*$*

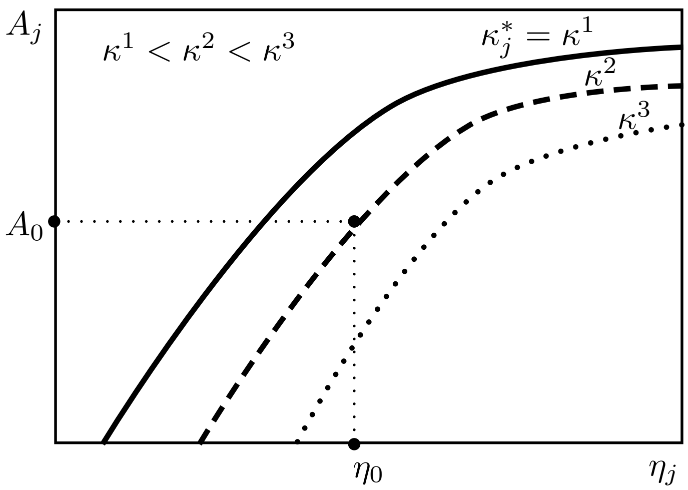
where $\underline{A}(\eta_j; \kappa_0^*) : \kappa^*(\underline{A}, \eta_j) = \kappa_0^*$ and $\bar{\eta}(A_j; \kappa_0^*) : \kappa^*(A_j, \bar{\eta}) = \kappa_0^*$

The proposition above implies that depending on the distribution of risk and productivity across sectors in a given economy, capital requirements may even be non-monotonic in bank risk.

In Figure 2, we plot the set of sectors sorted by productivity (vertical axis) and risk (horizontal axis) and identify the combinations that yield the same optimal level of capital requirements.

As the figure indicates, the optimal capital requirements increase as productivity declines and risk increases (towards bottom right of the figure). Each line in the figure represents an iso-κ curve given by the function $\underline{A}(\eta_j; \kappa^*)$ (or $\bar{\eta}(A_j; \kappa^*)$). Thus, in an economy where the sectors exhibit a negative relation

Figure 2: Optimal capital requirements



The figure plots iso-κ curves given by $\underline{A}(\eta_j, \kappa^*)$ with risk on the horizontal axis and productivity on the vertical axis.

between productivity (A_j) and risk (η_j), the set of optimal capital requirements can be characterized as decreasing in productivity and increasing in risk. On the other hand, when the correlation is positive, a one-dimensional ordering is no longer possible. Consequently, current risk-weighted capital requirement schemes which focus only on risk factors need not coincide nor produce the same relative ordering as that implied by our model. We explore this possibility in the next subsection.

3.3 Optimal against purely risk-based capital requirements

We now consider a regulatory scheme which aims to capture the principle behind the current regulation of requiring more capital for riskier assets. In the spirit of this scheme, we consider a policy rule which sets capital requirements such that bank failure probabilities across sectors are equalized. We henceforth refer to this scheme as the *Basel-IRBA* requirements for brevity.[15]

Suppose a regulator wants to set $\Psi_j = \Psi_k = \Psi^*$:

$$\max_{\Psi^*} \int \tilde{K} - \gamma \Psi^* R^b B$$
$$\text{s.t. } \tilde{\kappa}_j \equiv log\left(\frac{R^b}{(1-\kappa)R^d}\right) = \frac{\eta_j^2}{2} - \eta_j \Phi^{-1}(\Psi^*)$$

where $\kappa_j = \frac{(exp(\tilde{\kappa}_j)-1)R^d}{\rho+(exp(\tilde{\kappa}_j)-1)R^d}$. The resulting relative values of the *Basel-IRBA* requirements depend only on the sectoral portfolio risk and where Ψ^* solves: $\int_j -\eta \frac{\partial\kappa}{\partial\tilde{\kappa}}\left[-\gamma\theta K \frac{\partial\Psi}{\partial\kappa} + (1+\Phi(\theta-\bar{\epsilon}^d) - \gamma\theta\Psi)\frac{\partial K}{\partial\kappa}\right] = 0.$

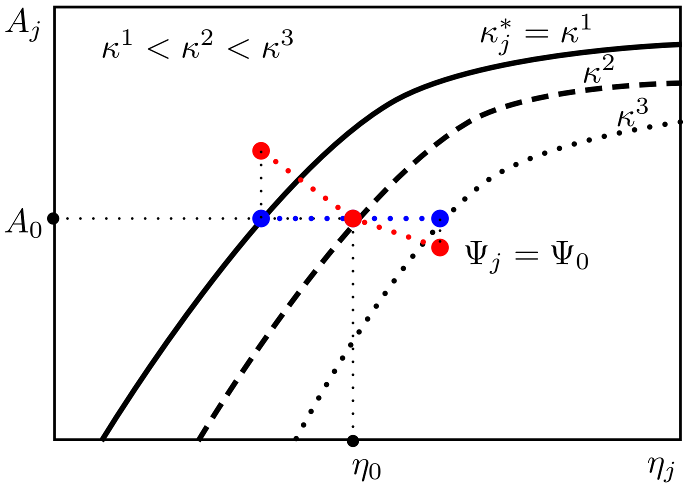
Proposition 2 (Equalizing bank failure probabilities)**.** *All else equal, a policy rule in which bank failure probabilities are equalized generates steeper than the optimal capital requirements: in other words, the requirement is too high for the risky sector and too low for the safe sector.*

Proposition 2 is illustrated in Figure 3. Consider three otherwise identical sectors but with different loan portfolio risks η. The optimal capital requirements are on three different curves representing $\kappa^* \in \{\kappa^1, \kappa^2, \kappa^3\}$ and highlighted by blue dots. The capital requirements imposed by the proposed policy rule are identified by the red dots which is the same as the optimal one for the benchmark sector with risk η_0. For the lower risk sector (leftmost blue and red dots), the optimal capital requirement (κ^1) is higher than that imposed by the policy rule and in the opposite case, for the riskier sector with the optimal capital

[15] The closest real-world counterpart in which this principle is applied is the Basel capital adequacy framework's Internal Ratings Based Approach. In its original form in the Basel II agreement, any bank with the permission to use it was to have a minimum amount of capital against the loan portfolio which covers loan losses with an annual 99.9% probability. If the minimum were fulfilled with equity capital and the bank's only assets were the loan portfolio, then this would translate into a 0.1% annual probability of the bank's failure.

requirement κ^3, the proposed policy requires even more.

Figure 3: Optimal against *Basel-IRBA* requirements



The figure plots iso-κ curves given by $\underline{A}(\eta_j, \kappa^*)$ with risk on the horizontal axis and productivity on the vertical axis. The blue line depicts optimal capital requirements when sectors have the same productivity A_0. The red line depicts capital requirements when bank default probabilities are equalized.

The result that the optimal capital requirements do not equalize bank failure probabilities can be understood as follows. When a bank's capital requirement is raised, the effective borrowing constraint for a sector tightens and as a result, the relative size of the sector diminishes. Not only does this reduce production in the sector, it also reduces the size of the social cost of bank failure arising from that sector. On the other hand, we also have that a much larger increase in capital requirement is needed to induce the same reduction in bank failure probability for a riskier sector relative to a safer one. That is, since $\frac{\partial^2 \Psi_j}{\partial \kappa_j \partial \eta_j} < 0$, equalizing default probabilities entails a much larger increase in capital requirements for riskier sectors such that the trade-off between sustaining credit on the one hand and ensuring banking stability on the other is tilted too much in favor of the latter.

