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Peter McAdam, Anders Warne

Euro area real-time density forecasting with financial or labor market frictions

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Abstract: We compare real-time density forecasts for the euro area using three DSGE models. The benchmark is the Smets-Wouters model and its forecasts of real GDP growth and inflation are compared with those from two extensions. The first adds financial frictions and expands the observables to include a measure of the external finance premium. The second allows for the extensive labor-market margin and adds the unemployment rate to the observables. The main question we address is if these extensions improve the density forecasts of real GDP and inflation and their joint forecasts up to an eight-quarter horizon. We find that adding financial frictions leads to a deterioration in the forecasts, with the exception of longer-term inflation forecasts and the period around the Great Recession. The labor market extension improves the medium to longer-term real GDP growth and shorter to medium-term inflation forecasts weakly compared with the benchmark model.

Keywords: Bayesian inference, DSGE models, forecast comparison, inflation, output, predictive likelihood.

JEL Classification Numbers: C11, C32, C52, C53, E37.

Non-Technical Summary

Medium-size dynamic stochastic general equilibrium (DSGE) models—as exemplified by Smets and Wouters (2007)—have been widely used among central banks for policy analysis, forecasting and to provide a structural interpretation of economic developments; see, e.g., Del Negro and Schorfheide (2013) and Lindé, Smets, and Wouters (2016). Recent years have, however, constituted an especially challenging policy environment. Given the global financial crisis in late 2008, the Great Recession that followed, and the European sovereign debt crisis starting in late 2009, many economies witnessed sharp falls in activity and inflation, persistent increases in unemployment, and widening financial spreads. In such a severe downturn, large forecasts errors may be expected across all models.

In the case of DSGE models, two prominent criticisms additionally emerged: (i) that such models were lacking 'realistic' features germane to the crisis, namely financial frictions and involuntary unemployment; and (ii) that their strong equilibrium underpinnings made them vulnerable to forecast errors following a severe, long-lasting downturn. For such debates see, inter alia, Caballero (2010), Hall (2010), Ohanian (2010), Buch and Holtemöller (2014) and Lindé et al. (2016).

In this paper we compare real-time density forecasts for the euro area based on three estimated DSGE models. The benchmark is that of Smets and Wouters (2007), as adapted to the euro area, and its real-time forecasts of real GDP growth and inflation. These forecasts are compared with those from two extensions of the model. The first adds the financial accelerator mechanism of Bernanke, Gertler, and Gilchrist (1999) (BGG) and augments the list of observables to include a measure of the external finance premium. The second allows for an extensive labor margin, following Galí (2011) and Galí, Smets, and Wouters (2012), and, likewise, augments the unemployment rate to the set of observables. We label these models SW, SWFF and SWU, respectively. The euro area real-time database (RTD), on which these models are estimated and assessed, is described in Giannone, Henry, Lalik, and Modugno (2012). To extend the data back in time, we follow Smets, Warne, and Wouters (2014) and link the real-time data to various updates from the area-wide model (AWM) database; see Fagan, Henry, and Mestre (2005).

Against this background, we strive to make the following contributions. *First*, we report the density forecasting of the SW model and the two model variants, SWFF and SWU, over a period prior to and after the more recent crises; namely, 2001–2014. These additional variants, reflect 'missing' elements emphasized by some critics of the core model: namely, financial frictions, and 'extensive' labor-market modelling. The importance of combining models for predictive and policy-analysis purposes is an enduring topic in the literature, see e.g., Levine, McAdam, and Pearlman (2008), Geweke and Amisano (2011), or Amisano and Geweke (2017). In that respect, it is important to compare sufficiently differentiated models, to balance different and relevant economic mechanisms. Indeed, although the BGG extension to the core model has been much discussed, that allowing for extensive employment fluctuations has received less

attention. And yet, we know that unemployment was high by international comparisons in many euro area economies prior to the crisis, and slow to revert back to pre-crisis levels thereafter (European Commission, 2016). It is of interest therefore to assess the forecasting contribution of models which attempted to capture extensive fluctuations over the crisis. To our knowledge, this is the first time these three models have been estimated on a common basis and directly compared with an emphasis on their density forecasting comparisons. Accordingly, we can address the question of whether these extensions did or can improve the density forecasts of growth and inflation and their joint forecasts.

Our *second* contribution is that we focus on *real-time* forecasting performance. It has become standard to use real-time data when analyzing the out-of-sample forecast performances of competing models. In our exercises, we utilize the euro area RTD. It is also worth noting that forecasting applications of the RTD for the euro area have been relatively few. An important exception is Smets et al. (2014), who also use the SWU model for real-time analysis, and other examples are Conflitti, De Mol, and Giannone (2015), Jarociński and Lenza (2016) and Pirschel (2016). In view of the limited number of studies based on real-time euro area data, our paper can therefore be seen as building on and extending this important line of research.

Turning to our results, we note the following. Regarding the point forecasts based on the predictive mean, all models over-predict GDP growth over most of the sample, and in particular beyond 2009. The SWFF model has the largest (if most stable) forecast errors relative to SW and SWU, which are both relatively similar. Concerning the inflation forecasts the SWFF model under-predicts and especially at the shorter horizons while the SW and SWU models perform similarly with a tendency to over-predict at the longer horizons. In that regard, we concur with Kolasa and Rubaszek (2015) that a model with the basic financial accelerator mechanism is (in forecasting terms) not an obviously superior alternative. For a more detailed discussion of different modelling and methodological approaches to take in the wake of the crisis to improve models, see Lindé et al. (2016).

In terms of the density forecasts, where the predictive density evaluated at the actual outcome is used to compare models, we also find that the SW and SWU models yield similar density forecasts and dominate the SWFF model. Since the onset of the crisis there is a tendency for the SWU model to forecast better over the medium term, while the SW model tends to dominate weakly over the shorter term. All models display a drop in performance with the onset of the crisis in 2008Q4 and 2009Q1. At the same time, this drop looks like a one-time event and the ability of the models have otherwise not been notably affected.

1. Introduction

Medium-size dynamic stochastic general equilibrium (DSGE) models—as exemplified by Smets and Wouters (2007)—have been widely used among central banks for policy analysis, forecasting and to provide a structural interpretation of economic developments; see, e.g., Del Negro and Schorfheide (2013) and Lindé et al. (2016). Recent years have, however, constituted an especially challenging policy environment. Given the global financial crisis in late 2008, the Great Recession that followed, and the European sovereign debt crisis starting in late 2009, many economies witnessed sharp falls in activity and inflation, persistent increases in unemployment, and widening financial spreads. In such a severe downturn, large forecasts errors may be expected across all models.

In the case of DSGE models, two prominent criticisms additionally emerged: (i) that such models were lacking 'realistic' features germane to the crisis, namely financial frictions and involuntary unemployment; and (ii) that their strong equilibrium underpinnings made them vulnerable to forecast errors following a severe, long-lasting downturn. For such debates see, inter alia, Caballero (2010), Hall (2010), Ohanian (2010), Buch and Holtemöller (2014) and Lindé et al. (2016).

In this paper we compare real-time density forecasts for the euro area based on three estimated DSGE models. The benchmark is that of Smets and Wouters (2007), as adapted to the euro area, and its real-time forecasts of real GDP growth and inflation. These forecasts are compared with those from two extensions of the model. The first adds the financial accelerator mechanism of Bernanke et al. (1999) (BGG) and augments the list of observables to include a measure of the external finance premium. The second allows for an extensive labor margin, following Galí (2011) and Galí et al. (2012), and, likewise, augments the unemployment rate to the set of observables. We label these models SW, SWFF and SWU, respectively. The euro area real-time database (RTD), on which these models are estimated and assessed, is described in Giannone et al. (2012). To extend the data back in time, we follow Smets et al. (2014) and link the real-time data to various updates from the area-wide model (AWM) database; see Fagan et al. (2005).

More generally, while financial frictions had already been introduced into some estimated DSGE models prior to the financial crisis, such extensions of the 'core' model were not yet standard; see, e.g., Christiano, Motto, and Rostagno (2003, 2008) and De Graeve (2008). Since the crisis there has been an active research agenda exploring extensions to the core model. For instance, Lombardo and McAdam (2012) considered the inclusion of the financial accelerator on firms' financing side alongside constrained and unconstrained households; see also Kolasa, Rubaszek, and Skrzypczyński (2012). Del Negro and Schorfheide (2013) also integrated the effect of BGG financial frictions on the core (SW) model and compared its forecasting performance with non model-based ones. Moreover, Christiano, Trabandt, and Walentin (2011) considered the open-economy dimension to financial frictions in a somewhat larger-scale model, which also included labor market frictions.

However, in terms of forecasting performance and model fit, it is by no means clear whether these extensions have improved matters. For instance, while Del Negro and Schorfheide (2013) and Christiano et al. (2011) favorably report the forecasting performance of the SW model augmented by financial frictions, Kolasa and Rubaszek (2015) find that adding financial frictions can worsen the average quality of density forecasts, depending on the friction examined. Moreover, while Del Negro and Schorfheide (2013) use (US) real time data, the other two studies do not. Nor is there a common basis of forecast comparison across these papers: Kolasa and Rubaszek (2015) and Del Negro and Schorfheide (2013) emphasize density forecasts, whereas Christiano et al. (2011) use point forecasts.

Against this background, we strive to make the following contributions. *First*, we report the density forecasting of the SW model and the two model variants, SWFF and SWU, over a period prior to and after the more recent crises; namely, 2001–2014. These additional variants, reflect 'missing' elements emphasized by some critics of the core model: namely, financial frictions, and 'extensive' labor-market modelling. The importance of combining models for predictive and policy-analysis purposes is an enduring topic in the literature, see e.g., Levine et al. (2008), Geweke and Amisano (2011), or Amisano and Geweke (2017). In that respect, it is important to compare sufficiently differentiated models, to balance different and relevant economic mechanisms. Indeed, although the BGG extension to the core model has been much discussed, that allowing for extensive employment fluctuations has received less attention. And yet, we know that unemployment was high by international comparisons in many euro area economies prior to the crisis, and slow to revert back to pre-crisis levels thereafter (European Commission, 2016). It is of interest therefore to assess the forecasting contribution of models which attempted to capture extensive fluctuations over the crisis. To our knowledge, this is the first time these three models have been estimated on a common basis and directly compared with an emphasis on their density forecasting comparisons. Accordingly, we can address the question of whether these extensions did or can improve the density forecasts of growth and inflation and their joint forecasts.

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The paper is organized as follows. Section 2 provides sketches of the three DSGE models. Prior distributions of the parameters are discussed in Section 3, while the full sample set of posteriors, impulse responses and forecast error variance decompositions are covered in Section 4.[1] The RTD of the euro area is the main topic in Section 5, along with how this data is linked backward in time with various updates of the AWM database. Section 6 contains the empirical results on point and density forecasts of real GDP growth and GDP deflator inflation, including backcasts, nowcasts, and one-quarter-ahead up to eight-quarter-ahead forecasts. Section 7 summarizes the main findings, while a detailed exposition of the DSGE models is provided in the Appendix.

2. The Models

In this section, we sketch the three models. The baseline model is that of Smets and Wouters (2007), where the authors study shocks and frictions in US business cycles. To this we add two augmented models: a version with an external finance premium, and one with the modelling of an extensive employment margin. The Appendix describes the frameworks in greater detail including a treatment of their flexible price and wage analogues, and their steady state equations.

2.1. SW Model Equations

The log-linearized aggregate resource constraint of this closed economy model is given by

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{i}_t + z_y \hat{z}_t + \varepsilon_t^g, \tag{1}$$

where $\hat{y}_t$ is (detrended) real GDP. It is absorbed by real private consumption ($\hat{c}_t$), real private investments ($\hat{i}_t$), the capital utilization rate ($\hat{z}_t$), and exogenous spending (ε_t^g). The parameter c_y is the steady-state consumption-output ratio and i_y is the steady-state investment-output ratio, where

$$c_y = 1 - i_y - g_y,$$

and g_y is the steady-state exogenous spending-output ratio. The steady-state investment-output ratio is determined by

$$i_y = (\gamma + \delta - 1)\, k_y,$$

where k_y is the steady-state capital-output ratio, γ is the steady-state growth rate, and δ is the depreciation rate of capital. Finally,

$$z_y = r^k k_y,$$

where r^k is the steady-state rental rate of capital. The steady-state parameters are shown in Section A.5, but it is noteworthy already at this stage that $z_y = \alpha$, the share of capital in production.

The dynamics of consumption follows from the consumption Euler equation and is equal to

$$\hat{c}_t = c_1 \hat{c}_{t-1} + (1 - c_1)\, E_t \hat{c}_{t+1} + c_2 \left(\hat{l}_t - E_t \hat{l}_{t+1}\right) - c_3 \big(\hat{r}_t - E_t \hat{\pi}_{t+1}\big) + \varepsilon_t^b, \tag{2}$$

[1]The full sample is given by update 14 of the AWM database, covering the period 1980Q1–2013Q4.

where $\hat{l}_t$ is hours worked, $\hat{r}_t$ is the policy controlled nominal interest rate, and ε_t^b is proportional to the exogenous risk premium, i.e., a wedge between the interest rate controlled by the central bank and the return on assets held by households. It should be noted that in contrast to Smets and Wouters (2007), but identical to Smets and Wouters (2005) and Lindé et al. (2016), we have moved the risk premium variable outside the expression for the ex ante real interest rate. This means that $\varepsilon_t^b = -c_3 \epsilon_t^b$, where ϵ_t^b is the risk premium variable in Smets and Wouters (2007). Building on the work by Krishnamurthy and Vissing-Jorgensen (2012), Fisher (2015) shows that this shock can be given a structural interpretation, namely, as a shock to the demand for safe and liquid assets or, alternatively, as a liquidity preference shock. The parameters of the consumption Euler equation are:

$$c_1 = \frac{\lambda/\gamma}{1+(\lambda/\gamma)}, \quad c_2 = \frac{(\sigma_c - 1)(w^h l/c)}{\sigma_c (1+(\lambda/\gamma))}, \quad c_3 = \frac{1-(\lambda/\gamma)}{\sigma_c (1+(\lambda/\gamma))},$$

where λ measures external habit formation, σ_c is the inverse of the elasticity of intertemporal substitution for constant labor, while $w^h l/c$ is the steady-state hourly real wage bill to consumption ratio. If $\sigma_c = 1$ (log-utility) and $\lambda = 0$ (no external habit) then the above equation reduces to the familiar purely forward looking consumption Euler equation.

The log-linearized investment Euler equation is given by

$$\hat{i}_t = i_1 \hat{i}_{t-1} + (1 - i_1) E_t \hat{i}_{t+1} + i_2 \hat{q}_t + \varepsilon_t^i, \tag{3}$$

where $\hat{q}_t$ is the real value of the existing capital stock, while ε_t^i is an exogenous investment-specific technology variable. The parameters of (3) are given by

$$i_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}, \quad i_2 = \frac{1}{(1+\beta\gamma^{1-\sigma_c})\gamma^2 \varphi},$$

where β is the discount factor used by households, and φ is the steady-state elasticity of the capital adjustment cost function.

The dynamic equation for the value of the capital stock is

$$\hat{q}_t = q_1 E_t \hat{q}_{t+1} + (1 - q_1) E_t \hat{r}_{t+1}^k - \left(\hat{r}_t - E_t \hat{\pi}_{t+1}\right) + c_3^{-1} \varepsilon_t^b, \tag{4}$$

where $\hat{r}_t^k$ is the rental rate of capital. The parameter q_1 is here given by

$$q_1 = \beta\gamma^{-\sigma_c}(1-\delta) = \frac{1-\delta}{r^k + 1 - \delta}.$$

Turning to the supply-side of the economy, the log-linearized aggregate production function can be expressed as

$$\hat{y}_t = \phi_p \left[\alpha \hat{k}_t^s + (1-\alpha)\hat{l}_t + \varepsilon_t^a\right], \tag{5}$$

where $\hat{k}_t^s$ is capital services used in production, and ε_t^a an exogenous total factor productivity variable. As mentioned above, the parameter α reflects the share of capital in production, while ϕ_p is equal to one plus the steady-state share of fixed costs in production.