Therefore it is optimal to set capital requirements on banks with riskier portfolios to a level where the riskier banks fail more often than the safer ones. Recall that, in this example, all sectors have the same productivity. Even so, it is not optimal to adjust capital requirements so as to equalize bank failure probabilities. If the relationship between a sector's risk and productivity is negative (i.e., high productivity sectors have low loan portfolio risk and vice versa), it is possible that optimal capital requirements coincide with or are even *"steeper"* than the current (*Basel-IRBA*) capital requirements. On the other hand, if risk and productivity are positively correlated, then the "flattening" of optimal capital requirements is further amplified. Ultimately, the slope of the optimal capital requirements may be an empirical question, depending on the correlation between risk and productivity across sectors. Casual empirical evidence suggests that positive correlation between the two may well be relevant; sectors with higher productivity are often so because of investments in new and risky technologies. Further, precisely because they are more risky, risky firms tend to be more credit-constrained and end up having higher marginal products to their investment and borrowing than safer and less credit-constrained firms (see e.g. Hsieh and Klenow, 2009).

3.4 Upward revision of bank failure risk

After the global financial crisis, capital requirements based on the Basel framework have been increased considerably. In particular, the overall level of requirements has been increased but the risk-weights have largely remained the same (except for the addition of a modest leverage ratio requirement, which effectively sets a floor to the lowest risk-weights). Through the lens of our model, the impetus behind this move from the pre-crisis Basel II to the post-crisis Basel III is best interpreted as an upward revision in the perceived loan portfolio risk. In other words, although the actual crisis dynamics were complicated, the crisis revealed that bank asset risks were greater than had been thought. We next assess the regulatory reform against our model with the help of the following proposition.

Proposition 3 (Upward revision of portfolio risk)**.** *A proportional increase in portfolio risk across the board leads to higher capital requirements with more frequent bank failures and, if the increase is not too large, flatter capital requirements. Suppose* $\eta_j^{new} = c\eta_j \quad \forall : j$ *where* $c > 1$*. Then,*

- $\kappa_j^{*new} > \kappa_j^* \quad \forall\, j$
- $\Psi_j^{*new} > \Psi_j^* \quad \forall\, j$
- $\frac{\partial \kappa_j^{*new}}{\partial \eta_j} < \frac{\partial \kappa_j^*}{\partial \eta_j} \quad \forall\, j$ *if* c *is not too large.*

The first two parts of the proposition follow from Propositions 1 and 2 whereas the third result arises from concavity of the optimal capital requirements in portfolio risk (η).

Proposition 3 suggests that, following a view that risks were previously underestimated (before the Global Financial Crisis), the reform to raise capital requirements across the board (post-crisis) should entail a relative flattening of risk weights as well.

4 Quantitative evaluation

In this section, we match key features of the model to the data to evaluate the quantitative importance of credit mis-allocation arising from a purely risk-based risk-weighting scheme. To do so we use data on internal credit rating grades for commercial loans taken from the Federal Reserve Board survey of large banking organizations as used in Gordy (2000). The survey provides information in terms of shares and default probabilities across seven credit grade categories (using the S&P scale) in banks' commercial loan books. The shares and average (annualized) default probabilities are reported in the first two rows of Table 1.

To convert the default probabilities to our portfolio risk parameter η_j, we use the internal ratings based approach formula to calculate an individual borrower's probability of default conditional on the realization of a systemic risk shock z at the 99.9th percentile (see e.g. Repullo and Suarez, 2004),

$$PD_j(z_{0.999}) = \Phi\left(\frac{1}{\sqrt{(1-\rho)}}\Phi^{-1}(\overline{PD}_j) + \frac{\sqrt{\rho}}{\sqrt{(1-\rho)}}z_{0.999}\right) \tag{19}$$

where $(\overline{PD}_j)$ is the unconditional default probability of borrower class j from the data, ρ is the systemic risk loading of borrower class j which we set to 20 percent and z is the realization of the systemic risk factor which we set equal to $\Phi^{-1}(0.999)$.

Given this probability of default and an assumed loss given default of 45 percent, as is the default value used by the Basel/IRBA framework, we then calculate the size of the unexpected loss as the difference between the default probability conditional on the systemic risk shock and unconditional default probability times the loss given default. We set the η_j parameter such that the loss in our bank's portfolio is equivalent to the unexpected loss at the 99.9th percentile.

$$\Phi^{-1}(\eta_j; 0.999) = LGD * (PD_j(z_{0.999}) - \overline{PD}_j) \tag{20}$$

The resulting values for the portfolio risk parameter are reported in the third row of Table 1. Next we set the borrower price shock variance σ_j to match the unconditional default probabilities across sectors (fourth row of Table 1),

$$\Phi^{-1}(\overline{PD}_j; -0.5\sigma_j^2, \sigma_j^2) = log(\theta) \tag{21}$$

Table 1: Commercial loans by credit grade

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**
Default Probability (%)	0.01	0.02	0.06	0.18	1.06	4.94	19.14
Share (%)	3	5	13	29	35	12	3
Implied Portfolio Risk (η_j)	0.0006	0.0011	0.0027	0.0061	0.0205	0.0480	0.0754
Implied Price Risk (σ_j)	0.2653	0.2777	0.3014	0.3320	0.4073	0.5329	0.8021
Constant Productivity ($\bar{A}$)				1.5			
Variable Productivity (A_j)	1.027	1.185	1.343	1.500	1.658	1.815	1.973

As the data does not provide a joint distribution of borrower risk and productivity, we consider two cases for the distribution of borrower productivities A_j. First, we assume that all borrowers are equally productive and calibrate the average level of productivity, $\bar{A}$, to match the average total asset to equity ratio of non-financial firms over the last two decades. Second, we assume that riskier borrowers are more productive than safer borrowers and set A_j to seven equally-spaced values such that the average is equal to $\bar{A}$. We set the average productivity $\bar{A}$ to 1.50 to match the average leverage (Total Assets /Equity) of non-financial firms of 3 as reported in Rauh and Sufi (2010). Further, we set the collateral constraint parameter θ to 0.36 to match the debt-to-equity ratio of non-financial firms of roughly one (Rauh and Sufi, 2010).

For the case where the riskiest sectors are more productive than the safest sectors, we set the ratio of productivity between the riskiest and safest sectors to 1.92 which is equivalent to estimates of the 90th to 10th percentile dispersion in U.S. manufacturing productivity in Syverson (2004).[16] The productivities for the intermediate risk sectors are then linearly interpolated such that the median sector has a productivity of $\bar{A}$ (last row of Table 1).

The rest of the model is calibrated as follows. We set the costs of bank funding sources equal to the the average over the last two decades of the effective federal funds rate (2 percent) and average annual return on a US banking sector index (8.5 percent) for the deposit rate R^d and expected return on equity ρ respectively. For the social cost of bank failure scale parameter γ we take a range of estimates of the cost of banking and financial crises in the literature as a fraction of output and set that equal to our model equivalent of $\theta\gamma$. The literature provides estimates ranging from 15 percent to several multiples of output.[17] We pick a low value of 15 percent an intermediate value of 25 percent

[16]See also Syverson (2011) and Bartelsman and Wolf (2017) for a review of the literature on productivity dispersion in the U.S. and Euro area. Hsieh and Klenow (2009) find the equivalent measure for productivity dispersion to be as high as five for China and India.