The capital services variable is used to reflect that newly installed capital only becomes effective with a one period lag. This means that

$$\hat{k}_t^s = \hat{k}_{t-1} + \hat{z}_t, \tag{6}$$

where $\hat{k}_t$ is the installed capital. The degree of capital utilization is determined from cost minimization of the households that provide capital services and is therefore a positive function of the rental rate of capital. Specifically,

$$\hat{z}_t = z_1 \hat{r}_t^k, \tag{7}$$

where $z_1 = \dfrac{1-\psi}{\psi}$ and ψ is a positive function of the elasticity of the capital adjustment cost function and normalized to lie between 0 and 1. The larger ψ is the costlier it is to change the utilization of capital. The log-linearized equation that specifies the development of installed capital is

$$\hat{k}_t = k_1 \hat{k}_{t-1} + (1-k_1)\,\hat{i}_t + k_2 \varepsilon_t^i. \tag{8}$$

where $k_1 = \dfrac{1-\delta}{\gamma}$ and $k_2 = (\gamma + \delta - 1)\left(1 + \beta\gamma^{1-\sigma_c}\right)\gamma\varphi$.

From the monopolistically competitive goods market, the price markup ($\hat{\mu}_t^p$) is equal to minus the real marginal cost ($\hat{\mu}_t^c$) under cost minimization by firms. That is,

$$\hat{\mu}_t^p = \alpha\left(\hat{k}_t^s - \hat{l}_t\right) - \hat{w}_t + \varepsilon_t^a, \tag{9}$$

where the real wage is given by $\hat{w}_t$. Similarly, the real marginal cost is

$$\hat{\mu}_t^c = \alpha \hat{r}_t^k + (1-\alpha)\,\hat{w}_t - \varepsilon_t^a, \tag{10}$$

where (10) is obtained by substituting for the optimally determined capital-labor ratio in equation (12).

Due to price stickiness, and partial indexation to lagged inflation of those prices that cannot be re-optimized, prices adjust only sluggishly to their desired markups. Profit maximization by price-setting firms yields the log-linearized price Phillips curve,

$$\begin{aligned} \hat{\pi}_t &= \pi_1 \hat{\pi}_{t-1} + \pi_2 E_t \hat{\pi}_{t+1} - \pi_3 \hat{\mu}_t^p + \varepsilon_t^p \\ &= \pi_1 \hat{\pi}_{t-1} + \pi_2 E_t \hat{\pi}_{t+1} + \pi_3 \hat{\mu}_t^c + \varepsilon_t^p, \end{aligned} \tag{11}$$

where ε_t^p is an exogenous price markup process. The parameters of the Phillips curve are given by

$$\pi_1 = \frac{\imath_p}{1+\beta\gamma^{1-\sigma_c}\imath_p}, \quad \pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\imath_p}, \quad \pi_3 = \frac{(1-\xi_p)\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)}{(1+\beta\gamma^{1-\sigma_c}\imath_p)\,\xi_p\left((\phi_p - 1)\varepsilon_p + 1\right)}.$$

The degree of indexation to past inflation is determined by the parameter $\imath_p$, ξ_p measures the degree of price stickiness such that $1-\xi_p$ is the probability that a firm can re-optimize its price, and ε_p is the curvature of the Kimball (1995) goods market aggregator.

Cost minimization of firms also implies that the rental rate of capital is related to the capital-labor ratio and the real wage according to.

$$\hat{r}_t^k = -\left(\hat{k}_t^s - \hat{l}_t\right) + \hat{w}_t. \tag{12}$$

In the monopolistically competitive labor market the wage markup is equal to the difference between the real wage and the marginal rate of substitution between labor and consumption

$$\hat{\mu}_t^w = \hat{w}_t - \left(\sigma_l \hat{l}_t + \frac{1}{1-(\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right]\right), \tag{13}$$

where σ_l is the elasticity of the labor input with respect to real wages.

Due to wage stickiness and partial wage indexation, real wages respond gradually to the desired wage markup

$$\hat{w}_t = w_1 \hat{w}_{t-1} + (1-w_1)\left[E_t \hat{w}_{t+1} + E_t \hat{\pi}_{t+1}\right] - w_2 \hat{\pi}_t + w_3 \hat{\pi}_{t-1} - w_4 \hat{\mu}_t^w + \varepsilon_t^w, \tag{14}$$

where ε_t^w is an exogenous wage markup process. The parameters of the wage equation are

$$w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}, \quad w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\imath_w}{1+\beta\gamma^{1-\sigma_c}},$$
$$w_3 = \frac{\imath_w}{1+\beta\gamma^{1-\sigma_c}}, \quad w_4 = \frac{(1-\xi_w)\left(1-\beta\gamma^{1-\sigma_c}\xi_w\right)}{\left(1+\beta\gamma^{1-\sigma_c}\right)\xi_w\left(\left(\phi_w-1\right)\varepsilon_w+1\right)}.$$

The degree of wage indexation to past inflation is given by the parameter $\imath_w$, while ξ_w is the degree of wage stickiness. The steady-state labor market markup is equal to $\phi_w - 1$ and ε_w is the curvature of the Kimball labor market aggregator.

The sticky price and wage part of the model is closed by adding the monetary policy reaction function

$$\hat{r}_t = \rho \hat{r}_{t-1} + (1-\rho)\left[r_\pi \hat{\pi}_t + r_y\left(\hat{y}_t - \hat{y}_t^f\right)\right] + r_{\Delta y}\left[\Delta\hat{y}_t - \Delta\hat{y}_t^f\right] + \varepsilon_t^r, \tag{15}$$

where $\hat{y}_t^f$ is potential output measured as the level of output that would prevail under flexible prices and wages in the absence of the two exogenous markup processes, whereas ε_t^r is an exogenous monetary policy shock process.

As productivity is written in terms of hours worked, we introduce an auxiliary equation with Calvo-rigidity to link from observed total employment $(\widehat{e}_t)$ to unobserved hours worked:

$$\hat{e}_t - \hat{e}_{t-1} = \beta\left(E_t \hat{e}_{t+1} - \hat{e}_t\right) + \frac{\left(1-\beta\xi_e\right)\left(1-\xi_e\right)}{\xi_e}\left(\hat{l}_t - \hat{e}_t\right), \tag{16}$$

where $1-\xi_e$ is the fraction of firms that are able to adjust employment to its desired total labor input; see Adolfson, Laséen, Lindé, and Villani (2005, 2007a).

2.1.1. The Exogenous Variables

There are seven exogenous processes in the Smets and Wouters (2007) model. These are generally modelled as AR(1) process with the exception of the exogenous spending process (where the process depends on both the exogenous spending shock η_t^g and the total factor productivity shock η_t^a) and the exogenous price and wage markup processes, which are treated as ARMA(1,1) processes. This means that

$$\begin{aligned}
\varepsilon_t^a &= \rho_a \varepsilon_{t-1}^a + \sigma_a \eta_t^a, \\
\varepsilon_t^b &= \rho_b \varepsilon_{t-1}^b + \sigma_b \eta_t^b, \\
\varepsilon_t^g &= \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g + \rho_{ga} \sigma_a \eta_t^a, \\
\varepsilon_t^i &= \rho_i \varepsilon_{t-1}^i + \sigma_i \eta_t^i, \\
\varepsilon_t^p &= \rho_p \varepsilon_{t-1}^p + \sigma_p \eta_t^p - \mu_p \sigma_p \eta_{t-1}^p, \\
\varepsilon_t^r &= \rho_r \varepsilon_{t-1}^r + \sigma_r \eta_t^r, \\
\varepsilon_t^w &= \rho_w \varepsilon_{t-1}^w + \sigma_w \eta_t^w - \mu_w \sigma_w \eta_{t-1}^w.
\end{aligned} \tag{17}$$

The shocks η_t^j, $j = \{a, b, g, i, p, r, w\}$, are $N(0, 1)$, where η_t^b is a preference shock (proportional to a risk premium shock), η_t^i is an investment-specific technology shock, η_t^p is a price markup shock, η_t^r is a monetary policy or interest rate shock, and η_t^w is a wage markup shock.

2.2. The SWFF Model

Lombardo and McAdam (2012) and Del Negro and Schorfheide (2013) introduce financial frictions into variants of the SW model based on the financial accelerator approach of Bernanke et al. (1999); see also Christiano et al. (2003, 2008) and De Graeve (2008). This amounts to replacing the value of the capital stock equation in (4) with

$$E_t \hat{r}_{t+1}^e - \hat{r}_t = \zeta_{sp,b} \big(\hat{q}_t + \hat{k}_t - \hat{n}_t \big) - c_3^{-1} \varepsilon_t^b + \varepsilon_t^e, \tag{18}$$

and

$$\hat{r}_t^e - \hat{\pi}_t = \big(1 - q_1\big) \hat{r}_t^k + q_1 \hat{q}_t - \hat{q}_{t-1}, \tag{19}$$

where $\hat{r}_t^e$ is the gross return on capital for entrepreneurs, $\hat{n}_t$ is entrepreneurial equity (net worth), and ε_t^e captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs, a spread shock. The parameter q_1 is here given by

$$q_1 = \frac{1 - \delta}{r^k + 1 - \delta},$$

where r^k is generally *not* equal to $\beta^{-1} \gamma^{\sigma_c} + \delta - 1$ since the spread between gross returns on capital for entrepreneurs and the nominal interest rate need not equal zero. The spread shock is assumed to follow the AR(1) process

$$\varepsilon_t^e = \rho_e \varepsilon_{t-1}^e + \sigma_e \eta_t^e. \tag{20}$$

The parameters $\zeta_{sp,b}$ is the steady-state elasticity of the spread with respect to leverage. It may be noted that if $\zeta_{sp,b} = \sigma_e = 0$, then the financial frictions are shut down and equations (18) and (19) yield the original value of the capital stock equation (4).

The log-linearized net worth of entrepreneurs equation is given by

$$\hat{n}_t = \zeta_{n,e}\big(\hat{r}_t^e - \hat{\pi}_t\big) - \zeta_{n,r}\big(\hat{r}_{t-1} - \hat{\pi}_t\big) + \zeta_{n,q}\big(\hat{q}_{t-1} + \hat{k}_{t-1}\big) + \zeta_{n,n}\hat{n}_{t-1} - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\varepsilon_{t-1}^e, \tag{21}$$

where $\zeta_{n,e}$, $\zeta_{n,r}$, $\zeta_{n,q}$, $\zeta_{n,n}$, and ζ_{n,σ_ω} are the steady-state elasticities of net worth with respect to the return on capital for entrepreneurs, the interest rate, the cost of capital, lagged net worth, and the volatility of the spread shock. Furthermore, ζ_{sp,σ_ω} is the steady-state elasticity of the spread with respect to the spread shock. Expressions for these elasticities are given in Appendix Section A.10.

2.3. The SWU Model

The Galí, Smets, and Wouters (2012, GSW) model is an extension of the standard SW model which explicitly provides a mechanism for explaining unemployment. This is accomplished by modelling the labor supply decisions on the extensive margin (whether to work or not) rather than on the intensive margin (how many hours to work). As a consequence, the unemployment rate is added as an observable variable, while labor supply shocks are admitted. This allows the authors to overcome the lack of identification of wage markup and labor supply shocks raised by Chari, Kehoe, and McGrattan (2009) in their critique of new Keynesian models. From a technical perspective the GSW model is also based on the assumption of log-utility, i.e. the parameter σ_c is assumed to be unity, but the equations presented below will instead be written as if this is a free parameter and therefore treat σ_c as in Section A.1 and A.7.

Smets, Warne, and Wouters (2014) present a variant of the GSW model aimed for the euro area and this version is presented below. The log-linearized aggregate resource constraint is given by equation (1). The consumption Euler equation may be expressed as

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \big(\hat{r}_t - E_t\hat{\pi}_{t+1}\big) - c_3^{-1}\varepsilon_t^b, \tag{22}$$

where $\hat{\lambda}_t$ is the log-linearized marginal utility of consumption, given by

$$\hat{\lambda}_t = -\frac{\sigma_c}{1-(\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right] + \frac{(\sigma_c-1)(w^h l/c)}{1-(\lambda/\gamma)}\hat{l}_t, \tag{23}$$

while ε_t^b is the preference shock. Making use of (22) and (23), the consumption Euler equation can be written in the more familiar form in equation (2), and where c_3 is given by the expressions below this equation. However, below we have use for the expression for $\hat{\lambda}_t$.

Concerning the Phillips curve, it is similar to equation (11), but differs in the way that the price markup shock enters the model:

$$\hat{\pi}_t = \pi_1\hat{\pi}_{t-1} + \pi_2 E_t\hat{\pi}_{t+1} - \pi_3\big(\hat{\mu}_t^p - \hat{\mu}_t^{n,p}\big). \tag{24}$$

The expressions for π_i below equation (11) hold and the natural price markup shock $\hat{\mu}_t^{n,p} = 100\epsilon_t^p$. The average price markup and the real marginal cost variables are given by equation (9) and (10), respectively. Relative to equation (11), the price markup shock $\varepsilon_t^p = \pi_3 100\epsilon_t^p$. Smets, Warne, and Wouters (2014) uses the shock ϵ_t^p, while we wish to treat the price markup shock symmetrically in the three models and therefore keep equation (11) such that our price markup shock is given by ε_t^p.

In the GSW model, the wage Phillips curve in equation (14) is replaced with the following expression for real wages

$$\hat{w}_t = w_1 \hat{w}_{t-1} + (1 - w_1) \left[E_t \hat{w}_{t+1} + E_t \hat{\pi}_{t+1} \right] - w_2 \hat{\pi}_t + w_3 \hat{\pi}_{t-1} - w_4 \left[\hat{\mu}_t^w - \hat{\mu}_t^{n,w} \right], \qquad (25)$$

where $\hat{\mu}_t^w$ is the average wage markup and $\hat{\mu}_t^{n,w}$ is the natural wage markup. Notice that the w_i parameters are given by the expressions below equation (14) for $i = 1, 2, 3$, while

$$w_4 = \frac{\left(1 - \xi_w\right)\left(1 - \beta\gamma^{1-\sigma_c}\xi_w\right)}{\left(1 + \beta\gamma^{1-\sigma_c}\right)\xi_w\left(1 + \varepsilon_w \sigma_l\right)}.$$

In addition, GSW and Smets, Warne, and Wouters (2014) let the curvature of the Kimball labor market aggregator be given by $\varepsilon_w = \dfrac{\phi_w}{\phi_w - 1}$.