[17]See Hoggarth et al. (2002) who estimate a 15-20 percent fall in output in response to banking crises and Boyd and Heitz (2016) for estimates around 22-27 percent. See also Laeven and Valencia (2008, 2010) for estimates ranging from 16-25 percent of output (in terms of output deviation from trend or debt to GDP) resulting from past banking crises. On the extreme end of estimates, Andrew Haldane (in his 2010 address *The $100 billion question* at the Institute of Regulation and Risk) posits a cost of about 1 to 5 times GDP for systemic crises due to their persistent effects.

and a high value of 33 percent.[18] Table 2 reports the calibrated values of these parameters.

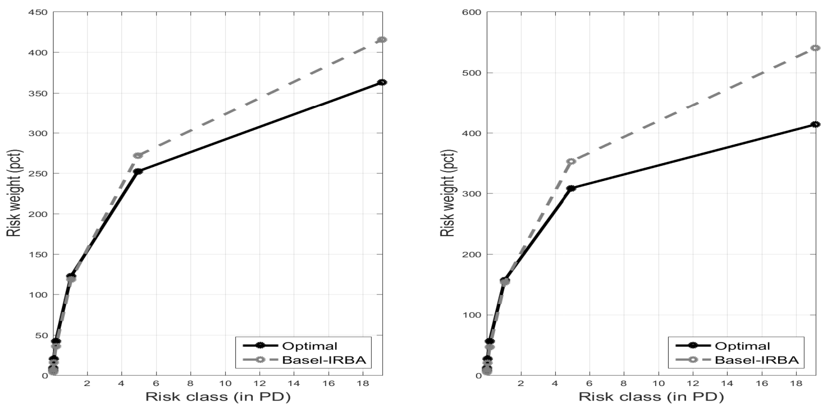
Table 2: Other parameters

ρ	R^d	θ	γ given bank failure cost as % of output		
			15%	25%	33%
1.085	1.02	0.36	0.417	0.694	0.917

4.1 Optimal vs Purely risk-based regulation

Figure 4 plots the set of optimal capital requirements against *Basel-IRBA* requirements given the calibration at a cost of bank failures equal to 15 percent of output. The left plot has productivities constant while the right plot has the riskier borrowers more productive.

Figure 4: Risk weights: Optimal vs. Basel-IRBA



Left panel reports risk weights in percent under the scenario of equal productivity. The right panel reports risk weights in percent when the riskier borrowers are more productive. Risk weights are defined as the ratio of the capital requirements in a given sector relative to the average capital requirement for all sectors. Risk categories, in terms of default probabilities, are on the horizontal axes.

[18] Increasing the value of this parameter leads to smaller differences between the optimal and *Basel-IRBA* requirements.

These and other calibration results are reported in Table 3. The welfare cost of adopting the purely risk-based requirements appear to be quite modest at up to 0.03 percent change in welfare. As we have previously shown, this cost is increasing in the correlation between borrower risk and productivity. Also, the relative difference between the two regulatory schemes becomes smaller the larger the cost of bank failures.

Table 3: Comparison of risk weighting schemes to optimal requirements

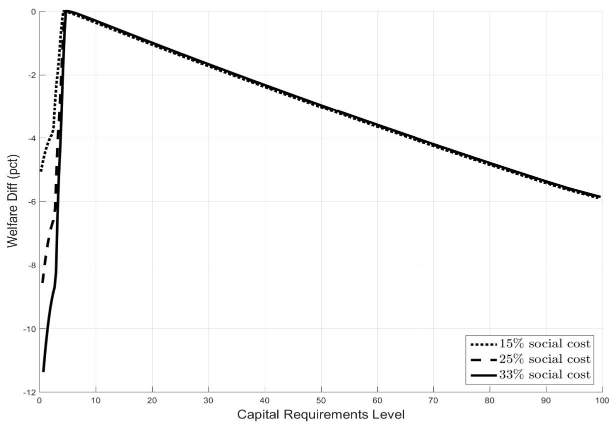
		Failure cost 15 %		Failure cost 25 %		Failure cost 33 %	
	Productivity	**Equal**	**Increasing**	**Equal**	**Increasing**	**Equal**	**Increasing**
Basel-IRBA	**Average Capital Requirement**	4.51	3.33	4.78	3.55	4.92	3.67
	MAD	0.40	0.48	0.37	0.44	0.36	0.43
	Bank Failure rate	0.23	0.33	0.13	0.19	0.09	0.14
	Welfare difference	0.02	0.03	0.01	0.03	0.01	0.03
Leverage Ratio	**Capital Requirement**	6.89	6.52	8.41	8.07	9.17	8.85
	MAD	4.09	4.06	5.14	5.28	5.67	5.89
	Average Bank Failure rate	1.27	0.88	0.71	0.48	0.52	0.35
	Welfare difference	0.34	0.29	0.42	0.37	0.46	0.41

The first four rows report results when comparing the purely risk-based requirements with the optimal. The first row reports the target bank failure rate under this policy. The second row gives the average capital requirement. The third row reports the mean absolute difference (MAD) in effective capital requirements. The fourth row reports the Welfare loss in percentage terms. The fifth to eighth rows report the same values under a pure leverage ratio requirement. The columns reflect different assumptions with regard to the bank failure costs (from 15 to 33 percent of output) and productivity differences across sectors. All averages are weighted by sector shares.

In addition, we also calculate the welfare cost of adopting a flat regulatory scheme where all borrowers are charged the same constrained-optimal capital requirements. This scheme, equivalent to a simple leverage ratio requirement on banks, generates higher welfare losses an order of magnitude larger and up to almost half a percent of output.

How large are the welfare losses of getting the risk weights wrong relative to getting the level of capital requirements wrong? In the following exercise, we calculate welfare losses when capital requirements are set using the optimal risk weights but the level of capital requirements wrong. Figure 5 plots these losses for the case where the sectors are equally productive.[19] The welfare losses on getting the average level of requirements wrong are asymmetric and suggests that it is better to err on the side of too high capital requirements rather than too low. The loss from capital requirements too high by three quarters of a percent is about 0.03 percent which is roughly equivalent to the welfare loss of adopting the Basel-IRBA risk weighting scheme at the right level of capital requirements. On the other hand, the welfare loss for the right risk weights but the level of requirements too low by three quarters percent is much larger at between two to four percent of welfare.

Figure 5: Welfare losses at various levels of capital requirements



These are welfare losses (in percent) of adopting capital requirements using the right risk weights but at different levels (horizontal axis) when sectors have equal productivity. Each line reports losses under different assumed values for the social cost of bank failures.

4.2 Response to increased risk

In a second exercise, we simulate a change that motivates the need for higher capital requirements. This is done by raising the riskiness of all borrowers by

[19]Similar results are obtained for the case when the riskiest sectors are more productive. See also Firestone et al. (2017) for asymmetric loses on the level of capital requirements.

the same factor such that the average of the new optimal risk-weighted capital requirements is five percentage points higher. This mimics the elevated requirement for common equity ratios following the implementation of Basel III requirements. This is done by raising the correlation factor from 20 to 44 percent in the calculation of η_j.

We consider three schemes along with the new optimal set of requirements as a benchmark. These are (1) the new Basel-IRBA requirements which sets a new constrained-optimal target Ψ^* (New IRBA), (2) a simple increase of five percent of the previous set of equal-default-probability requirements (Level Shift), and (3) a flat rate regulatory regime equal to the average level of the new optimal capital requirements (Leverage Ratio). Table 4 reports welfare losses for the various schemes relative to the optimal.

We find that, as was shown in Section 3.4, the increase in riskiness has led to an increase in the optimal average bank failure rates. This also leads to a small increase in the welfare losses of adopting the various alternative schemes. The set of risk-weighted capital requirements under the New IRBA requirements are also still too *steep*. The credit spread (lending rate of riskiest to safest sectors) under this scheme is between 22-37 basis points higher than the one under the optimal set of risk weights to the detriment of borrowing by the riskiest sector.

On the other hand, if regulation did not change (bottom rows of Table 4) then the welfare loss is between 1.8 to 3.6 percent of output with much higher bank failure rates (as much as 18 percent) and too much aggregate lending. Based on these results, the data suggests that given the level of variation and risk across commercial loans, credit mis-allocation arising from a purely risk-based risk-weighted capital regulation does not generate significant welfare losses.

Table 4: Risk weighting schemes vs optimal under increased risk

		Failure cost 15 %		Failure cost 25 %		Failure cost 33 %	
		Equal A	**Incr A**	**Equal A**	**Incr A**	**Equal A**	**Incr A**
New IRBA	**Ave Req**	8.97	6.91	9.60	7.45	9.92	7.73
	MAD	0.77	0.79	0.70	0.71	0.67	0.68
	Bank Failure rate	0.58	0.84	0.32	0.47	0.24	0.34
	Loan vol	-0.05	-0.09	-0.05	-0.08	-0.04	-0.07
	Welfare diff	-0.02	-0.05	-0.02	-0.05	-0.02	-0.04
Level Shift	**Ave Req**	9.45	8.26	10.12	8.92	10.45	9.25
	MAD	1.96	2.06	2.20	2.35	2.32	2.49
	Bank Failure rate	0.84	1.02	0.56	0.66	0.45	0.52
	Loan vol	-0.12	-0.14	-0.12	-0.13	-0.12	-0.13
	Welfare diff	-0.10	-0.08	-0.11	-0.09	-0.12	-0.10
Leverage Ratio	**Ave Req**	11.61	11.17	13.55	13.08	14.66	14.17
	MAD	5.25	5.52	6.45	6.87	7.14	7.65
	Bank Failure rate	1.91	2.25	1.14	1.35	0.84	1.00
	Loan vol	-0.41	-0.33	-0.58	-0.49	-0.68	-0.59
	Welfare diff	-0.39	-0.31	-0.48	-0.40	-0.53	-0.45
Unchanged	**Ave Req**	4.51	3.33	4.78	3.55	4.92	3.67
	MAD	4.05	3.43	4.46	3.76	4.66	3.94
	Bank Failure rate	16.01	17.93	14.42	16.11	13.64	15.22
	Loan vol	0.54	0.56	0.60	0.64	0.62	0.67
	Welfare diff	-1.78	-1.87	-2.84	-2.99	-3.64	-3.83

The first five rows report results when comparing the new purely risk-based requirements with the optimal. The first row reports the average risk-weighted capital requirement; the second row gives the mean absolute difference (MAD) in effective capital requirements with respect to the optimal; the third row provides the average bank failure rate; the fourth row provides the difference (in percent) in aggregate loan volumes with respect to the optimal; and finally the fifth row reports the difference (in percent) in welfare. The next set of rows report the same results for a level shift in the old purely-risk based requirements, the optimal requirements under a completely flat and risk-insensitive regime (i.e. a leverage ratio), and outcomes when the purely risk-based requirements are left unchanged. The columns reflect different assumptions with regard to the bank failure costs (from 15 to 33 percent of output) and productivity differences across sectors. All averages are weighted by sector shares.

Instead, a completely risk-insensitive regulatory regime as well as one where the average level of capital requirements are inappropriately low seem to be more costly.

Who suffers or gains the most from these changes? In Table 5, we report changes in loan volumes, loan rates, and welfare for the safest and riskiest sectors as well as the overall economy under the various regulatory adjustment schemes. In all but the leverage ratio requirement case, where borrowing rates for the riskiest sector actually falls, lending to the riskiest sectors shrinks the most. The change in welfare is also smallest (and mostly negative) for the riskiest sector.

These results corroborate the claims made by several parties mentioned in the beginning of this paper that the push for improving banking stability is likely to have resulted in leaving the riskiest sectors worse off (or the least better off).

Table 5: Adjustment to new requirements across sectors

		Equal A			A Diff		
		Safest	**Riskiest**	**Overall**	**Safest**	**Riskiest**	**Overall**
New Optimal	**Loans**	-0.06	-0.23	-0.54	-0.05	0.14	-0.50
	Loan rates	0.03	0.11	0.26	0.03	-0.05	0.22
	Welfare	2.46	0.03	1.80	2.65	0.01	2.08
New IRBA	**Loans**	-0.04	-0.66	-0.60	-0.03	-0.96	-0.54
	Loan rates	0.02	0.32	0.29	0.02	0.31	0.23
	Welfare	2.40	-0.05	1.78	2.55	-0.31	2.03
Level Shift	**Loans**	-0.67	-0.65	-0.66	-0.49	-0.99	-0.68
	Loan rates	0.32	0.32	0.32	0.32	0.32	0.32
	Welfare	2.15	-0.04	1.70	2.49	-0.33	1.99
Leverage Ratio	**Loans**	-1.54	0.95	-0.95	-1.09	1.39	-1.00
	Loan rates	0.74	-0.46	0.46	0.72	-0.44	0.51
	Welfare	1.69	-1.01	1.41	2.27	-0.62	1.76

All numbers, except for loan rates which are differences in percent, are percentage changes relative to when requirements are unchanged at the Basel-IRBA *values. The assumed cost of bank failures is 15% and the first three columns pertain to the case where all sectors have equal productivity while the latter three columns are for the case when the riskiest sector is also more productive. Overall changes are averaged using sector shares as weights. Nevertheless, overall welfare is improved relative to keeping capital requirements unchanged in all cases considered.*

5 Concluding Remarks

We have revisited the question of optimal capital requirements for banks from the viewpoint of risk weights. In the current regulation, a key principle is to set the risk weights applied to bank loans such that the risk weight reflects a loan's contribution to the bank's loan portfolio risk. However, recent literature has suggested several reasons on why a purely risk-based capital requirement may not be optimal, and have argued for less "risk-sensitive" capital requirements. The current risk-weighting system may be prone to manipulation; it may spur excessive growth in unregulated "shadow banking"; and it may distort the allocation of credit. We focus on the latter argument to formalize and shed light on policy-oriented discussions that have called for less risk-sensitive capital requirements motivated by concerns regarding business lending and economic growth.

We show that the pure risk perspective of setting capital requirements is too narrow and that the optimal risk weights take into account borrowing constraints and productivity differences across firms. We find that the optimal capital requirements are flatter or less risk-sensitive than the current regulation. This result obtains even if there are no productivity differences across sectors. The flattening effect is amplified when productivity and risk are positively correlated across sectors, when the bank equity premium is large, or when the social cost of bank failures is small. Nevertheless, a quantitative evaluation of the model indicates that the welfare loss from adopting current regulation is of second-order importance. Further, even though overall welfare is improved when requirements are sufficiently high and risk-sensitive, there are *winners* (safe borrowers) and *losers* (risky borrowers).

As regards future research, our model has assumed a competitive banking sector which implies that the cost of higher capital requirements is fully transmitted to borrower loan rates and hence credit allocation. An extension which allows for imperfect bank competition, and thus a more benign effect of capital requirements on credit allocation, is an area for future work. Other areas to explore include allowing for endogenous bank equity premia and non-linearities in the social cost of bank failures. For instance, convex costs could be one way to incorporate risk-aversion in the welfare function. Finally, the questions on whether bank equity premia responds (negatively) to increased capital regulation and whether there is indeed some asymmetry in the social losses associated with too high or too low capital requirements are left for future empirical research.