The average wage markup is defined as the difference between the real wage and the marginal rate of substitution, which is a function of the adjusted smoothed trend in consumption, $\hat{x}_t$, the marginal utility of consumption $\hat{\lambda}_t$, total employment, $\hat{e}_t$, and the labor supply shock. This expression is equal to the elasticity of labor supply times the unemployment rate, i.e.

$$\begin{aligned} \hat{\mu}_t^w &= \sigma_l \hat{u}_t \\ &= \hat{w}_t - \left(\hat{x}_t - \hat{\lambda}_t + \varepsilon_t^s + \sigma_l \hat{e}_t\right), \end{aligned} \qquad (26)$$

where unemployment is defined as labor supply minus total employment:

$$\hat{u}_t = \hat{l}_t^s - \hat{e}_t. \qquad (27)$$

The labor supply shock is assumed to follow an AR(1) process such that

$$\varepsilon_t^s = \rho_s \varepsilon_{t-1}^s + \sigma_s \eta_t^s. \qquad (28)$$

The natural wage markup shock is expressed as $100\epsilon_t^w$ and is, in addition, equal to the elasticity of labor supply times the natural rate of unemployment. Accordingly,

$$\hat{\mu}_t^{n,w} = 100\epsilon_t^w = \sigma_l \hat{u}_t^n. \qquad (29)$$

The natural rate of unemployment, $\hat{u}_t^n$, is defined as the unemployment rate that would prevail in the absence of nominal wage rigidities, and is here proportional to the natural wage markup. Finally, we here let the wage markup shock, ε_t^w, be defined such that:

$$\varepsilon_t^w = w_4 \hat{\mu}_t^{n,w} = w_4 100 \epsilon_t^w.$$

Hence, the wage markup shock ε_t^w enters equation (25) suitably re-scaled and, apart from the definition of the w_4 parameter, the wage Phillips curve is identical to equation (14). This has the advantage of allowing us to treat the wage markup shock as symmetrically as possible in the three models.

The adjusted smoothed trend in consumption is given by

$$\hat{x}_t = \hat{\kappa}_t - \frac{1}{1-(\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right], \tag{30}$$

where the second term on the right hand side is the adjustment, while the smoothed trend in consumption is given by

$$\hat{\kappa}_t = \big(1-\upsilon\big)\hat{\kappa}_{t-1} + \frac{\upsilon}{1-(\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right]. \tag{31}$$

Making use of equation (23), we find that $\hat{x}_t - \hat{\lambda}_t = \hat{\kappa}_t$ when $\sigma_c = 1$, thereby simplifying the expression of the average markup in (26). Provided that $\sigma_c = 1$, the parameter υ measures the weight on the marginal utility of consumption of the smooth trend in consumption. Notice that if $\sigma_c = \upsilon = 1$ and $\varepsilon_t^s = 0$, then the average wage markup in (26) is very similar to the wage markup in equation (13) of the SW model, with the only difference being that $\hat{e}_t$ replaces $\hat{l}_t$.

2.4. The Measurement Equations

The Smets and Wouters (2007) model is consistent with a balanced steady-state growth path (BGP) driven by deterministic labor augmenting technological progress. The observed variables for the euro area are given by quarterly data of the log of real GDP for the euro area (y_t), the log of real private consumption (c_t), the log of real total investment (i_t), the log of total employment (e_t), the log of quarterly GDP deflator inflation (π_t), the log of real wages (w_t), and the short-term nominal interest rate (r_t) given by the 3-month EURIBOR rate. Except for the nominal interest rate, the natural logarithm of each observable is multiplied by 100 to obtain a comparable scale. The measurement equations are given by

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ \Delta w_t \\ \Delta e_t \\ \pi_t \\ r_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma}+\bar{e} \\ \bar{\gamma}+\bar{e} \\ \bar{\gamma}+\bar{e} \\ \bar{\gamma} \\ \bar{e} \\ \bar{\pi} \\ 4\bar{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{e}_t - \hat{e}_{t-1} \\ \hat{\pi}_t \\ 4\hat{r}_t \end{bmatrix}. \tag{32}$$

Since all observed variables except the short-term nominal interest rate (which is already reported in percent) are multiplied by 100, it follows that the steady-state values on the right hand side are given by

$$\bar{\gamma} = 100\,(\gamma-1)\,, \quad \bar{\pi} = 100\,(\pi-1)\,, \quad \bar{r} = 100\left(\frac{\pi}{\beta\gamma^{-\sigma_c}} - 1\right),$$

where π is steady-state inflation while $\bar{e}$ reflects steady-state labor force growth. The interest rate in the model, $\hat{r}_t$, is measured in quarterly terms in the model and is therefore multiplied by 4 in (32) to restore it to annual terms for the measurement.

Apart from the steady-state exogenous spending-output ratio only six additional parameters are calibrated. These are $\delta = 0.025$, $\phi_w = 1.5$, $\varepsilon_p = \varepsilon_w = 10$, and $\mu_p = \mu_w = 0$. The latter two parameters are estimated by Smets and Wouters (2007) on US data, but we have here opted to treat all exogenous processes in the model symmetrically. The remaining 20 structural and 15 shock process parameters are estimated. When estimating the parameters, we make use of the following transformation of the discount factor

$$\beta = \frac{1}{1 + \left(\bar{\beta}/100\right)}.$$

Following Smets and Wouters (2007), a prior distribution is assumed for the parameter $\bar{\beta}$, while β is determined from the above equation.

2.4.1. Measurement Equations: SWFF

The set of measurement equations is augmented in Del Negro and Schorfheide (2013) by

$$s_t = 4\bar{s} + 4E_t\left[\hat{r}^e_{t+1} - \hat{r}_t\right], \tag{33}$$

where $\bar{s}$ is equal to the natural logarithm of the steady-state spread measured in quarterly terms and in percent, $\bar{s} = 100\ln(r^e/r)$, while s_t is a suitable spread variable. The parameter s is linked to the steady-state values of the model variable variables according to

$$\frac{r^e}{r} = (1 + s/100)^{1/4}, \quad \frac{r}{\pi} = \beta^{-1}\gamma^{\sigma_c}, \quad \frac{r^e}{\pi} = r^k + 1 - \delta.$$

If there are no financial frictions, then $r^e = r$, with the consequence that the steady-state real interest rate is equal to the steady-state real rental rate on capital plus one minus the depreciation rate of capital, i.e., the Smets and Wouters steady-state value; see Section A.5.

Del Negro and Schorfheide (2013) estimate s, $\zeta_{sp,b}$, ρ_e, and σ_e while the parameters $\bar{F}$ and κ^e are calibrated. The latter two parameters will appear in the next Section on the steady-state, but it is useful to know that they represent the steady-state default probability and survival rate of entrepreneurs, respectively, with $\bar{F}$ determined such that in annual terms the default probability is 0.03 (0.0075 in quarterly terms) and $\kappa^e = 0.99$. These values are also used by Del Negro, Giannoni, and Schorfheide (2015). Finally, the financial frictions extension also involves the following calibrated parameters: $\delta = 0.025$, $\phi_w = 1.5$, $\varepsilon_p = \varepsilon_w = 10$, and $\mu_p = \mu_w = 0$.

2.4.2. Measurement Equations: SWU

The steady-state values of the capital-output ratio, etc., are determined as in Section A.5 for the SW model. The model is consistent with a BGP, driven by deterministic labor augmenting trend growth, and the vector of observed variables for the euro area is augmented with an equation for unemployment, denoted by u_t. Specifically,

$$u_t = \bar{u} + \hat{u}_t. \tag{34}$$

The steady-state parameter $\bar{u}$ is given by

$$\bar{u} = 100 \left(\frac{\phi_w - 1}{\sigma_l} \right), \tag{35}$$

where $(\phi_w - 1)$ is the steady-state labor market markup and σ_l is the elasticity of labor supply with respect to the real wage. Apart from the parameter σ_c, four additional structural parameters are calibrated. These are $g_y = 0.18$, $\delta = 0.025$, and $\varepsilon_p = 10$ as in the SW model. Unlike Galí et al. (2012) and Smets et al. (2014) we estimate the persistence parameter of the labor supply shock, ρ_s, and calibrate $\phi_w = 1.5$. The latter parameter can also be estimated and yields posterior mean and mode estimates very close to 1.5 when using the same prior as in Galí et al. (2012) and Smets et al. (2014). We have opted to calibrate it in this study in order to treat it in the same way across the three models.

3. Prior Distributions

The details on the prior distributions of the structural parameters of the three models are listed in Table 1. For the Smets and Wouters (SW) model and the extension with financial frictions (SWFF) the prior parameters have typically been selected as in Del Negro and Schorfheide (2013), where US instead of euro area data are used. In the case of ξ_e we use the same prior as in Smets and Wouters (2003) and in Smets et al. (2014). Before we go into further details, it should be borne in mind that we use exactly the same priors for the models for all data vintages. Moreover, the priors have been checked with the real-time data vintages to ensure that the posterior draws are well behaved.

Turning first to the structural parameters, the priors are typically the same across the three models. One difference is the prior mean and standard deviation of ξ_p (the degree of price stickiness) for the SWFF model, which has a higher mean and a lower standard deviation than in the other two models. The prior standard deviation of φ (the steady-state elasticity of the capital adjustment cost function) is unity in Galí et al. (2012) and Smets et al. (2014) while it is 1.5 in Del Negro and Schorfheide (2013). We have here selected the somewhat more diffuse prior for the SW and SWU models, while the SWFF has the tighter prior. In the case of the elasticity of labor supply with respect to the real wage, σ_l, we have opted for a more informative prior for the SWFF model, whose prior standard deviation is half the size of the prior in the other two models.[2]

[2]The decision to use different prior distributions for a few of the structural parameters was based on the need to obtain well-behaved (convergent) posterior distributions of the parameters not only for the full sample, discussed in Section 4, but also over the RTD vintages. To achieve this was more difficult for the SWFF model, which explains why we opted to make use of tighter priors for certain parameters for this model. Moreover, convergence was mainly assessed by inspecting the raw posterior draws from single chains and checking that simple statistics, such as recursive posterior mean estimates and cusum plots, were consistent with convergence; see, e.g., Warne (2017) and references therein.

Compared with Del Negro and Schorfheide (2013) and Galí et al. (2012), regarding $\bar{\gamma}$ (the steady-state per capita growth rate) we have opted for a prior with a lower mean and standard deviation for all the models. This is in line with the prior selected by Smets et al. (2014) when the sample includes data after 2008, i.e., after the onset of the financial crisis. Furthermore, and following Galí et al. (2012) and Smets et al. (2014), the σ_c parameter is calibrated to unity (inverse elasticity of intertemporal substitution) for the SWU model, while it has a prior mean of unity and a standard deviation of 0.25 for the SW model, and higher mean and lower standard deviation for the SWFF models. The priors for $\zeta_{sp,b}$ and s are taken from Del Negro and Schorfheide (2013).

The parameters of the shock processes are displayed in Table 2. The autoregressive parameters all have the same prior across models and shocks, except for the spread shock whose prior has a higher mean and a lower standard deviation than for the priors of the other shock processes; see also Del Negro and Schorfheide (2013). Following this article, we have also opted to use the beta prior for the shock-correlation parameter ρ_{ga} in the three models.[3] Regarding the standard deviations we have followed Del Negro and Schorfheide (2013) and have an inverse gamma prior for the SW and SWFF models, and a uniform prior for the SWU model, as in Galí et al. (2012) and Smets et al. (2014). The prior of the standard deviation of the spread shock is, like the autoregressive parameter for this shock process, taken from Del Negro and Schorfheide (2013). Finally, we have opted to calibrate the moving average parameters of the price (μ_p) and wage (μ_w) markup shocks to zero so that all shock processes have the same representation.

4. Full Sample Posterior Parameter Distributions for the Models

The euro area data for the full sample estimation of the three models have been obtained from the AWM database; see Fagan et al. (2005). Since year 2000, the database is (with a few exceptions) updated annually during the third quarter and we have employed Update 14, released in September 2014. Although all the variables of the SW and SWU models are available from 1970Q1, we use the sample 1980Q1–2013Q4, where the observations prior to 1985Q1 are treated as a training sample for the underlying Kalman filter. In this study we take advantage of the Chandrasekhar recursions, implemented as in Herbst (2015); see also Warne (2017). Compared with the standard Kalman filter, it is our experience that these recursions speed up the calculation of the log-likelihood for the three models by roughly 50 percent. However, once we move to the real-time data for the density forecasts the Chandrasekhar recursions cannot be used since these datasets have missing observations. We then resort to a Kalman filter which is consistent with this property.

The observations of the data that are used for the full sample estimation are shown in Figure 1. The observed variables for the euro area are given by quarterly data of the log of real GDP for the euro area (y_t), the log of real private consumption (c_t), the log of real total investment (i_t),

[3] Well-informed readers may recall that ρ_{ga} has a normal prior in Galí et al. (2012) and Smets et al. (2014), with mean 0.5 and standard deviation 0.25.

the log of total employment (e_t), the log of quarterly GDP deflator inflation (π_t), the log of real wages (w_t), and the short-term nominal interest rate (r_t) given by the 3-month EURIBOR rate.[4]

The observed variable for the spread, s_t, is given by the total lending rate minus a short-term nominal interest rate. Following Lombardo and McAdam (2012), the latter is equal to the 3-month EURIBOR rate from 1999Q1 onwards. Prior to EMU, synthetic values of this variable has been calculated as GDP-weighted averages of the available country data.[5] The growth rates of this synthetic data were then used to create the backtracked history for a given official starting point. The historical data on the total lending rate from 1980Q1–2002Q4 is identical to the data constructed and used by Darracq Paries, Kok Sørensen, and Rodriguez-Palenzuela (2011), while the data from 2003Q1 onwards is also available from the Statistical Data Warehouse (SDW) at the European Central Bank.[6]

4.1. Marginal Posterior Distributions

The posterior draws have been obtained using the random-walk Metropolis (RWM) sampler with a normal proposal density for the three models; see An and Schorfheide (2007) and Warne (2017) for details. Based on 750,000 draws, where the first 250,000 are used as a burn-in sample, estimates of the mean and mode location parameters as well as 5 and 95 percent quantiles from the posterior distributions of the structural parameters are shown in Table 3 for the euro area sample. Similarly, Table 4 provides the location and quantile estimates of the parameters of the shock processes. In addition, kernel density estimates of the marginal posteriors of all the parameters are plotted in Figures 2–4. In these graphs, the marginal mode is obtained from the marginal posterior, while the joint mode is computed from the joint posterior distribution of all estimated parameters.

Concerning the structural parameters it is interesting to note that the steady-state elasticity of the capital adjustment cost function, φ, is the largest for the SW model with posterior mean and mode around 4.8, while the smallest estimates are provided under the SWFF model, where they are roughly half in magnitude. Since ψ is a positive function of this adjustment cost function elasticity, it is interesting that the estimated values across models are in line with this property when comparing the SW and SWU models, but not for the SWFF model where the estimates of φ and ψ are the lowest and highest, respectively, among the three models. The larger ψ is, the costlier it is to change the utilization of capital. In addition, the investment-specific technology shock is more persistent (ρ_i) in the SWFF model than in the other two models.

[4]Except for the nominal interest rate, the natural logarithm of each observable is multiplied by 100 to obtain a comparable scale.

[5]In cases of full data availability, this means values for Germany, France, Italy, Spain and the Netherlands.

[6]The entry point for the SDW is located at www.ecb.europa.eu/stats/ecb_statistics/sdw/html/index.en.html. Specifically, the total lending rate from 2003Q1 is computed from the outstanding amounts weighted total lending rate for non-financial corporations (SDW code: MIR.M.U2.B.A2I.AM.R.A.2240.EUR.N) and households for house purchases (MIR.M.U2.B.A2C.AM.R.A.2250.EUR.N). The outstanding amounts for the lending rates are given by BSI.Q.U2.N.A.A20.A.1.U2.2240.Z01.E for NFCs and BSI.Q.U2.N.A.A20.A.1.U2.2250.Z01.E for households.

The mean and the mode of the inverse elasticity of intertemporal substitution for constant labor, σ_c, is moderately larger than unity in the SW model, and well above unity for the SWFF model. For the former model, the posterior distribution of this parameter covers values below unity with a fairly high probability, while the distribution for the latter model suggests has the probability of this parameter being below or equal to unity is zero.[7]

Turning to the parameters representing external habit formation, λ, it is noteworthy that it is roughly equal across the SW and SWU models with a location estimate around 0.6, while estimated habit is only half in the SWFF model.

Another interesting difference between these models is the elasticity of labor supply with respect to the real wage, σ_l. Business-cycle models tend to give a higher value to this elasticity relative to micro studies.[8] Moreover, studies which account for the extensive labor-participation margin yield yet higher estimates of this elasticity (Peterman, 2016; Chetty et al., 2011). Indeed, while the posterior mean of σ_l is somewhat larger than unity for the SW model, well below unity for the SWFF model, it is above five for the SWU model (which precisely admits the extensive margin). From equation (35) it can be seen that the population mean of unemployment in the SWU model is directly determined from and negatively related to σ_l.[9] The posterior mean estimate of 5.2 implies a mean unemployment rate of approximately 9.67 percent, which is quite close to the sample mean from the data of 9.45 percent.

Concerning the price markup related parameters, it is interesting to note that for the SWFF model prices appear to be the least flexible and in the SW model they are the most flexible. Specifically, the estimated probability that firms can re-optimize the price $(1 - \xi_p)$ is about 15 percent and the degree of indexation to past inflation $(\imath_p)$ is also around 15 percent for the SWFF model, while the same parameters are estimated to be roughly 25 percent and 15 percent, respectively, for the SW model, and about 25 and 20 percent for the SWU model. Turning to the price markup process parameters in Table 4 we find that price markup shocks are marginally less persistent (ρ_p) in the SWFF model than in the SW model, and the least persistent in the SWU model.

For wages, the picture is more complex as wage indexation to past inflation $(\imath_w)$ is estimated to be roughly equal in the SWFF and SWU models and that the probability that firms to re-optimize the wage $(1 - \xi_w)$ is slightly lower in the former case. For the underlying wage markup process, the estimated persistence is substantially larger in the SWFF than in the SWU model. This may very well be a consequence of adding labor supply shocks to the latter model, where such shocks are estimated to be highly persistent and more volatile than the other shocks. For the SW model, wage indexation is estimated to be higher and the probability to re-optimize to

[7]This is a result of the very tight prior around 1.5 on this parameter for the SWFF model.

[8]For example Chetty, Guren, Manoli, and Weber (2011) cites a typical macro-model average value of 2.84 for this elasticity as against a typical micro value of 0.82.

[9]It should be recalled that the steady-state wage markup ϕ_w is calibrated. However, when allowing this parameter to be estimated, its posterior mean and mode are close to 1.5, while the mean of σ_l is marginally higher than when ϕ_w is calibrated, while the mode is somewhat lower.

be lower than in the SWFF and SWU models. At the same time, the persistence of the wage markup shocks is almost as high as in the SWFF model.

Although the estimated interest rate smoothing parameter (ρ) is quite similar across models, the other parameters in the monetary policy rule are quite different. The weight on inflation (r_π) is the largest in the SW model and the lowest in the SWFF model. The weight on the output gap level (r_y) is the highest in the SWU model and the lowest in the SWFF model, while the weight on output gap growth ($r_{\Delta y}$) shows a reverse pattern. The degree of persistence of the monetary policy shocks (ρ_r) is highest for the SWFF model and the lowest for the SWU model. The estimate in the SW model is close to that of the SWFF model, but it should be kept in mind that the monetary policy shock process is among the less persistent among the shock processes in these estimated DSGE models.

The seven observables of the SW are shared across all models, and their population means are mainly determined by the four parameters with a "bar".[10] In particular, the location estimates of $\bar{e}$ and $\bar{\gamma}$ differ across the models with the effect that, for example, the quarterly growth rate of real GDP is estimated to be very different. If we add the marginal posterior mean of these parameters, the estimate from the SWFF model is approximately 0.51 percent, while the estimates from the SW and SWU models are roughly 0.37 and 0.32 percent, respectively. On the other hand, the estimated population mean of GDP deflator inflation do not vary a great deal across the models, with an estimate from the SW model at 0.60 percent, 0.53 percent for the SWFF model, and 0.58 percent from the SWU model. Keeping in mind that the estimates of these parameters are likely to vary over the vintages of the real-time data, these full sample estimates nevertheless suggest that the SW and SWU models may be better able to predict low growth, while the SWFF model may be better adapted to high growth. Whether this conjecture holds in real time or not shall be investigated in our forecast comparison exercise.

Based on the results in Table 4, it can also be noted that the estimated volatilities of the seven common shocks are quite similar across models. The preference (risk premium) shock has the lowest volatility (σ_b) in all models, but is about half as volatile in the SWFF model compared with the SW and SWU models. This is probably related to the addition of a spread shock in this model; see equation (18).

The marginal prior and posterior densities of the parameters are plotted in Figures 2–4.[11] Turning our attention first to the estimated densities for the SW model in Figure 2 it can be seen that some of the priors are very similar to the posteriors, such as for the $\bar{\pi}$ and $\bar{\beta}$ parameters. It is tempting to assume that this means that the parameters are poorly identified in the sense that the data are uninformative about them. The $\bar{\pi}$ parameter is the population mean of the inflation rate, measured by the GDP deflator. From the measurement equations in (32) we also

[10]The population mean of the interest rate is also determined by σ_c, the inverse elasticity of intertemporal substitution for constant labor.

[11]The marginal mode in these graphs refers to the mode of the marginal posterior density, while the joint mode refers to the mode of the posterior of all estimated parameters.

find that $\bar{\pi}$ via π affects the population mean of the nominal interest rate, where the latter is also influenced by β, γ, and σ_c. This suggests that $\bar{\pi}$ and $\bar{\beta}$ need not be poorly identified from the data, but simply that the priors were tailored to data with similar means of the inflation rate and the nominal interest rate.

A heuristic approach to the identification problem from a Bayesian perspective is to compare the surface of the log-posterior kernel and the log-likelihood in a region around the mode of the joint log-posterior.[12] Such an investigation can be carried out for each parameter *conditional* on all the other parameters being evaluated at the mode. Furthermore, it is useful to add the value of the prior at the mode to the log-likelihood, with the effect that the corresponding "scaled" log-likelihood and the log-posterior kernel are equal at the mode.

Figure 5 shows such plots for the SW model, where *mode* in the legend refers to the mode of the joint posterior kernel. Compared with an investigation of the estimated information matrix of the log-likelihood, these plots can be used to not only detect poor local identification, represented by a flat log-likelihood around the mode, but they also reveal when the mode of the log-likelihood differs visibly from the mode of the log-posterior. This gives us information about how the prior influences the estimate of the parameter relative to the data, and is a different issue from identification. It can be seen that the log-likelihood around the mode of both $\bar{\pi}$ and $\bar{\beta}$ is indeed flatter than the log-posterior of each one of these parameters, but the modes of these two functions are not very different. In the case of $\bar{\pi}$, the mode value based on the log-likelihood appears to be somewhat smaller than the mode of the log-posterior.

Another parameter in Figure 2 with a marginal prior and posterior density that are visually close is the weight on the output gap in the monetary policy rule, r_y. It can be seen in Figure 5 that the log-likelihood and the log-posterior are also similar. Since the modes are almost equal with similar curvature, these observations suggest that this parameter is *not* poorly identified and that the prior is not in disagreement with the data. Turning to the weight on inflation in the monetary policy rule, r_π, this parameter also appears to be well identified, but the data (log-likelihood) appears to prefer a larger value than the log-posterior. For some of the other parameters, the prior and data also seem to disagree, such as for the indexation parameters of the wage and price Phillips curves, where the data prefers lower values.

Turning to the SWFF model in Figures 3 and 6, the parameters $\zeta_{sp,b}$ and s are of particular interest. The marginal posterior density of $\zeta_{sp,b}$ is close to its prior with a somewhat higher mean, and where the data seems to agree better with higher values than do the prior and the posterior. The parameter determining the mean spread, s, on the other hand displays a posterior with a slightly lower mean than the prior, and where the data consequently appears to prefer an even lower value. For both cases, the prior disagrees slightly with the data concerning the mode

[12] See, e.g., Canova and Sala (2009), Consolo, Favero, and Paccagnini (2009), Iskrev (2010), Komunjer and Ng (2011) and references therein for discussions of identifiability issues in DSGE models and how these can be analysed.

while the parameter appear to be well identified, although the log-likelihood is flatter around its mode than the log-posterior.

The υ parameter in the SWU model in Figures 4 and 7, reflecting the weight on the marginal utility of consumption of the smooth trend in consumption, has a lower marginal posterior than prior mode. The overall downward shift of this parameter when the data is taken into account is also reflected in the log-likelihood increasing with lower υ values, where the log-likelihood appears to be convex around the mode. The selected tightness of this parameter matters for the posterior mode estimate, as well as the choice to set σ_c to unity.[13]

4.2. Forecast-Error-Variance Decompositions

In Table 5 we provide details on the long-run contributions of the structural shocks of the three models to the forecast error variances of the observed variables: real GDP growth, consumption and investment growth; employment and real wage growth; price inflation; the nominal interest rate; and (where applicable) the spread and unemployment rate.[14] All contributions sum to unity. To ease interpretation, we embolden the contributions that account for 80 percent or more of the total variance to a maximum of three such contributions.

Looking across the models, we see that preference, TFP and monetary shocks tend to be key drivers of the observables. Although for the SWFF model, preference shocks tend to be less important at least for the real variables. An interesting difference also emerges in that the interest rate and the spread are almost entirely determined by preference shocks in the SWFF model, but is somewhat more evenly split in the two other models. Monetary policy shocks have a strong effect on output, consumption and employment across models. On the nominal side, the markup shocks have a naturally large impact on real wages and inflation. Regarding the new observables, unemployment is mostly driven by shocks to preferences and wage markups, and the spread by preferences and spread shocks.[15]

Table 6 shows the equivalent decomposition at the short run (1 quarter) horizon and similar patterns emerge. However, across models the price and wage markup shocks tend to be relatively more important for the variance decomposition of the nominal variables. Also, quite naturally, some of the 'own' shocks (i.e., the effect of the monetary policy shock on the variance decomposition of r_t, the labor supply shock on u_t, the spread shock on s_t) play a more prominent role in the short run (accounting for $30 - -40$ percent of the variance in the short run but being less strong in the long run). Some final takeaways from Table 6 are that for SWFF, the investment-specific shock is equally powerful for investment growth in the short and long

[13] If we add σ_c to the vector of estimated parameters for the SWU model, the prior $\sigma_c \sim N(1.5, 0.375)$ gives a posterior mode estimate of υ around 0.02. However, this modelling choice also leads to an unstable posterior sampler, as represented by for instance a trending acceptance rate, and to multiple modes of the resulting marginal posterior densities for several parameters. A tightening of the prior to $\sigma_c \sim N(1, 0.1)$ gives similar results and we therefore opted to follow Galí et al. (2012) and Smets et al. (2014) and set $\sigma_c = 1$ for the SWU model.

[14] The main reason for our interest in the long-run forecast error variance is that it is equal to the variance.

[15] Recall that the natural rate of unemployment in the SWU model is proportional to the wage markup shock; see equations (A.54) and (A.55).

run. The same may be roughly said for preference shocks on the two interest rate observables (around $60--80$ percent).

4.3. Impulse Responses

Figures 8 to 16 depict the dynamic posterior mean responses of the observables across the three models over a 40-quarter horizon to unit increases in the innovations (the η_t's) relating to the structural shocks, along with 90 percent equal-tails credible bands. All impulse responses are reported as percentage deviations from the model's non-stochastic steady state, except for those of the inflation and interest rates which are reported as annualised percentage-point deviations. The spread and labor-supply shocks (respectively, Figures 12 and 16) only apply to the SWU and SWFF models, respectively.

Most of the common shocks have a fairly similar qualitative pattern with differences reflected by the size and persistence of the particular shock processes and the parameterized general equilibrium interactions of the models. In the following we tend to focus on qualitative features; in some cases, the effects of the shocks on model variables, such as on policy rates and inflation, is in absolute terms often quite small.

Our analysis is split into demand shocks (exogenous spending, preference, investment-specific, monetary policy and spread shocks) and supply shocks (TFP, wage and price markup, and labor supply shocks). Demand shocks imply a positive co-movement between output and inflation, while supply shocks generate a counter-cyclical response of inflation. Moreover for all demand shocks (excluding the interest rate shocks), nominal policy rates initially rise, whereas for supply shocks policy rates tend to respond in line with the inflation response, consistent with the Taylor principle.

4.3.1. Demand Shocks

In the case of the expenditure shock, Figure 8, there is an immediate injection into aggregate demand which, despite the crowding out of private consumption, expands employment and, where admissible, reduces unemployment. The resulting expansion in demand prompts a slight tightening of nominal policy rates and, provided the shock is sufficiently large, the expansion of employment reduces wages. An exception is the SWU model, where there is a rise in extensive labor demand, and thus a temporary boost in wage growth. A key qualitative difference is that the expenditure shock also crowds out investment in the SWU model. This reflects the relatively less expansionary effects of the shock in that model, the initially stronger reaction of the policy rate, and that because higher output can now be produced substituting into labor and, thus, less capital.

Next, a positive preference shock affects the discount rate determining households' inter-temporal substitution; see Figure 9. Qualitatively there are limited differences between the models, except that SWFF generates a far more persistent response following the shock. The expansion of real activity and higher inflation increases firms' net worth, reflected in the protracted fall in the spread.

The investment-specific technology shock is depicted in Figure 10. This is a shock to investment adjustment costs which facilitates a short-lived expansion of investment, consumption and employment (and wage growth). This demand stimulus invokes a short-run monetary tightening, then protracted loosening. The spread rises, reflecting the expansion of firms' leverage following the shock which, in turn, moderates investment growth.

The monetary policy shock in Figure 11 leads to a rise in the nominal and real short-term interest rate and depresses activity, employment and prices. The effects are consistent with the stylized facts following a monetary policy shock. The contractionary effects are more acute in the SWFF model given the counter-cyclicality of the spread, reflecting firms' declining net worth; the heightened sensitivity of SWFF to this shock is consistent with comparative importance of monetary policy shocks; recall Table 6.

The final demand shock pertains to the SWFF model: the spread shock (Figure 12). A unit innovation increases the spread by just under 30 basis points on impact, followed by a protracted return to base. The higher funding costs of capital contracts investment directly with consequent effects on employment, prices and output. Policy makers respond to this demand shock by cutting the policy rate. Consumption is initially higher given this reduction in real policy rates, as well as the increase in real wages following the slight tightening in the labor market. The rise in private consumption however is dominated by the fall in investment, leaving overall output lower.

4.3.2. Supply Shocks

The TFP shock shown in Figure 13 leads to higher real output and falling prices, reflecting lower real marginal costs. Given the supply-driven deflation and the negative output gap, policy rates decline. In line with sticky-price New Keynesian models, employment falls producing higher wage growth: i.e., nominal rigidities prevent demand from expanding sufficiently to match the increased supply.[16] In the SWFF model, the spread rises in line with the higher real user costs of capital.

As regards the wage markup shock in Figure 14, by directly raising wages and thus firms' costs, the effect of the shock is generally to lower output and employment. This heightened inflationary pressure produces a policy tightening which further depresses demand, employment and wage growth. SWFF, however has a small initial increase in output following the response of investment. The inflationary reaction of that model to the shock is the mildest of the three, leaving no initial fall in employment, which further supports output. A rise in the price of capital, which ceteris paribus weakens the investment motive, boosts firms' net worth due to the higher returns on capital leading to a slight decline in spreads, further buttressing investment.

[16] For a discussion, see Cantore, León-Ledesma, McAdam, and Willman (2014).

Similarly, for the price markup shock in Figure 15, by directly raising prices, we see higher policy rates which depresses demand, employment and real wage growth. The relatively high real interest rates exhibited in the SWFF reduces the economy's net worth and thus leads to a protracted fall in the spread. The main qualitative difference between the two markup shocks concerns the response of real wages: the wage markup shock by definition temporarily increases real wage growth, while the price markup shock does the opposite.