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A Proofs

A.1 Proof of Proposition 1

We first prove the first two lemmas. Note that the first and second order conditions for optimality given an interior solution require that,

$$\begin{aligned} \frac{\partial w_j}{\partial \kappa_j^*} &= 0 \\ \frac{\partial^2 w_j}{\partial \kappa_j^{*2}} &< 0 \end{aligned}$$

where

$$w_j = \left[1 + \Phi_j(\theta_j - \bar{\epsilon}^d) - \gamma \theta_j \Psi_j\right] K_j$$

For any parameter x whenever $\frac{\partial^2 w_j}{\partial \kappa_j \partial x} > 0 \Rightarrow \partial \kappa^* / \partial x > 0$.[20] It is useful to note the following,

$$\begin{aligned} \frac{\partial K}{\partial \kappa} &= -\theta(\rho - R^d)\left[\frac{K}{R^b}\right]^2 < 0 \\ \frac{\partial \Psi}{\partial \kappa} &= -\frac{\psi(\cdot)}{\eta}\left[\frac{\partial \tilde{\kappa}}{\partial \kappa}\right] < 0 \\ \frac{\partial \tilde{\kappa}}{\partial \kappa} &= \frac{\rho}{(1-\kappa)R^b} > 0 \end{aligned}$$

where $\psi(\cdot) \equiv (2\pi)^{-\frac{1}{2}} exp(-(\frac{\eta}{2} - \frac{\tilde{\kappa}}{\eta})^2/2)$ is the normal pdf evaluated at the

[20] Implicit function theorem.

standardized value of the capital requirement. We now show that $\frac{\partial^2 w}{\partial \kappa \partial \eta} > 0$ since

$$\begin{aligned}
\frac{\partial^2 w_j}{\partial \kappa_j \partial \eta} &= -\gamma\theta \left[\frac{\partial K}{\partial \kappa} \frac{\partial \Psi}{\partial \eta} + K \frac{\partial^2 \Psi}{\partial \kappa \partial \eta} \right] \\
&> 0 \quad \text{since} \\
\frac{\partial \Psi}{\partial \eta} &= \psi(\cdot)(\frac{1}{2} + \frac{\tilde{\kappa}}{\eta^2}) > 0 \\
\frac{\partial^2 \Psi}{\partial \kappa \partial \eta} &= \frac{\partial \tilde{\kappa}}{\partial \kappa} \frac{\psi(\cdot)}{\eta^2} \left[1 + \frac{\eta^2}{4} - \frac{\tilde{\kappa}^2}{\eta^2} \right] \\
&< 0 \qquad \text{if} \quad \tilde{\kappa}^2 > \frac{\eta^4}{4} + \eta^2 \\
&\Leftrightarrow \\
\frac{\partial \kappa^*}{\partial \eta} &> 0
\end{aligned}$$

where $\tilde{\kappa}^2 > \frac{\eta^4}{4} + \eta^2$ is satisfied for reasonable (i.e. low) values of portfolio risk that generate low optimal failure probabilities $\Psi(\kappa^*)$. Let Ψ^* be the bank failure probability under the optimal capital requirement. Then,

$$\begin{aligned}
\tilde{\kappa}^2 &= \left(\frac{\eta^2}{2} - \eta \Phi^{-1}(\Psi^*) \right)^2 \\
&= \frac{\eta^4}{4} + \eta^2 + \eta^2 \left[(\Phi^{-1}(\Psi^*))^2 - 1 - \eta \Phi^{-1}(\Psi^*) \right] \\
&> \frac{\eta^4}{4} + \eta^2 \quad \text{iff} \\
&\quad (\Phi^{-1}(\Psi^*))^2 - 1 - \eta \Phi^{-1}(\Psi^*) > 0 \\
&\Leftrightarrow \\
\Psi^* &< \Phi(\frac{\eta}{2} - \sqrt{\frac{\eta}{4} + 1})
\end{aligned}$$

where the last inequality is trivially satisfied when $\Psi^* < \Phi(-1) = 0.1587$. Note that this is a sufficient and not necessary condition for the lemma.

Similarly We now show that $\frac{\partial^2 w}{\partial\kappa\partial A} < 0$,

$$\begin{aligned}
\frac{\partial^2 w_j}{\partial\kappa_j\partial A} &= (R^b - \theta A)^{-2}\left[2\theta K_j(\rho - R^d)\left(\gamma\theta\Psi - (1+\Phi(\theta-\bar{\epsilon}^d))\right) - \gamma\theta R^{b2}\frac{\partial\Psi}{\partial\kappa}\right] \\
&< 0 \quad \text{iff} \\
&\quad \left[1+\Phi(\theta-\bar{\epsilon}^d) - \gamma\theta\Psi\right]\frac{\partial K}{\partial\kappa} < \frac{\gamma\theta K}{2}\frac{\partial\Psi}{\partial\kappa} \\
&\quad \frac{\partial w_j}{\kappa_j} = 0 < -\frac{\gamma\theta K}{2}\frac{\partial\Psi}{\partial\kappa} \\
&\Leftrightarrow \\
\frac{\partial\kappa^*}{\partial A} &< 0
\end{aligned}$$

Consequently, we can write the optimal capital requirement as a function of two arguments $\kappa^*(A,\eta)$ which is decreasing in A and increasing in η. The proof of Proposition 1 arises from defining the thresholds $\underline{A}(\eta_j;\kappa_0^*)$ such that $\kappa^*(\underline{A},\eta_j) = \kappa_0^*$ and $\bar{\eta}(A_j;\kappa_0^*)$ such that $\kappa^*(A_j,\bar{\eta}) = \kappa_0^*$.

Finally, an interior solution exists when,

$$\begin{aligned}
1+\Phi(\theta-\bar{\epsilon}^d) - \theta\gamma\Psi > 0 \quad \text{for some } \kappa \in [0\ 1] \quad &\Rightarrow \quad \kappa^* < 1 \\
\frac{\partial\Psi}{\partial\kappa} < 0 \quad &\Rightarrow \quad \kappa^* > 0
\end{aligned}$$

A.2 Proof of Proposition 2

The proof follows from the previous Proposition. The first lemma showed that $\kappa_j^* > \kappa_k^*$ whenever $\eta_j^2 > \eta_k^2$. Next we show that $\Psi(\kappa_j^*) > \Psi(\kappa_k^*)$

Let sector 0 have $\Psi(\kappa_0^*) = \Psi^*$. Consider now the equal bank failure probability capital requirement scheme where $\tilde{\kappa}_j^{epd} = \frac{\eta_j^2}{2} - \eta_j\Phi^{-1}(\Psi^*)$ and sector 0 be such that $\Psi(\tilde{\kappa}_0^{epd}) = \Psi^*$ where Ψ^* solves

$$\int_j -\eta\frac{\partial\kappa}{\partial\tilde{\kappa}}\left[-\gamma\theta K\frac{\partial\Psi}{\partial\kappa} + (1+\Phi(\theta-\bar{\epsilon}^d) - \gamma\theta\Psi)\frac{\partial K}{\partial\kappa}\right] = 0$$