Finally, the (negative) labor-supply shock in Figure 16, which only appears in the SWU model, reduces output, consumption, investment, and employment. Given the reduction in the economy's potential, a positive output gap opens up and unemployment increases which pushes up prices, prompting a persistent tightening of policy rates.

5. The Euro Area Real-Time Database

Following Croushore and Stark (2001), it is standard to utilize real-time data when comparing and evaluating out-of-sample forecasts of macroeconomic models for the USA; see Croushore (2011) for a literature review. Much less real-time analysis has been undertaken with euro area data, mainly since it has only more recently become more readily accessible; see, however, Coenen, Levin, and Wieland (2005), Coenen and Warne (2014), Conflitti et al. (2015), Pirschel (2016) and Smets et al. (2014).

The RTD of the ECB is described in Giannone et al. (2012) and data from the various vintages can be downloaded from the SDW. The RTD covers vintages starting in January 2001 and has been available on a monthly basis, covering a large number of monthly, quarterly, and annual data for the euro area, until early 2015 when the vintage frequency changed from three to two per quarter. The original monthly frequency of the RTD largely followed the publication of the Monthly Bulletin of the ECB since 2001 and was therefore frozen at the beginning of each month. The latter ECB publication was replaced in 2015 by the Economic Bulletin, published in the second and third month of each quarter, and the vintage frequency of the RTD has changed accordingly. Specifically, the two vintages per quarter since 2015 are dated in the middle of the first month and in the beginning of the third month. The latter vintage is therefore timed similarly to the third month vintages prior to 2015.

In this study we use the last vintage of each quarter and consider the vintages from 2001Q1–2014Q4 for estimation and forecasting. As actuals for the density forecast calculations, we have opted for *annual revisions*, meaning that the assumed actual value of a variable in year Y and quarter Q is taken from this time period in the RTD vintage dated year $Y+1$ and quarter Q, i.e. we also require the vintages 2015Q1–2015Q4 in order to cover actuals up to 2014Q4. The data on real GDP, private consumption, total investment, the GDP deflator, total employment and real wages are all quarterly. For the last vintage per quarter, the first four variables are typically published with one lag while the two labor market variables lag with two quarters, leading to an unbalanced end point of the data, also known as a *ragged edge*; see, e.g. Wallis (1986).

The unemployment and nominal interest rate series (three-month EURIBOR) are available on a monthly frequency, while the lending rate is neither included in the RTD vintages nor in the AWM updates; our treatment of this variable is discussed in Section 5.2. Since the last vintage per quarter is frozen early during the third month, it covers interest rate data up to the second month, while the unemployment rate lags one month. For our quarterly series of these variables we take the monthly averages. This means that for the last quarter of each vintage we have two monthly observations of the interest rate (first and second month of the quarter) and one of the unemployment rate (first month of the quarter). More details on this issue and the ragged edge property of each vintage are given in Section 5.3.

5.1. Linking the RTD Vintages to the AWM Updates

The RTD vintages typically only cover data starting in the mid-1990s and to extend the data back in time we follow Smets et al. (2014) and make use of the updates from the AWM database; see also Section 4. The data on all observables except for the spread are constructed as sketched in Table 7, and the vintages are portrayed for these variables in Figure 17.

The AWM updates include data on the eight observables in the SW and SWU models from 1970Q1. As in Smets et al. (2014) we only consider data from 1979Q4, such that the growth rates are available from 1980Q1. To link the AWM updates to the RTD vintages we have followed a few simple rules. First, for each variable except the unemployment rate the AWM data on a variable is multiplied by a constant equal to the ratio of the RTD and AWM values for a particular quarter. For each RTD vintage up to 2013Q4 this quarter is 1995Q1, while it is 2000Q1 for the RTD vintages from 2014Q1 onwards. Regarding the unemployment rate, the AWM data are adjusted for updates 2 and 3 only. These updates are employed in connection with RTD vintages 2001Q1 until 2003Q2. For these vintages, the AWM data on unemployment is multiplied by a constant equal to the ratio between its RTD and AWM values in 1995Q1.[17] Second, when prepending the AWM updates to the RTD vintages, the common start dates in the second column of Table 7 are used. That is, for each linked vintage the AWM data is taken up to the quarter prior to the common start date, while RTD data is taken from that date.

The data from the various vintages on the eight variables taken from the RTD and the AWM updates are plotted in Figure 17. It is noteworthy that the medium-term paths of the variables do not change considerably over the 56 vintages, while details on the revisions of the data are displayed in Figure 18. Blue lines represent the maximum values and red lines give the minimum values in Panel A, while the difference between these lines is plotted below in Panel B. Based on

[17]This adjustment avoids having a jump in the unemployment rate for the vintages up to 2003Q2. For the data in 1995Q1, the RTD vintage in 2001Q1 gives 11.27 percent while the AWM update 2 provides 11.38 percent. Similarly, for RTD vintage 2003Q2 the unemployment rate in 1995Q1 is 10.60 percent while the AWM update 3 gives a value of 11.38 percent, a difference close to 0.8 percent. Once we turn to RTD vintage 2003Q3 and later, this discrepancy between the AWM update and RTD vintage values is close to zero. For this reason we do not adjust the unemployment rate in the AWM for updates 4 and later. The interested reader may also consult the notes below Table 7 concerning changes in the definition of unemployment which may explain the discrepancies between the early RTD vintages and AWM updates 2 and 3.

Figure 17 and Panel A in Figure 18 it appears that the nominal interest rate is hardly revised. However, in Panel B of the latter figure, it can be seen that some of the revisions can be quite large. For the differences since 2001Q1, the revisions are primarily explained by the last data point. That is, the first vintage when data is available covers only 2 of the 3 three months for the vintage identifier quarter, while the vintages thereafter cover all three months of the same quarter. Concerning the unemployment rate, it can be seen that the revisions gradually become smaller over the sample and that for the data until the mid-90s the time series is typically revised downward over the AWM updates.[18]

Turning to the six variables in *first differences* of the natural logarithms, Panel B of Figure 18 indicates that the average revisions are generally larger since the mid-90s, reflecting the revisions to the RTD, with the total investment revisions on average being the largest. It is also noteworthy that the revisions to the total employment growth series are more volatile for the earlier vintages compared to the more recent, especially during the 80s; see Figure 17.

5.2. The Lending Rate

The lending rate is not covered by the RTD and the AWM and we have therefore chosen to use the data on this variable discussed in the introduction to Section 4. For each given vintage we take the observation from 1980Q1 until the quarter prior to the vintage date. An important reason for excluding this data point from each pseudo real-time vintage for the SWFF model is that the outstanding amount weights for the total lending rate is only available at a quarterly frequency, while the sectorial lending rates for non-financial corporations and households for house purchases are available at a monthly frequency. In principle, it is possible to take the weights from the last available quarter, but as the sectorial data are averages of the monthly rates and we do not have access to the latter prior to 2003, we refrain from such calculations.[19]

5.3. The Ragged Edge of Real-Time Data

The ragged edge property of real-time data means that it is unbalanced or incomplete at the end of the sample for each vintage. When forecasting, we refer to the vintage date as the nowcast period, and to the quarter prior to it as the backcast period. Table 8 provides details on the vintage dates when there is missing data on the variables covered by the RTD. The table also lists the number of missing observations for each variable for the backcast and the nowcast period, respectively.

Concerning the backcast period, it can be seen from Table 8 that employment and wage data are missing for all vintages, while unemployment and interest rate data always exist. For real GDP, there are three vintages with missing backcasts, all occurring during the first five vintages.

[18] It should be kept in mind that the euro area is a time-varying aggregate with new countries being added to the area over the RTD vintages; see Table 7 for some more information.

[19] It is interesting to note that the sectorial lending rates in the SDW in January 2017 are equal to the sectorial number we have, which were collected in July 2015. Hence, there is some support for the hypothesis that the sectorial lending rates are not subject to substantial revisions over our sample.

In the cases of private consumption and total investment, the total number of missing backcast period observations is nine, most of which occur in the first third of the sample, while the GDP deflator has a total of 16 such missing observations, likewise mainly located in the first third of the forecast sample. Note that the GDP deflator is always missing when the consumption and investment data for the backcast period is not available.

For the nowcast period, we find that data on all variables except for the nominal interest rate and the unemployment rate are missing. For these latter variables, the nominal interest rate is never missing, while the unemployment rate nowcast is missing in five vintages, all but one occurring during the first half of the sample.

6. Density Forecasting with Ragged Edge Real-Time Data

In this section we compare marginalized h-step-ahead forecasts for real GDP growth and inflation using the three DSGE models. These models are re-estimated annually using the Q1 vintage for each year. For example, the 2001Q1 vintage is used to obtain posterior draws of the parameters for all vintages with year 2001.[20] Section 6.1 outlines the methodology for the density forecasts based on log-linearized DSGE models and using Monte Carlo (MC) integration to estimate the predictive likelihood. Point forecasts obtained from the predictive density are discussed in Section 6.2, while density forecasts for the full sample as well as recursive estimates using the MC estimator are discussed in Section 6.3. In Section 6.4, we turn our attention to comparing the MC estimates of the predictive likelihood with those obtained using a normal density approximation based on the mean and covariance matrix of the predictive density. If the resulting normal predictive likelihood approximates the MC estimator based predictive likelihood well, it makes sense to decompose the former into a forecast uncertainty term and a quadratic standardized forecast error term, as suggested by Warne, Coenen, and Christoffel (2017), to analyze the forces behind the ranking of models from the predictive likelihood.

6.1. Estimation of the Predictive Likelihood

Density forecasting with Bayesian methods in the context of linear Gaussian state-space models is discussed by Warne et al. (2017). Although they do not consider real-time data with back/nowcasting, it is straightforward to adjust their approach to deal with such ragged edge data issues.

To this end, let $\mathcal{Y}_T = \{y_1, y_2, \ldots, y_T\}$ be a real-valued time series of an n-dimensional vector of observables, y_t. Given a vintage $\tau \geq t$, let $y_t^{(\tau)}$ denote the "observation" (measurement) in vintage τ of this vector of random variables dated time period t, while $\mathcal{Y}_T^{(\tau)} = \{y_1^{(\tau)}, y_2^{(\tau)}, \ldots, y_T^{(\tau)}\}$. The ragged edge property of, e.g., vintage $\tau = T$ means that some elements of $y_t^{(T)}$ have missing values when $t = T$, and possibly also for, say, $T-1$; see Table 8 where, for example, real wages has missing values for periods $\tau - 1$ and τ for all vintages τ. In addition, we let $y_t^{(a)}$ denote

[20] This has the advantage of speeding up the underlying computations considerably and also mimics well how often such models are re-estimated in practise within a policy institution.

the *actual* observed value of y_t, which is used to assess the density forecasts of y_t. This value is given by $y_t^{(t+4)}$ in our empirical study.

To establish some further notation, let the observable variables y_t be linked to a vector of state variables ξ_t of dimension m through the equation

$$y_t = \mu + H'\xi_t + w_t, \quad t = 1, \ldots, T. \tag{36}$$

The errors, w_t, are assumed to be i.i.d. $N(0, R)$, with R being an $n \times n$ positive semi-definite matrix, while the state variables are determined from a first-order VAR system:

$$\xi_t = F\xi_{t-1} + B\eta_t. \tag{37}$$

The state shocks, η_t, are of dimension q and i.i.d. $N(0, I_q)$ and independent of w_τ for all t and τ, while F is an $m \times m$ matrix, and B is $m \times q$. The parameters of this model, (μ, H, R, F, B), are uniquely determined by the vector of model parameters, θ.

The system in (36) and (37) is a state-space model, where equation (36) gives the measurement or observation equation and (37) the state or transition equation. Provided that the number of measurement errors and state shocks is large enough and an assumption about the initial conditions is added, we can calculate the likelihood function with a suitable Kalman filter.

With E_t being the expectations operator, a log-linearized DSGE model can be written as:

$$A_{-1}\xi_{t-1} + A_0\xi_t + A_1 E_t\xi_{t+1} = D\eta_t. \tag{38}$$

The matrices A_i $(m \times m)$, with $i = -1, 0, 1$, and D $(m \times q)$ are functions of the vector of DSGE model parameters. Provided that a unique and convergent solution of the system (38) exists, we can express the model as the first order VAR system in (37).

To estimate the parameters θ of a log-linearized DSGE model for vintage $\tau = T$ we use the data $\mathcal{Y}_T^{(T)}$. Since this real-time vintage is subject to missing observations, we employ a filter which is consistent with this property; see, e.g., Harvey (1989, Chapter 3.4.7) or Durbin and Koopman (2012, Chapter 4.10).

Suppose we have N draws from the posterior distribution of θ using vintage $\tau = T$. These draws are denoted by $\theta^{(i)} \sim p(\theta|\mathcal{Y}_T^{(T)})$ for $i = 1, 2, \ldots, N$. When back/now/forecasting we make use of the smoothed estimates of the state variables $\xi_{t|T}^{(i)} = E[\xi_t|\mathcal{Y}_T^{(T)}, \theta^{(i)}]$ and the corresponding covariance matrix $P_{t|T}^{(i)}$ for $t \leq T$. For the nowcast this means $t = T$ and the backcast, say, $t = T - 1$. In forecasting mode, the smooth estimates for $t = T$ are used as initial conditions for the state variables.

Furthermore, if the model has measurement errors $(R \neq 0)$ we also need smooth estimates of the measurement errors as well as their covariance matrix when back/nowcasting. Since there exists an equivalent representation to the state-space setup where all measurement errors are moved to the state equations, we henceforth assume without loss of generality that $R = 0$. It may also be noted that all of the models in our empirical study are without measurement errors ($R = 0$).

Suppose that we are interested in the density forecasts of a subset of the observable variables, denoted by $x_t = S'y_t$, where the selection matrix S is $n \times s$ with $s \leq n$, and that we wish to examine each horizon h individually. In the empirical study, we let the matrix S select inflation and real GDP growth jointly or individually, while $h = -1, 0, 1, \ldots, 8$, where $h = 0$ represents the nowcast, a positive number a forecast, while a negative horizon is a backcast.

The nowcast of x_T for a fixed value of $\theta = \theta^{(i)}$ is Gaussian with mean $x^{(i)}_{T|T}$ and covariance matrix $\Sigma^{(i)}_{x,T|T}$, where

$$
\begin{aligned}
x^{(i)}_{T|T} &= S'\big(\mu + H'\xi^{(i)}_{T|T}\big),\\
\Sigma^{(i)}_{x,T|T} &= S'H'P^{(i)}_{T|T}HS.
\end{aligned}
$$

It should be kept in mind that the state-space parameters (μ, H, F, B) depend on $\theta^{(i)}$, but we have suppressed the index i here for notational convenience.

The predictive likelihood of $x^{(a)}_T$—the actual value of the variables of interest—conditional on $\theta^{(i)}$ exists if $\Sigma^{(i)}_{x,T|T}$ has full rank s and is then given by the usual expression for Gaussian densities, while the covariance matrix has full rank when $S'y^{(T)}_T$ contains only missing data. The predictive likelihood of, for example, $x^{(a)}_{T-1}$ conditional on $\theta^{(i)}$ can be computed analogously once the smooth estimates of the state variables and their covariance matrix are replaced by $\xi^{(i)}_{T-1|T}$ and $P^{(i)}_{T-1|T}$, respectively.

The forecast of x_{T+h} conditional on $\theta = \theta^{(i)}$ is also Gaussian, but with mean $x^{(i)}_{T+h|T}$ and covariance matrix $\Sigma^{(i)}_{x,T+h|T}$, where

$$
\begin{aligned}
x^{(i)}_{T+h|T} &= S'\big(\mu + H'\xi^{(i)}_{T+h|T}\big),\\
\Sigma^{(i)}_{x,T+h|T} &= S'H'P^{(i)}_{T+h|T}HS.
\end{aligned}
$$

for $h = 1, 2, \ldots, h^*$. In addition, the forecasts of the state variables and their covariance matrix are given by

$$
\begin{aligned}
\xi^{(i)}_{T+h|T} &= F^h\xi^{(i)}_{T|T},\\
P^{(i)}_{T+h|T} &= F^hP^{(i)}_{T|T}\big(F'\big)^h.
\end{aligned}
$$

The predictive likelihood of $x^{(a)}_{T+h}$ conditional on $\theta^{(i)}$ can directly be evaluated using these recursively computed means and covariances.

The objective of density forecasting with vintage $\tau = T$ is to estimate the predictive likelihood of $x^{(a)}_{T+h}$ for $h = -1, 0, 1, \ldots, h^*$ by integrating out the dependence on the parameters from the predictive conditional likelihood. One approach is to use MC integration, which means that

$$
\hat{p}_{MC}\big(x^{(a)}_{T+h}|\mathcal{Y}^{(T)}_T\big) = \frac{1}{N}\sum_{i=1}^{N} p\big(x^{(a)}_{T+h}|\mathcal{Y}^{(T)}_T, \theta^{(i)}\big). \tag{39}
$$

Under certain regularity conditions (Tierney, 1994), the right hand side of equation (39) converges almost surely to the predictive likelihood $p(x^{(a)}_{T+h}|\mathcal{Y}^{(T)}_T)$; see Warne et al. (2017) and the references therein for further discussions.

To compare the density forecasts of the three DSGE models, we use their log predictive score. For each horizon h and model, the log predictive score is the sum of the log of the predictive likelihood in equation (39) over the different vintages. This well-known scoring rule is optimal in the sense that it uniquely determines the model ranking among all local and proper scoring rules; see Gneiting and Raftery (2007) for a survey on scoring rules. However, there is no guarantee that it will pick the same model as the forecast horizon or the selected subset of variables changes.

6.2. Point Forecasts of Real GDP Growth and Inflation

The point forecast is given by the mean of the predictive density. It is computed by averaging over the mean forecast conditional on the parameters using a sub-sample of the 500,000 post-burn-in posterior parameter draws; see, e.g., Warne et al. (2017, equation 12). Specifically, we use 10,000 of the available 500,000 draws, taken as draw number 1, 51, 101, etc, thereby combining modest computational costs with a lower correlation between the draws and a sufficiently high estimation accuracy. The recursively estimated paths of these point forecasts are shown using *so called* spaghetti-plots for real GDP growth (chart A) and inflation (chart B) in Figure 19 along with the actual values of the variables. Each chart contains three sub-charts corresponding to the three DSGE models, where the forecast paths are red for the SW model, blue for the SWFF model, and green for the SWU model.

Turning first to real GDP growth, it is noteworthy that the SWFF model over-predicts during most of the forecast sample. Moreover, the other two models also tend to over-predict since early 2009 and therefore the beginning of the Great Recession. The mean errors for the backcasts ($h = -1$), nowcasts ($h = 0$), and the forecasts up to eight-quarters-ahead for the full forecast sample 2001Q1–2014Q4 are listed in Table 9 and they confirm the ocular inspection. It is interesting to note that the mean errors for the SWFF model are fairly constant over the forecast horizons with $h \geq 1$ and the largest in absolute terms, while those of the other two models are smaller but vary more. Over the one- and two-quarter-ahead horizons as well as for back- and nowcasts, the SW point forecasts have smaller mean errors than those of the SWU model, while the latter model has smaller mean errors from the three-quarter-ahead horizon. Nevertheless, the real GDP growth mean errors are substantial, also for the shorter horizons, especially as the SW and SWU models seem to produce higher forecasts since the onset of the crisis, while the actual real GDP growth data tends to move in the opposite direction.

The mean errors up to 2008Q3 are shown in Table 10. It is notable that the ordering of the models based on the mean errors is virtually unaffected compared with the full sample and that most mean errors are closer to zero, with the exception of the inflation forecasts errors with $h \leq 1$. Moreover, the mean errors since 2008Q4 are shown in Table 11, where it can be seen that the errors for real GDP growth are considerably more problematic for all models, as already suggested by the spaghetti-plots in Figure 19.

Concerning the inflation point forecasts in Chart B of Figure 19, the SWFF model under-predicts and especially at the shorter horizons with $h \geq 1$. On the other hand, the SW and SWU models have similar mean errors and share the tendency to over-predict once $h \geq 2$. Moreover, the inflation mean forecast errors are much less affected by the Great Recession and are only marginally larger since 2008Q4 than they were up to 2008Q3; see Table 10 and 11. It is also noteworthy from Figure 19 that the SW and SWU models often have upward sloping forecast paths, while the SWFF model tends to provide v-shaped paths with a lower end-point than starting-point.

From Table 9 it can be seen that mean errors are the largest for the SWFF model at the shorter horizons and the smallest at the longer horizons, while those of the SW and SWU models are increasing with the horizon and roughly equal, with the SW model errors being somewhat smaller (larger) than the SWU model errors for $h \geq 2$ ($h \leq 1$).

The spaghetti-plots of the point forecasts are also plotted in Figure 20, but now the paths are compared with the recursive posterior mean estimates of mean real GDP growth ($\bar{\gamma} + \bar{e}$) and inflation ($\bar{\pi}$), respectively. For real GDP growth in Chart A, the SW and SWU models both have downward sloping recursive mean growth estimates with the point estimates being close to but below 0.50 percent per quarter in 2001. Towards the end of the forecast sample estimated mean real GDP growth is around 0.35 and 0.30 percent for the SW and SWU models, respectively. By contrast, the SWFF model has a hump-shaped path with an estimated quarterly mean growth rate of around 0.65 percent just prior to the Great Recession and never below 0.50 percent.

In the case of inflation, the SWFF model has the lowest point estimates of mean inflation and the SW model the highest. Following the fall in the path of actual inflation in 2009, there is a permanent fall in the point estimate paths for the SWFF model, while the effects on the SW and SWU models are moderate and temporary. Furthermore, it can be deduced that the upward-sloping forecast paths of the SW and SWU models can be explained by their mean reversion properties, where the point nowcasts are typically below the estimate of mean inflation.

In view of the downward sloping paths for the recursive posterior mean estimates of real GDP growth, it is curious that the point forecasts seem to jump up to higher rates with the onset of the Great Recession for, in particular, the SW and SWU models.[21] Table 12 lists the point nowcasts and h-step-ahead forecasts of real GDP growth for the SWU model based on the 2008Q4 and 2009Q1 vintages. Not only are the point forecasts for each horizon substantially higher for the 2009Q1 vintage than for the 2008Q4 vintage, but also when forecasts for the same quarter are compared. There can be three possible sources for this upward shift:

[21] Lindé et al. (2016) show that (annual) real GDP growth forecasts for the U.S. overshoot actual growth during the crisis in the fall of 2008 and the slow and pro-longed recovery that followed. This feature is not just present in their benchmark DSGE model, but also in their Bayesian VAR model (Figure 4), which uses the same observables as the DSGE model.

(i) the posterior distribution of the parameters has changed between the two vintages;
(ii) revisions to the data available in both vintages; and
(iii) the new data in the 2009Q1 vintage.

A complete separation of these three sources of change is not possible, especially since changes in the posterior distribution depends on revisions to common data period observations and the new data points, i.e., (i) is an indirect effect on the point forecasts from changes to the latter two direct sources.

With this caveat in mind, the first source can be investigated by simply using the posterior distribution from the 2008Q4 vintage when forecasting with the 2009Q1 vintage. In Table 12 this case is referred to as *Parameters* and the effect on the point forecasts in 2009Q1 from this source of change is in fact weakly positive. In other words, the real GDP growth path is somewhat higher than when the posterior distribution from the 2009Q1 vintage is used. Hence, the upward shift in the projected path of real GDP growth is not due to a shift of the posterior distribution.

The second source can be investigated by using all the available data from the 2008Q4 vintage instead of the corresponding values from the 2009Q1 vintage, while the very latest data points of the latter vintage remain in the information set. The data from the two vintages that applies to the SWU model are shown in Chart A of Figure 21, while the revisions are plotted in Chart B.[22] In Table 12 this case is called *Revisions* and this change to the input in the computation of the point forecasts indeed lowers the path substantially. Apart from the nowcast, which drops to zero, the new path is quite flat with values from 0.4 percent to 0.3 percent growth and therefore much closer to the estimated mean growth rate.

The third source is examined by treating all the new data points in the 2009Q1 vintage as unobserved. From the *New data* case in Table 12 we find that the point forecast path is lower than the original 2009Q1 path, but also somewhat higher than the path for the *Revisions* case. These two sources of change may also be compared with the case when the 2009Q1 vintage data is completely replaced with the 2008Q4 vintage and the latest data points are treated as unobserved, called *Old data* in Table 12. Compared with the New data case it can be seen that the point forecast path is lowered, especially for the nowcast and shorter-term forecasts. Similarly, when comparing the Old data case to the Revisions case, we learn about the impact that the last data points have on the forecast path. The large negative real GDP, private consumption, and total investment growth rate data for the 2008Q4 quarter and taken from the 2009Q1 vintage in the Revisions case are likely to explain the decrease on the nowcast of real GDP growth relative to the Old data case, while the large drop in the short-term nominal

[22]There are not any revisions prior to 1995 for these two vintages as their data prior to 1995 are taken from the same AWM update; see Table 7 for further details. It is also noteworthy that the variables which has been subject to many relatively larger revisions are private consumption growth, total investment growth, and unemployment. In addition, the nominal interest rate is subject to one large revision for 2008Q4. For this quarter, the 2008Q4 vintage data point is computed as the average of the monthly observations for October and November, while the 2009Q1 vintage data point is the average of all three months.

interest rate from 4.2 percent to 2.2 percent in 2009Q1 acts as a likely catalyst to raise the shorter-term (one to three-quarter-ahead) point forecast path of real GDP growth.

To summarize, the evidence in the Table suggests that the source for the upward jump of the real GDP growth point forecast path is the combination of revisions of the historical data and the use of the latest observations of the model variables. At the same time, the effect these data changes have on the posterior distribution of the model parameters indirectly leads to a sobering impact on the projected path, i.e. had the posterior been unaffected then this path would have been even higher.

6.3. Density Forecasts: Empirical Evidence Using the MC Estimator

The log predictive scores for the full forecast sample (2001Q1–2014Q4) are shown in Figure 22 over the various forecast horizons. The results based on the MC estimator are displayed in Chart A, while those calculated with a normal density based on the mean vector and covariance matrix of the predictive density are plotted in Chart B; see Warne et al. (2017, Section 3.2.2) for details. The discussion here will focus on the MC estimator and we shall return to the latter normal approximation in Section 6.4.

Figure 22 includes three charts where those in the top row show the log score for the density forecasts of real GDP growth (left) and GDP deflator inflation (right), while the chart in the bottom row provides the log score of the joint density forecasts. The most striking feature of the charts is that the SWFF model is overall outperformed by the SW and SWU models and, given the findings in Section 6.2, this result is not surprising. The exceptions concern inflation at the eight-quarter-ahead horizon and real GDP growth for the back- and nowcasts. Concerning the other two models, it is interesting to note that the SWU model has a higher log score than the SW model for the inflation forecasts up to six quarters ahead and for real GDP growth from the four-quarters-ahead forecasts. Turning to the joint density forecasts, the SW model has a higher log score for the shorter-term, while the SWU model wins from the two-quarters-ahead. It should be kept in mind that numerically, the differences in log score between the SW and SWU models are quite small; see Table 13 for details. For example, for the four-quarter-ahead real GDP forecasts, the difference is approximately 1 log-unit in favor of the SWU model, while for inflation the difference at the same horizon is about 1.5 log-units, and for the joint forecast roughly 2 log-units.[23] Finally, and omitting the backcast period, since all the models have a larger log score for inflation than for real GDP growth they are better at forecasting the former variable than the latter.

The recursive estimates of the average log score for the real GDP growth, inflation, and joint density forecasts are shown in Figure 23. Concerning real GDP growth in Chart A, it is noteworthy that all models display a drop in average log score with the onset of the crisis in 2008Q4 and 2009Q1 when growth fell to about -1.89 and -2.53 percent per quarter, respectively.

[23] The exponential function value of 1.5 is about 4.5, corresponding to the posterior predictive odds ratio when the models are given equal prior probability.

The ranking of the models for the different forecast horizons is to some extent affected by this two-quarter event, where the nowcast ordering of the SWU model switches from being at the top to the bottom of the three, while the SW and SWU trade places for the h-quarter-ahead forecasts from $h = 4$.

Continuing with the recursive average log scores for inflation in Chart B, the ranking of the models for the h-quarter-ahead forecasts with $h \geq 3$ is stable over the sample. For the two-quarter-ahead forecasts and onwards, there is an increasing impact on the scores with the onset of the crisis for the SW and SWU models, while the paths of SWFF model display little change. As a consequence, the SWFF model jumps from the third and last to the first rank for the eight-quarter-ahead forecasts in early 2010. A possible explanation is that the forecast errors of the SWFF model are smaller than those of the others models from this point in time. We shall return to the question of the impact of the forecast errors on the density forecasts in Section 6.4.

The recursive average log scores for both variables are displayed in Chart C and are overall in line with the finding that the SW and SWU models yield similar density forecasts and are better than the SWFF model. Since the onset of the crisis there is a tendency for the SWU model to forecast better over the medium and longer term, while the SW model tends to dominate these horizons weakly before the crisis. It is also interesting to note that for the nowcasts, the SWU model ranks first before the crisis and trades places with the SW model after the crisis.

Table 14 displays the log predictive scores for the whole sample since the onset of the economic crisis in 2008Q4. In line with the observations made above, the ranking of the models at the end of this smaller sample is not much affected compared with the full forecast sample. However, turning to the recursive average log scores in Figure 24 it is interesting to note that the SWFF model is competitive for the real GDP growth and inflation forecasts individually as well as jointly during and after the Great Recession for the euro area. The finding that a variant of the SWFF model plays an important role around the Great Recession in the US has previously been shown by Del Negro and Schorfheide (2013) and Del Negro et al. (2015). Hence, while the SWFF model does not help to sharpen the density forecasts in normal times, there is evidence also for the euro area that it can improve the density forecasts during times of financial turbulence.

6.4. Evidence Based on the Normal Approximation

Let us now turn to the issue of how well the predictive density can be approximated by a normal density when the purpose is to compute the log predictive score. The estimated log predictive score for the full sample using the normal approximation are depicted in Chart B of Figure 22; see Warne et al. (2017, Section 3.2.2). It is striking how similar the plots in the Chart are to those in Chart A, suggesting that the predictive likelihood is very well approximated by a normal likelihood. Indeed, the numerical differences between the MC estimator and the normal approximation of the log predictive score are listed in Table 15. Overall, the differences are positive and for the individual variables well below unity. Recall from Table 8 that there are three backcasts of real GDP growth and 16 backcasts of inflation, while the number of nowcasts is 56 for both variables. We can therefore deduce that the number of h-quarter-ahead forecasts is equal to $56 - h$ for $h \geq 0$. The differences in average log score for the full forecasting sample are therefore minor.

For completeness, the recursively estimated differences in average log score are plotted in Figure 25. It is notable for real GDP growth and all $h \geq 1$ that there is an upward jump at the beginning of the economic crisis in 2008Q4, indicating that the difference between the MC estimator and the normal approximation is greater when the log predictive likelihood is very low. For example, in the case of the SW model and the 2008Q3 vintage, the MC estimator of the predictive likelihood for $h = 1$ (2008Q4) is equal to -3.983 with a numerical standard error of 0.033 and for $h = 2$ it is -6.788 with a standard error of 0.056. The normal approximation here gives the values -4.021 and -7.019, respectively, indicating that the normal approximation differs more substantially from the MC estimator as the log predictive likelihood becomes small. Recalling that the actual values of real GDP growth are -1.89 and -2.53 percent for these two quarters, we find that the SW model produces mean forecasts of 0.03 and 0.16 percent, respectively, with predictive standard deviations of 0.74 and 0.75.