That is, both the optimal capital requirement and the equal bank failure probability capital requirement scheme coincide for sector 0. Then, note that

$$\begin{aligned}
\tilde{\kappa}_j^{epd} - \frac{\eta_j^2}{2} &= \frac{\eta_j}{\eta_0}(\tilde{\kappa}_0^{epd} - \frac{\eta_0^2}{2}) \\
&\text{and} \\
\frac{\partial \tilde{\kappa}^{epd}}{\partial \eta} &= \frac{\eta_j}{2} + \frac{\tilde{\kappa}_j^{epd}}{\eta_j}
\end{aligned}$$

where the derivative assumes that $\frac{\partial \Psi^*}{\partial \eta_j} \to 0$ or a marginal increase in capital requirements for a given sector does not significantly change the target failure probability. On the other hand, consider the sensitivity of the optimal capital requirements to portfolio risk. Using the implicit function theorem,

$$\begin{aligned}
\frac{\partial \tilde{\kappa}^*}{\partial \eta} &= -\left[\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j \partial \eta_j}\right]\left[\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j^2}\right]^{-1} \\
&\text{where} \\
\frac{\partial^2 w}{\partial \tilde{\kappa} \partial \eta} &= \frac{\theta \gamma \psi(\cdot)}{\eta}\left[\frac{K}{\eta}\left(\frac{\tilde{\kappa}_j^2}{\eta^2} - \frac{\eta_j^2}{4} - 1\right) - \frac{\partial K}{\partial \tilde{\kappa}}(\frac{\eta_j}{2} + \frac{\tilde{\kappa}_j}{\eta_j})\right] \\
&\text{thus} \\
\frac{\partial \tilde{\kappa}^*}{\partial \eta} &= -\theta \gamma \frac{\psi(\cdot)}{\eta}\left[\left(\frac{\tilde{\kappa}_j^2}{\eta^2} - \frac{\eta_j^2}{4} - 1\right)\frac{K}{\eta} - (\frac{\eta_j}{2} + \frac{\tilde{\kappa}_j}{\eta_j})\left(\frac{\partial K}{\partial \tilde{\kappa}}\right)\right]\left[\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j^2}\right]^{-1} \\
&\text{where} \\
\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j^2} &= (1 + \Phi(\theta - \bar{\epsilon}^d) - \theta \gamma \Psi)\frac{\partial^2 K}{\partial \tilde{\kappa}^2} - 2\theta \gamma \frac{\partial K}{\partial \tilde{\kappa}}\frac{\partial \Psi}{\partial \tilde{\kappa}} - \theta \gamma K \frac{\partial^2 \Psi}{\partial \tilde{\kappa}^2}
\end{aligned}$$

Since $\frac{\partial w}{\partial K} > 0$ and $\frac{\partial K}{\partial \tilde{\kappa}} < 0$, it must be the case that

$$\frac{\partial \tilde{\kappa}^*}{\partial \eta} \leq \frac{\partial \tilde{\kappa}^*}{\partial \eta}\Big|_{\frac{\partial K}{\partial \tilde{\kappa}}=0}$$

That is, if investment is inelastic to capital requirements then the optimal capital requirement is more sensitive to portfolio risk. This hypothetical sensitivity is given by

$$\begin{aligned}
\frac{\partial \tilde{\kappa}^*}{\partial \eta}\Big|_{\frac{\partial K}{\partial \tilde{\kappa}}=0} &= \left[-\frac{\theta\gamma K\psi(\cdot)}{\eta^2}\left(\frac{\tilde{\kappa}_j^2}{\eta^2}-\frac{\eta_j^2}{4}-1\right)\right]\left[\frac{\theta\gamma K\psi(\cdot)}{\eta^2}\left(\frac{\eta}{2}-\frac{\tilde{\kappa}}{\eta}\right)\right]^{-1} \\
&= \left[\left(\frac{\eta}{2}+\frac{\tilde{\kappa}}{\eta}\right)\left(\frac{\eta}{2}-\frac{\tilde{\kappa}}{\eta}\right)+1\right]\left[\left(\frac{\eta}{2}-\frac{\tilde{\kappa}}{\eta}\right)\right]^{-1} \\
&= \left(\frac{\eta}{2}+\frac{\tilde{\kappa}}{\eta}\right)+\frac{2\eta}{\eta^2-2\tilde{\kappa}} \\
&< \left(\frac{\eta}{2}+\frac{\tilde{\kappa}}{\eta}\right)=\frac{\partial \tilde{\kappa}^{epd}}{\partial \eta}
\end{aligned}$$

where the last inequality follows from the assumption that $\tilde{\kappa}^2 > \frac{\eta^4}{4} + \eta^2$ (i.e. parameters are such that the optimal failure probabilities are sufficiently low). Thus we have shown that,

$$\frac{\partial \tilde{\kappa}^*}{\partial \eta} \leq \frac{\partial \tilde{\kappa}^*}{\partial \eta}\Big|_{\frac{\partial K}{\partial \tilde{\kappa}}=0} < \frac{\partial \tilde{\kappa}^{epd}}{\partial \eta}$$

That is, the optimal capital requirements are less sensitive to increases in portfolio risk than the equal bank failure probability scheme. This implies that, all else equal, for $\eta_j > \eta_0$ we have $\kappa_0^* < \kappa_j^* < \kappa_j^{epd}$ and $\Psi(\kappa_j^*) > \Psi(\kappa_j^{epd})$.

Similarly, the capital requirement given by this policy scheme for a sector with a lower portfolio risk is lower than the optimal capital requirement hence the Proposition.

A.3 Proof of Proposition 3

In this scenario, we have that $\eta_j^{new} = c\eta_j \quad \forall\, j$ with $c > 1$. From Proposition 1, we know that higher portfolio risk leads to higher capital requirements,

$$\kappa_j^{*new} > \kappa_j^* \quad \forall\, j$$

From Proposition 2, we also know that the higher capital requirements will not completely offset the rise in failure probability such that,

$$\Psi_j^{*new} > \Psi_j^* \quad \forall\, j$$

We only need to show that the new and higher capital requirements are also relatively flatter than the previous set of capital requirements. Here, it is sufficient to show that the optimal capital requirements are (increasing and) concave in portfolio risk.

From Proposition 2, we know that $\frac{\partial \tilde{\kappa}^*}{\partial \eta} < \frac{\partial \tilde{\kappa}^{epd}}{\partial \eta}$. We now show that the equal bank failure probability capital requirement scheme is also concave in portfolio risk.

$$\begin{aligned} \frac{\partial \tilde{\kappa}^{epd}}{\partial \eta} &= \left(\frac{\eta}{2} + \frac{\tilde{\kappa}^{epd}}{\eta}\right) \\ \Rightarrow & \\ \frac{\partial^2 \tilde{\kappa}^{epd}}{\partial \eta^2} &= \frac{1}{\eta}\left(\frac{\eta}{2} - \frac{\tilde{\kappa}^{epd}}{\eta}\right) \\ &< 0 \end{aligned}$$

where the last inequality holds for any bank failure probability less than one half (well above our previous assumption of $\Psi < \Phi(-1)$). Thus we have that (1) the equal bank failure probability capital requirement scheme is increasing and concave in portfolio risk (η) and (2) its slope is larger than that of the optimal capital requirement scheme. Thus, it must be the case that the optimal capital requirements are also concave in portfolio risk. Finally, given that the optimal capital requirements are concave in portfolio risk, a proportional increase in risk across the board would also lead to flatter requirements for so long as the increase does not lead to a violation of our assumption regarding the upper bound on bank failure probabilities: $max(\Psi(\kappa_j^{*new})) < \Phi(-1)$.