The previous log predictive likelihood estimates may be compared with the estimates from the 2008Q1 vintage. This vintage represents a less extreme case where the MC estimator for $h = 1$ yields -0.770 (0.004) and for $h = 2$ it yields -0.991 (0.008), while the normal approximation delivers -0.769 and -0.986 for these forecast horizons. This suggests that while the normal approximation is generally very accurate, its performance deteriorates when the actuals lie in the tails of the predictive density.[24]

Turning to inflation in Chart B, it is interesting to note that for the h-quarter-ahead forecasts with $h \geq 4$ the differences between the two estimators appear to be smaller with the onset of the crisis, especially in the case of the SW model, and simultaneously the average log score falls for the MC estimator. The actual value of the quarterly inflation rate in 2008Q4 is 0.56 percent and therefore close to the steady-state value of all three models, while the actual value in 2009Q1 is 0.08 percent. From the SW model and the 2007Q4 vintage, the predictive four-quarter-ahead (2008Q4) mean forecast is 0.535 with an estimated standard deviation of 0.343, while the mean five-quarter-ahead (2009Q1) mean forecast is 0.565 with a standard deviation of 0.350. The MC estimator of the log predictive likelihood is now 0.168 and -0.859, respectively, while the normal approximation results in 0.148 and -0.821. The 2009Q1 period is accordingly the first period in

[24] It is implicitly assumed here that the MC estimator in the current paper is accurate. It is shown by Warne et al. (2017, Online Appendix, Part D) that the numerical standard error is small for the number of parameter draws used for estimation in the current paper (10,000 posterior draws out of a 500,000 available post burn-in draws) and for dimensions of the set of predicted variables that are greater than two, but also that this standard error is close to the across chain variation of the point estimate of the log predictive likelihood. Since the models considered in that study have greater dimensions (number of observables, number of parameters, number of state variables, shocks, and so on) than the models in the current paper, we expect the numerical precision of the MC estimator in the current case to be at least as good as found by Warne et al. (2017).

Chart B where the difference in average log score falls and at the same time the actual inflation value for this quarter is not very unusual.[25]

By construction, the only source of non-normality of the MC estimator is the posterior distribution of the parameters.[26] One indicator for checking if the normal density will approximate the MC estimator well is the share of parameter uncertainty of the total uncertainty when represented by the predictive covariance matrix; see, e.g., Warne et al. (2017, equation 13) for a decomposition of this matrix.[27] In this decomposition parameter uncertainty is represented by the covariance matrix of the mean predictions of the observed variables conditional on the parameters. The more these conditional mean predictions vary across parameter values, the larger the share of parameter uncertainty is, with the consequence that the MC estimator mixes normal densities that potentially lie far from each other. Hence, the greater the parameter uncertainty share is, the more likely it is that the predictive density is *not* well approximated by the normal density.

The recursive estimates of the parameter uncertainty share are plotted in Figure 26 for real GDP growth (Chart A) and inflation (Chart B) over the different forecast horizons. Overall, the share is less than ten percent and in the case of real GDP growth typically less than five percent with little volatility. In addition, there is a tendency for the share to be slightly lower at the longer forecast horizons. The parameter uncertainty share for inflation appears to be somewhat more variable over time and, on average, somewhat higher than for real GDP growth. For the one-quarter-ahead forecasts, the share increases by roughly 5 percent with the onset of the Great Recession, while the rise is less pronounced as the horizon increases. It is also interesting to note that following the crisis the share is larger for the SWFF and SWU models than for the SW model. However, parameter uncertainty is generally not large and may therefore explain why the normal approximation yields predictive likelihood estimates that are close to those from the MC estimator.

Since the normal density provides a good approximation to the MC estimator of the predictive likelihood, it can meaningfully be utilized to examine which moments of the predictive density that matter most for the ranking of models. Following Warne et al. (2017) we decompose the log of the normal density into a forecast uncertainty (FU) term based on minus one-half times the log determinant of the predictive covariance matrix and a quadratic standardized forecast error (FE) term. For the univariate case, it should be kept in mind that a lower predictive variance yields a higher FU-term value. The decomposition is shown for the average log predictive score of

[25]For example, based on the normal approximation and the SW model for the five-quarter-ahead forecasts based on the 2007Q4 vintage, the probability of observing a value less than or equal to 0.08 for inflation in 2009Q1 is approximately 8.3 percent.

[26]From equation (39) we find that each individual density on the right hand side is normal, yielding an estimator which is mixed normal.

[27]See also Adolfson, Lindé, and Villani (2007b) and Geweke and Amisano (2014) for further discussions on decomposing the estimated predictive covariance matrix.

real GDP growth in Figure 27, with the FU term in Panel A and the FE term in Panel B.[28] The FU term is very smooth for all models and slowly rising over the forecast sample for all $h \geq 0$, reflecting that the predictive variance is slowly getting lower as the information set increases. For all positive h, the SW model has the highest value, while the SWU model tends to have the lowest value after the onset of the crisis. The FE term, on the other hand, is more volatile and determines the rank of the SWFF model when $h \geq 1$. This term is also decisive when ranking the SW and SWU model, especially for longer horizons and after the crisis.

The decompositions of the average log predictive score for inflation are shown in Figure 28. The behavior of the FU term is very different from the real GDP growth case. While it remains quite smooth for the SW and SWU models, it tends to trend weakly downward for the SWFF model and all $h \geq 1$ prior to 2005, when it begins to trend upward strongly until it levels out around 2009–2010. Moreover, the FU term is falling weakly in the early years of the forecast sample for the SW and SWU models, and thereafter remains quite stable for the latter model while it begins to drift downward again for the former model around 2007. Towards the end of the forecast sample, the FU terms of the SW and SWFF models are very close, while the SWU model has slightly higher values. Concerning the FE term, it is notable that for $h \leq 5$ it is trending downward strongly for the SWFF model due to larger standardized forecast errors, and consequently the ranking of the SWFF model over these horizons is due to both terms. For the longer term forecasts, the SWFF model is competitive when looking at the FE term, where in particular it is fairly constant from 2008–2009 onwards. This is mainly in line with the time profile of the FU term, suggesting that the actual longer term forecast errors are quite similar not only since but also for some years prior to the onset of the Great Recession.

The time profiles of the FE term for the SW and SWU models are quite similar and for the shorter term forecasts nearly equal. For these forecasts, the SWU model seems to have a slight advantage over the SW model, suggesting that the ranking of these two models for such horizons is manly determined by the FU term. Turning to the medium and longer term forecasts, it can be seen that the FE term is typically greater for the SW model than for the SWU model, with the effect that the ranking of these models prior to the crisis is due to the FE term, while the sum of the FU and FE terms even out after the crisis. In other words, the lower predictive variances of the SWU model compared with the SW model after the crisis go hand in hand with higher standardized forecast errors, indicating that the actual forecast errors from these two models are quite similar for the medium and longer term inflation forecasts.

Finally, the decompositions of the average log predictive score of the joint real GDP growth and inflation forecasts are depicted in Figure 29. It is noteworthy that the path of the FU term for the SWFF model takes a similar shape as the path of the same term for inflation and is overall lower than for the other two models. The paths of the FU terms of the SW and SWU

[28]It should here be kept in mind that dating along the horizontal axis in the Figure follows the dating of the log predictive score. This means that 2004Q4 for h-quarter-ahead forecasts concerns the outcome in 2004Q4 for the density forecast made h quarters earlier.

models are fairly constant and quite close. The SW model has a slightly higher FU term than the SWU model for the medium term forecasts, especially since 2005 up to the end of the forecast sample when these two terms are nearly equal.

Turning to the FE term, the SWFF model again typically yields lower values, suggesting that not only its predictive variances are higher than the variances for the SW and SWU models, but also that its forecast errors are larger. It is also striking that although the levels of the FE term paths differ across the models, the paths are otherwise quite similar, especially since the large fall in real GDP growth in 2008Q4 and 2009Q1. Moreover, after the crisis the SWU model is often ranked ahead of the SW model for the longer term forecasts and this is mainly due to its smaller standardized forecast errors.

7. Summary and Conclusions

In this paper we compare real-time density forecasts of real GDP growth and inflation for the euro area using a version of the Smets and Wouters (2007) model as a benchmark. We consider two extensions of this model that focus on (i) financial frictions using the BGG financial accelerator framework (Bernanke et al., 1999), and (ii) extensive labor market variations following Galí et al. (2012), respectively. The former model expands the vector of observables with a measure of the external finance premium (SWFF model), while the latter adds the unemployment rate (SWU model).

The euro area RTD typically covers data starting in the mid-1990s and to extend the data back in time we have followed Smets et al. (2014) and made use of the updates from the AWM database back to 1980. The forecast comparison sample begins with the first RTD vintage in 2001 and ends in 2014, thereby covering the period of wage moderation, the global financial crises, the Great Recession that followed, the European sovereign debt crisis, and the period of policy rates close to the effective lower bound towards the end of the forecast sample. Consequently, this is a time period that we expect to be uniquely challenging for any model to forecast well.

The density forecasts of the three models are compared using the log predictive score over the forecast sample 2001Q1–2014Q4 for the backcasts, the nowcasts and the one to eight-quarter-ahead horizons for real GDP growth and inflation, separately and jointly. In addition, we use two methods for estimating the underlying predictive likelihoods, the MC integration estimator and the normal approximation based on the mean and covariance of the predictive density. While the former estimator is consistent under standard assumptions, the latter estimator allows for a more detailed analysis of the density forecasting properties provided that errors from using the approximation are small.

One important finding from the empirical forecast comparison exercise is that adding financial frictions of the BGG-type overall leads to a deterioration of the forecasts. In fact, this shortcoming is not only present in the density forecasts but also in the point forecasts, where the SWFF model typically over-predicts real GDP growth and under-predicts inflation. The finding that the SWFF model does not improve the forecasts relative to the benchmark is also supported by Kolasa and Rubaszek (2015), who use U.S. data and where the forecasts are not based on real-time data.[29]

One exception to this finding concerns the longer-term (eight-quarter-ahead) inflation forecasts, where the mean errors of the SWFF model are fairly close to zero and its density forecasts are the highest among the three models we have studied. Moreover, there is evidence that the SWFF model at least for some horizons improves the density forecasts for real GDP growth and inflation after the onset of the Great Recession, as previously shown for US data by Del Negro and Schorfheide (2013) and Del Negro et al. (2015). Hence, it is likely that the SWFF model may still be useful once pooling of forecasts is considered; see, e.g., Amisano and Geweke (2017) and the references therein.

In line with the results in Warne et al. (2017) for the euro area and Kolasa and Rubaszek (2015) for the U.S., we find that the predictive density at the actual values, represented by the MC estimator of the predictive likelihood, is generally well approximated with a normal density. However, we also find that the approximation is less accurate when the actuals give very low values for the height of the predictive density, such as at the onset of the Great Recession.

Since the normal density provides a very good approximation of the predictive density when estimating the predictive likelihood, we have also utilized the formulation of the former density to decompose the predictive likelihood into a forecast uncertainty (FU) term, given by minus one-half times the log determinant of the predictive covariance matrix, and a quadratic standardized forecast error (FE) term. This decomposition allows for an analysis of which moment of the density is mainly contributing to the ranking of models in the forecast comparison exercise. A similar exercise was also conducted by Warne et al. (2017) who found that their results were mainly driven by differences in forecast uncertainty among their models, while the FE term was the main factor when the ranking of models changed. In the present study, the FE term often plays a much more prominent role for the ranking of the three models.

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[29]Kolasa and Rubaszek (2015) also consider a second type of financial friction where, following Iacoviello (2005), the extension to the benchmark model instead incorporates housing and collateral constraints into the household sector. This type of financial friction outperforms the benchmark model as well as their SWFF-type model during times of financial turbulence. Coenen, Karadi, Schmidt, and Warne (2018) adds financial frictions to the New Area-Wide Model (NAWM) of the ECB by extending it with a rich financial intermediary sector. The financial extension of the NAWM is setup such that banks engage in maturity transformation by offering long-term loans to the private sector to finance investment projects and hold long-term government bonds (Carlstrom, Fuerst, and Paustian, 2017), and where banks fund these assets with short-term household deposits and with their equity (net worth) accumulated through retained earnings. Furthermore, the extension has imperfect financial markets, where the option to abscond ("agency problem") limits the leverage of banks (Gertler and Karadi, 2011, 2013), banks' capital position influences the transmission of shocks (a "financial accelerator" mechanism), and where foreign trade is intermediated by banks. Other key elements include bank-based intermediation with delayed pass-through to lending rates, as in Gerali, Neri, Sessa, and Signoretti (2010), and that the presence of nominal debt gives rise to a debt-deflation channel. The central bank can purchase long-term private sector loans and government bonds as relief of banks' balance sheets/leverage constraints ("stealth recapitalisation"), and where banks' holdings of foreign currency-denominated bonds accounts for an exchange-rate channel of asset purchases. The financial frictions of this environment may better describe the financing conditions in the euro area than the BGG framework and may therefore be another important avenue to consider when comparing density forecasting with DSGE models for the euro area.

In the wake of the Great Recession, two apparent shortcomings of DSGE models were often raised in the discussions. First, DSGE models lack an endogenous treatment of financial frictions and, furthermore, do not allow for involuntary unemployment. Second, it has been argued that the strong equilibrium mechanisms of these models have made them vulnerable to forecast errors following a severe and long-lasting economic downturn. Our evidence suggests that while financial frictions of the BGG-type may improve the forecasts of real GDP growth and inflation around such an event, these frictions typically lead to a deterioration in the forecasting performance for the euro area. Modelling and measuring unemployment, on the other hand, has overall improved the density forecasts compared with the benchmark model since the onset of the crisis, albeit not dramatically.

Concerning the second criticism, we note that all three DSGE models display larger mean forecast errors since the onset of the Great Recession than before this event. However, it is not possible to prove or disprove that this is a result of the equilibrium mechanisms of the models. In fact, the mean reversion properties of the models are not exceptionally strong compared with reduced-form models with fixed parameters and, hence, how quickly the point forecasts return to the means of the variables depends mainly on which shocks are important at the various forecast horizons. For inflation, we have found in the full sample analysis that price markup shocks are particularly important in the short-term and these shocks are overall not very persistent. This may explain why the mean forecasts tend to return to values around the estimated mean inflation rate within two years for the SW and SWU models. In the case of the SWFF model, however, the point forecasts are far from mean inflation after two years. A potential explanation is that preference shocks are more important in this model within the two-year horizon and that such shocks are estimated to be highly persistent.

The possibility of improving the forecasts of the models through predictions pools has been mentioned above and is an interesting avenue for future research. In view of the finding that the DSGE models typically over-predict real GDP growth since the onset of the Great Recession, it may also be important to consider additional information about euro area developments. In particular, Smets et al. (2014) make use of the ECB's Survey of Professional Forecasters (SPF) which is conducted during the first month of each quarter since 2001. Although the question of how to best utilize the survey data from the SPF is open, it should be kept in mind that Smets et al. caution against replacing the mean real GDP growth rate with the "five-year-ahead" SPF average on the basis that it worsens their point forecasts.[30] An examination of these issues is beyond the scope of this paper, but is an appealing area for future research.

[30] For analyses on the optimal combination of individual survey forecasts and how this combination compares with the standard equal weight (average) combination for the ECB's SPF, see Conflitti et al. (2015).

Tables

Table 1. Prior distributions for the structural parameters of the Smets and Wouters (SW) model, extended with financial frictions (SWFF), or with unemployment (SWU).