B Deposit insurance

We assume deposit insurance to protect depositors in case of bank failure and financed by premia charged on banks. We depart from a standard deposit insurance scheme whereby an insurer collects a fixed, potentially risk-sensitive, premium on banks to finance the shortfall in failing banks' assets to meet deposit liabilities. Instead, our scheme consists of state-contingent premia collected from surviving banks which resembles the payoffs of two call options, one long and one short, on bank assets at different strike prices. Our scheme circumvents the problem that a standard deposit insurance with an actuarially fair but fixed premium typically yields a fixed-point problem which jointly determines the size of the premium and the likelihood of bank failures.[21] Our scheme delivers a parsimonious description of bank failure probabilities along with an actuarially fair deposit insurance scheme. In this section we show that such a scheme generates enough funds to sufficiently insure deposits and is consistent with the characterization of the return on bank equity detailed in the main text.

Our scheme is analogous to those in Merton (1977); Ronn and Verma (1986). A deposit insurance fund is assumed to cover deposits held at retail banks by guaranteeing a return R^d in case of bank failure. The scheme is funded by insurance premia collected from surviving banks and the liquidation of assets from failed banks. Relative to a standard deposit insurance scheme where insurance premia are fixed, the state-contingent nature of our premia allows us to generate a simple description of bank failure probabilities along with a fair insurance scheme. Further, to ensure feasibility of our scheme we assume that the deposit insurer observes each bank's loan portfolio and capital structure (see Chan et al., 1992).[22]

Our deposit insurance scheme may be thought of as follows. Consider an Insurer who owns all the claims to a risky asset of unit size in period one which yields ξR^b in the second period for a per unit return of $\tilde{\xi} + r^b \equiv log(\xi R^b)$ where $r^b \equiv R^b - 1$ and $\tilde{\xi} \sim \mathcal{N}(-\eta^2/2, \eta^2)$. The Insurer then sells a risk free bond of size equal to a fraction $(1-\kappa)$ of the asset to Depositors at the risk-free rate of return $r^d \equiv R^d - 1$. Finally, the Insurer sells a call option on the underlying asset with strike value of $(R^d(1-\kappa)+s)$ at the price κ to a Banker where s is

[21] For instance, one may have that the default probability of a bank is given by $\Psi = \Phi(\frac{(R^d+s)(1-\kappa)}{R^b})$ where Φ is the Normal cdf, κ is the share of bank equity financing, R^d and R^b are the deposit and lending rates respectively, and s is the deposit insurance premium. In turn, we would also have that $s = \frac{\Psi(\mathbb{E}^-[\xi R^B - R^d(1-\kappa)])}{1-\Psi}$ where ξ is the shock to bank asset returns and the expectation is conditional on bank failure.

[22] In the data, only about 30 percent of deposit insurance schemes around the world have some form of risk-sensitivity (Demirguc-Kunt et al., 2015).

determined by,

$$
\begin{aligned}
-\int_{-\infty}^{r^d(1-\kappa)-r^b} (\tilde{\xi} + r^b - r^d(1-\kappa)) f(\tilde{\xi}) \delta\tilde{\xi} &= \int_{r^d(1-\kappa)-r^b}^{\infty} (\tilde{\xi} + r^b - r^d(1-\kappa)) f(\tilde{\xi}) \delta\tilde{\xi} \\
&\quad - \int_{r^d(1-\kappa)-r^b+s}^{\infty} \tilde{\xi} + r^b - (r^d(1-\kappa) + s) f(\tilde{\xi}) \delta\tilde{\xi} \\
&\Leftrightarrow \\
P(R^d(1-\kappa)) &= C(R^d(1-\kappa)) - C(R^d(1-\kappa) + s)
\end{aligned}
$$

The above defines an actuarially fair and state-contingent premium s where the expected payout to the Depositor, with a present value equal to that of a put option at a strike of $R^d(1-\kappa)$, is exactly offset by the Insurer's remaining claims after paying out the call option to the Banker at a strike of $R^d(1-\kappa) + s$ such that the value left with the Insurer is zero. This arrangement is illustrated in Figure 6.

Figure 6: Bank asset returns

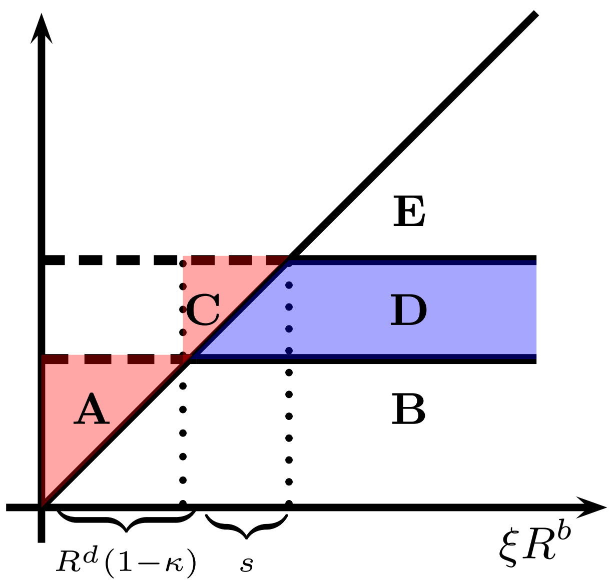

Figure 6 plots the realizations of the asset's return on the horizontal axis with the value of the asset as a $45°$ line. In this figure, the areas marked with A and B indicate the Depositor's claims over realizations of the Asset value on the horizontal axis. The blue space denoted with D represent the Insurer's claims (the insurance premia) which is equivalent in expectations to the red area marked with A which represents the Insurer's payouts to satisfy the Depositor's claims for the states where $\xi R^b < R^d(1-\kappa)$. Finally, the Banker's claims is represented by the remaining space under the $45°$ line denoted with E.

In the standard deposit insurance scheme where a premia is fixed at s, the premia collected would equal the spaces $C + D$ which would equal the space A if the scheme were actuarially fair. This implies that a bank fails when $\xi R^b < R^d(1-\kappa) + s$ and in turn, the value of the premia s depends on the bank failure probability. On the other hand, in our scheme the bank fails whenever $\xi R^b < R^d(1-\kappa)$ which is independent of the insurance premia.

The present value of the asset is divided into the following

$$\begin{aligned} 1 &= C(R^d(1-\kappa)) - P(R^d(1-\kappa)) + (1-\kappa) \\ &= \begin{cases} (1-\kappa)+ & \text{Depositor} \\ C(R^d(1-\kappa)) - P(R^d(1-\kappa)) - C(R^d(1-\kappa)+s)+ & \text{Insurer} \\ C(R^d(1-\kappa)+s) & \text{Banker} \end{cases} \end{aligned}$$

where the first line is just the *Put-Call Parity* and we also have that $C(R^d(1-\kappa)+s) = \kappa$. Finally, since the premium s is actuarially fair such that the Insurer has zero expected claims in the second period, the expected value of the Banker's holdings in the second period is given by,

$$\begin{aligned} \int_{R^d(1-\kappa)+s}^{\infty} (\xi R^b - R^d(1-\kappa) - s) f(\xi)\delta\xi &= \left(\int_0^{\infty} \xi R^b f(\xi)\delta\xi\right) - R^d(1-\kappa) - 0 \\ &= \mathbb{E}[\xi]R^b - R^d(1-\kappa) \\ &= \kappa\rho \end{aligned}$$

since $R^b = \kappa\rho + (1-\kappa)R^d$ and $\mathbb{E}[\xi] = exp(-\eta^2/2 + \eta^2/2) = 1$. Thus we have shown that with our actuarially fair deposit insurance scheme and the loan rate equation in 11, we have that a banker who puts in κ in the first period obtains an expected gross return of $\rho\kappa$ in the second period.

This insurance scheme is designed to produce an actuarially fair deposit insurance scheme as well as provide a simple characterization of the probability of bank default which is given by $\Psi = Pr(log(\xi) < log(R^d(1-k)/R^b)) = \Phi(\frac{\eta}{2} - \frac{\tilde{\kappa}}{\eta})$ where $\tilde{\kappa} = log(1 + \frac{\kappa}{1-\kappa}\frac{\rho}{r^d})$ and $\Phi(\cdot)$ is the Normal cumulative density function. Note that this probability is not the complement to the probability that the Banker makes a positive profit which is given by $1 - Pr(\xi < (R^d(1-\kappa)+s)/R^b)$.

C Endogenous portfolio risk

Consider now the case where the rest of the model is as before but the price shock that the entrepreneurs face are correlated within sectors or the economy as a whole. Consider the following processes for the entrepreneurs' price shock,

$$\begin{aligned}
log(\epsilon_{i,j}) &= e_{i,j} + \xi_j \\
\mathbb{E}[e_{i,j}\xi_j] &= 0 \\
e_{i,j} &\sim \mathcal{N}(-(\sigma_j^2 - \eta_j^2)/2, \sigma_j^2 - \eta_j^2) \\
\sigma_j &\geq \eta_j \quad \forall\, j \\
\xi_j &\sim \begin{cases} \mathcal{N}(-\eta_j^2/2, \eta_j^2) & \text{Independent sector-specific shocks} \\ -\eta_j^2/2 + \eta_j z\ ,\ z \sim \mathcal{N}(0,1) & \text{Loading on aggregate shock} \end{cases}
\end{aligned}$$

where the two cases for the sector-specific shock differ only in the interpretation of their aggregate consequences. Finally, each sector's retail banker is characterized with the ability to sell the specialized good at a price ϵ_j^b which reflects liquidation and other costs as a lower mean and at a price which is also subject to the sector-specific (or aggregate) shock,

$$\begin{aligned}
log(\epsilon_j^b) &= log(\theta_j) + \xi_j \\
&\sim \mathcal{N}(log(\theta_j) - \eta_j^2/2, \eta_j^2)
\end{aligned}$$

As before we have that $\mathbb{E}[log(\epsilon_{i,j})^2] = \sigma_j^2$ which determines the entrepreneur's unconditional default probability Φ_j. The expected cost to the bank of liquidating the entrepreneur's asset ($\theta_j < 1$) generates the borrowing constraint. That is, the banker's participation constraint is given by,

$$R_j^b B_j \leq \mathbb{E}[\epsilon_j^b] K_j = \theta_j K_j$$

Further, a risk-neutral banker's loan portfolio of sector j borrowers is now subject to sector-specific (or aggregate) risk η. In particular, the actual fraction of borrowers in default for a given sector is now a random variable with mean Φ_j. Further, the proceeds from liquidating the specialized good of defaulting borrowers is also a random variable with mean $\theta_j K_j$ and variance η_j^2.

The fraction $(1 - \Phi_j)$ of borrowers will repay their debt $(R_j^b B_j)$ and only the fraction Φ_j who default will subject the banker to risk as she will collect $\epsilon_j^b K_j$. Denote the fraction of deposits exposed to this risk after deducting repayments by borrowers with z_j,

$$\begin{aligned}
z_j d_j &\equiv d_j - (1 - \Phi_j) B_j \frac{R_j^b}{R^d} \\
&= d_j \left[1 - (1 - \Phi_j) \left(1 + \frac{\kappa_j}{1 - \kappa_j} \frac{\rho}{R^d} \right) \right] \\
&\Rightarrow \\
z_j &= 1 - (1 - \Phi_j) exp(\tilde{\kappa_j})
\end{aligned}$$

Then, a bank fails when the risky fraction of its assets are insufficient to cover the fraction of deposits exposed to risk:

$$\begin{aligned}
\Psi_j &= Pr(\Phi_j \epsilon_j^b K_j \leq R^d d_j z_j) \\
&= Pr(\frac{\epsilon_j^b}{\theta_j} \leq \frac{z_j}{\Phi_j} \left[1 + \frac{\kappa_j}{1 - \kappa_j} \frac{\rho}{R^d} \right]^{-1}) \\
&= Pr(log(\epsilon_j^b) - log(\theta_j) \leq log(\frac{z_j}{\Phi_j}) - \tilde{\kappa}_j) \\
&= \Phi(\frac{\eta_j}{2} - \frac{\tilde{\kappa}_j}{\eta_j} + \frac{1}{\eta_j} log(\frac{z_j}{\Phi_j})) \\
&= \Phi \left(\frac{\eta_j}{2} - \frac{\tilde{\kappa}_j}{\eta_j} + \frac{1}{\eta_j} log(\frac{1}{\Phi_j} - \frac{(1 - \Phi_j)}{\Phi_j} exp(\tilde{\kappa_j})) \right)
\end{aligned}$$

Note that in this alternative formulation, there is a third term determining the likelihood of bank failure which captures the share of bank assets exposed to risk (defaulting borrowers) relative to the share of deposits exposed to risk. This third term depends on both the riskiness of the borrowers ($\Phi(\sigma)$) and the bank's leverage which amplifies the effects of both in the setting of optimal capital requirements. On the one hand, a bank's failure probability from this alternative version of the model will be more sensitive to borrower risk than in the baseline model. On the other hand, capital requirements are also more effective in reducing the probability of bank failure in this alternative version.[23] On balance, we obtain qualitatively similar outcomes as in the main text.

As before, the optimal capital requirements trade off the cost of bank failure and the borrowers' credit constraint given by the following optimality condition:

$$\overbrace{(1+\Phi(\theta-\bar{\epsilon}^d))\frac{\partial K}{\partial \kappa}}^{\text{Entrep collateral constraint}} \;=\; \overbrace{\gamma\theta\left(K\underbrace{\frac{\partial \Psi}{\partial \kappa}}_{\text{Frequency}} + \Psi\underbrace{\frac{\partial K}{\partial \kappa}}_{\text{Size}}\right)}^{\text{Bank bankruptcy}}$$

This is the same condition in the main text with the subtle difference that the frequency of bank failures Ψ now has three terms (as outlined above) and the sensitivity of the frequency of bank failure to changes in the capital requirement is now given by

$$\frac{\partial \Psi}{\partial \kappa} \;=\; -\frac{\psi(\cdot)}{\eta}\frac{\partial \tilde{\kappa}}{\partial \kappa}\left[1+\frac{1-\Phi}{z}exp(\tilde{\kappa})\right]$$

where in the main text we have $\frac{\partial \Psi}{\partial \kappa} = -\frac{\psi(\cdot)}{\eta}\frac{\partial \tilde{\kappa}}{\partial \kappa}$. Here, since risk only affects a fraction of banks' assets, capital requirements are more effective in reducing the likelihood of bank failures which suggests that the level of optimal capital requirements under this alternative setup will be lower than those given in the main text.

[23] Note as well that in this alternative version, it is sufficient to set $\kappa_j \geq \left[1+\frac{1-\Phi_j}{\Phi_j}\frac{\rho}{R^d}\right]^{-1}$ to guarantee that a bank never fails.