		SW model		SWFF model		SWU model	
parameter	density	P_1	P_2	P_1	P_2	P_1	P_2
φ	N	4.0	1.5	4.0	1.0	4.0	1.5
σ_c	N	1.0	0.25	1.5	0.1	1.0^{c}	–
λ	β	0.7	0.1	0.7	0.1	0.7	0.1
σ_l	N	2.0	0.75	2.0	0.375	2.0	0.75
ξ_w	β	0.5	0.05	0.5	0.05	0.5	0.05
ξ_p	β	0.5	0.1	0.7	0.05	0.5	0.1
$\imath_w$	β	0.5	0.15	0.5	0.15	0.5	0.15
$\imath_p$	β	0.5	0.15	0.5	0.15	0.5	0.15
ϕ_p	N	1.25	0.125	1.25	0.125	1.25	0.125
ψ	β	0.5	0.15	0.5	0.15	0.5	0.15
$\zeta_{sp,b}$	β	–	–	0.05	0.005	–	–
ρ	β	0.75	0.1	0.75	0.1	0.75	0.1
r_π	N	1.5	0.25	1.5	0.25	1.5	0.25
r_y	N	0.125	0.05	0.125	0.05	0.125	0.05
$r_{\Delta y}$	N	0.125	0.05	0.125	0.05	0.125	0.05
ξ_e	β	0.5	0.15	0.5	0.15	0.5	0.15
υ	β	–	–	–	–	0.2	0.05
$\bar{\pi}$	Γ	0.625	0.1	0.625	0.1	0.625	0.1
$\bar{\beta}$	Γ	0.25	0.1	0.25	0.1	0.25	0.1
$\bar{e}$	N	0.2	0.05	0.2	0.05	0.2	0.05
$\bar{\gamma}$	N	0.3	0.05	0.3	0.05	0.3	0.05
s	Γ	–	–	2.0	0.1	–	–
α	N	0.3	0.05	0.3	0.05	0.3	0.05

Notes: The columns P_1 and P_2 refer to the mean and the standard deviation of the normal (N), standardized beta (β), and gamma (Γ) distributions. The superscript c means that the parameter is calibrated.

Table 2. Prior distributions for the parameters of the shock processes of the Smets and Wouters (SW) model, extended with financial frictions (SWFF), or with unemployment (SWU).

		SW model		SWFF model		SWU model	
parameter	density	P_1	P_2	P_1	P_2	P_1	P_2
ρ_g	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_{ga}	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_b	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_i	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_a	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_p	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_w	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_r	β	0.5	0.2	0.5	0.2	0.5	0.2
ρ_e	β	–	–	0.85	0.1	–	–
ρ_s	β	–	–	–	–	0.5	0.2
σ_g	Γ^{-1}/U	0.1	2	0.1	2	0	5
σ_b	Γ^{-1}/U	0.1	2	0.1	2	0	5
σ_i	Γ^{-1}/U	0.1	2	0.1	2	0	5
σ_a	Γ^{-1}/U	0.1	2	0.1	2	0	5
σ_p	Γ^{-1}/U	0.1	2	0.1	2	0	5
σ_w	Γ^{-1}/U	0.1	2	0.1	2	0	5
σ_r	Γ^{-1}/U	0.1	2	0.1	2	0	5
σ_e	Γ^{-1}/U	–	–	0.05	4	–	–
σ_s	Γ^{-1}/U	–	–	–	–	0	5

Notes: The columns P_1 and P_2 refer to the mean and the standard deviation of the standardized beta distribution, the upper and lower bound of the uniform (U) distribution, and the location and degree of freedom parameters of the inverse Gamma (Γ^{-1}) distribution; see, e.g., Del Negro and Schorfheide (2013) for the parameterization of the inverse Gamma. The uniform distribution is only used for the SWU model.

Table 3. Posterior distributions for the structural parameters of the Smets and Wouters (SW) model, extended with financial frictions (SWFF), or with unemployment (SWU) based on the sample 1985Q1–2013Q4 for the euro area.

	SW model				SWFF model				SWU model			
	mean	mode	5%	95%	mean	mode	5%	95%	mean	mode	5%	95%
φ	4.897	4.774	3.467	6.569	2.500	2.328	1.641	3.514	3.431	3.193	2.380	4.831
σ_c	1.047	1.013	0.918	1.222	1.454	1.438	1.311	1.600	1.0^{c}	1.0^{c}	–	–
λ	0.632	0.651	0.532	0.717	0.301	0.301	0.241	0.361	0.621	0.623	0.541	0.695
σ_l	1.204	1.085	0.556	1.986	0.268	0.156	−0.091	0.712	5.203	5.211	4.957	5.453
ξ_w	0.607	0.609	0.537	0.676	0.500	0.498	0.418	0.583	0.553	0.553	0.468	0.636
ξ_p	0.751	0.755	0.692	0.803	0.833	0.839	0.782	0.874	0.758	0.767	0.698	0.814
$\imath_w$	0.282	0.245	0.128	0.471	0.219	0.190	0.101	0.365	0.219	0.198	0.096	0.367
$\imath_p$	0.149	0.120	0.060	0.275	0.158	0.118	0.059	0.306	0.206	0.169	0.085	0.367
ϕ_p	1.462	1.452	1.318	1.612	1.527	1.518	1.377	1.682	1.510	1.512	1.405	1.651
ψ	0.795	0.810	0.671	0.902	0.920	0.937	0.852	0.968	0.620	0.613	0.496	0.744
$\zeta_{sp,b}$	–	–	–	–	0.055	0.055	0.048	0.063	–	–	–	–
ρ	0.890	0.893	0.858	0.918	0.863	0.863	0.813	0.910	0.856	0.862	0.821	0.887
r_π	1.913	1.926	1.586	2.240	1.320	1.292	1.028	1.675	1.477	1.473	1.226	1.749
r_y	0.125	0.124	0.065	0.192	0.088	0.084	0.035	0.144	0.175	0.166	0.127	0.227
$r_{\Delta y}$	0.155	0.155	0.114	0.199	0.274	0.270	0.222	0.328	0.033	0.029	0.010	0.060
ξ_e	0.736	0.739	0.696	0.772	0.735	0.738	0.689	0.778	0.681	0.687	0.635	0.724
υ	–	–	–	–	–	–	–	–	0.149	0.138	0.067	0.245
$\bar{\pi}$	0.604	0.592	0.478	0.739	0.532	0.522	0.425	0.643	0.585	0.588	0.465	0.712
$\bar{\beta}$	0.245	0.241	0.126	0.383	0.230	0.216	0.125	0.352	0.235	0.214	0.121	0.367
$\bar{e}$	0.147	0.149	0.120	0.172	0.212	0.206	0.160	0.271	0.182	0.184	0.167	0.197
$\bar{\gamma}$	0.225	0.225	0.166	0.284	0.294	0.298	0.192	0.383	0.143	0.142	0.094	0.191
s	–	–	–	–	1.886	1.892	1.748	2.030	–	–	–	–
α	0.254	0.258	0.222	0.287	0.321	0.321	0.282	0.360	0.233	0.232	0.206	0.260

Notes: The columns for each model display the mean, the mode, and the 5% and 95% quantiles, respectively, from the posterior distributions. The superscript c means that the parameter is calibrated.

TABLE 4. Posterior distributions for the parameters of the shock processes of the Smets and Wouters (SW) model, extended with financial frictions (SWFF), or with unemployment (SWU) based on the sample 1985Q1–2013Q4 for the euro area.

	SW model				SWFF model				SWU model			
	mean	mode	5%	95%	mean	mode	5%	95%	mean	mode	5%	95%
ρ_g	0.993	0.996	0.985	0.998	0.957	0.983	0.892	0.992	0.996	0.997	0.992	0.999
ρ_{ga}	0.128	0.117	0.054	0.212	0.159	0.137	0.056	0.281	0.190	0.180	0.098	0.291
ρ_b	0.853	0.861	0.796	0.903	0.958	0.959	0.941	0.974	0.873	0.875	0.823	0.918
ρ_i	0.294	0.285	0.158	0.430	0.415	0.419	0.284	0.542	0.229	0.223	0.103	0.364
ρ_a	0.983	0.985	0.972	0.992	0.987	0.990	0.975	0.996	0.979	0.979	0.972	0.985
ρ_p	0.465	0.478	0.281	0.614	0.436	0.449	0.231	0.630	0.264	0.284	0.072	0.444
ρ_w	0.896	0.902	0.848	0.935	0.920	0.932	0.879	0.952	0.739	0.765	0.545	0.879
ρ_r	0.398	0.395	0.264	0.535	0.420	0.419	0.331	0.502	0.308	0.316	0.158	0.452
ρ_e	–	–	–	–	0.962	0.965	0.937	0.982	–	–	–	–
ρ_s	–	–	–	–	–	–	–	–	0.979	0.980	0.968	0.988
σ_g	0.300	0.296	0.269	0.335	0.295	0.295	0.266	0.332	0.300	0.297	0.269	0.335
σ_b	0.069	0.067	0.054	0.087	0.033	0.032	0.028	0.038	0.070	0.069	0.052	0.092
σ_i	0.505	0.509	0.421	0.598	0.600	0.591	0.510	0.704	0.534	0.530	0.451	0.625
σ_a	0.555	0.538	0.456	0.672	0.397	0.390	0.334	0.469	0.506	0.488	0.414	0.614
σ_p	0.107	0.103	0.082	0.136	0.116	0.113	0.085	0.148	0.134	0.135	0.107	0.161
σ_w	0.074	0.072	0.059	0.092	0.082	0.080	0.066	0.102	0.086	0.082	0.064	0.115
σ_r	0.115	0.112	0.100	0.132	0.149	0.145	0.127	0.174	0.108	0.107	0.095	0.122
σ_e	–	–	–	–	0.062	0.062	0.052	0.073	–	–	–	–
σ_s	–	–	–	–	–	–	–	–	1.040	1.031	0.913	1.178

NOTES: See Table 3 for details.

TABLE 5. Long-run forecast error variance decompositions of observables at posterior mean.

Shock \ Variable	Δy_t	Δc_t	Δi_t	π_t	Δe_t	Δw_t	r_t	u_t	s_t
					SW model				
Spending	0.118	0.026	0.001	0.001	0.068	0.000	0.004	–	–
Preference	**0.325**	**0.453**	**0.198**	**0.116**	**0.364**	**0.121**	**0.654**	–	–
Investment-specific	0.134	0.004	**0.525**	0.001	0.047	0.006	0.012	–	–
TFP	**0.200**	**0.266**	0.087	0.034	0.088	0.108	0.079	–	–
Price markup	0.024	0.018	0.024	**0.280**	0.039	**0.130**	0.017	–	–
Wage markup	0.038	0.041	0.044	**0.457**	**0.168**	**0.561**	**0.178**	–	–
Monetary policy	**0.161**	**0.192**	**0.120**	0.110	**0.226**	0.073	0.056	–	–
					SWU model				
Spending	0.072	0.072	0.001	0.009	0.020	0.000	0.012	0.002	–
Preference	**0.456**	**0.519**	**0.339**	**0.214**	**0.600**	**0.195**	**0.620**	**0.423**	–
Investment-specific	0.086	0.002	**0.405**	0.002	0.037	0.004	0.007	0.010	–
TFP	**0.233**	**0.243**	**0.133**	0.122	**0.069**	0.076	**0.144**	0.021	–
Price markup	0.019	0.013	0.016	**0.400**	0.022	**0.137**	0.025	0.005	–
Wage markup	0.008	0.008	0.012	**0.199**	0.046	**0.532**	0.062	**0.390**	–
Monetary policy	**0.111**	**0.121**	0.086	0.044	**0.152**	0.047	**0.119**	0.106	–
Labor supply	0.013	0.022	0.008	0.008	0.052	0.008	0.010	0.042	–
					SWFF model				
Spending	0.108	0.013	0.001	0.001	0.064	0.001	0.004	–	0.003
Preference	**0.140**	**0.239**	0.063	**0.422**	**0.210**	0.089	**0.763**	–	**0.659**
Investment-specific	**0.263**	0.043	**0.556**	0.002	**0.112**	0.024	0.038	–	0.034
TFP	0.140	**0.254**	0.020	0.011	0.043	0.090	0.028	–	0.012
Price markup	0.020	0.021	0.014	**0.323**	0.041	**0.110**	0.007	–	0.004
Wage markup	0.005	0.033	0.050	**0.147**	0.016	**0.548**	**0.084**	–	0.023
Monetary policy	**0.317**	**0.363**	**0.246**	0.088	**0.497**	**0.131**	0.044	–	0.006
Spread	0.007	0.033	0.050	0.005	0.016	0.006	0.032	–	**0.258**

NOTES: This table reports posterior mean estimates for the forecast-error-variance decompositions of additional observed variables at the long-run-quarter horizon. The decomposition is conducted only for the structural shock part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped. We embolden the contributions that account for 80% or more of the total variance (to a maximum of three contributions).

TABLE 6. Short-run forecast error variance decompositions of observables at posterior mean.

Shock \ Variable	Δy_t	Δc_t	Δi_t	π_t	Δe_t	Δw_t	r_t	u_t	s_t
					SW model				
Spending	**0.166**	0.023	0.000	0.000	0.100	0.000	0.002	–	–
Preference	**0.334**	**0.525**	**0.167**	0.061	**0.377**	**0.115**	**0.475**	–	–
Investment-specific	**0.174**	0.001	**0.690**	0.000	0.064	0.006	0.018	–	–
TFP	0.163	**0.235**	0.038	0.021	**0.173**	0.059	**0.081**	–	–
Price markup	0.013	0.007	0.013	**0.659**	0.028	**0.209**	0.065	–	–
Wage markup	0.002	0.005	0.002	**0.206**	0.044	**0.549**	0.048	–	–
Monetary policy	0.147	**0.204**	0.084	0.052	**0.213**	0.061	**0.311**	–	–
					SWU model				
Spending	0.091	0.067	0.001	0.000	0.039	0.000	0.010	0.008	–
Preference	**0.463**	**0.564**	**0.298**	0.069	**0.595**	0.104	**0.126**	**0.337**	–
Investment-specific	**0.114**	0.000	**0.547**	0.000	0.055	0.003	0.013	0.021	–
TFP	**0.200**	**0.217**	0.066	0.028	**0.092**	0.020	**0.122**	0.031	–
Price markup	0.014	0.007	0.010	**0.810**	0.017	**0.223**	0.105	0.003	–
Wage markup	0.001	0.000	0.001	0.076	0.017	**0.615**	0.015	**0.123**	–
Monetary policy	0.106	**0.124**	0.073	0.015	**0.145**	0.025	**0.602**	0.080	–
Labor supply	0.010	0.020	0.004	0.001	0.039	0.009	0.008	**0.397**	–
					SWFF model				
Spending	0.132	0.013	0.000	0.000	0.089	0.001	0.007	–	0.000
Preference	0.142	**0.256**	0.049	0.081	**0.206**	0.069	**0.647**	–	**0.654**
Investment-specific	**0.278**	0.035	**0.676**	0.001	**0.137**	0.019	**0.157**	–	0.011
TFP	**0.149**	**0.260**	0.012	0.001	0.075	0.068	0.035	–	0.000
Price markup	0.005	0.004	0.004	**0.824**	0.026	**0.177**	0.056	–	0.002
Wage markup	0.002	0.026	0.039	0.060	0.002	**0.553**	0.026	–	0.004
Monetary policy	**0.292**	**0.377**	**0.183**	0.034	**0.460**	**0.110**	0.070	–	0.010
Spread	0.000	0.028	0.037	0.000	0.006	0.003	0.002	–	**0.318**

NOTES: This table reports posterior mean estimates for the forecast-error-variance decompositions of additional observed variables at the one-quarter horizon. See also Table 5.

Table 7. Linking the vintages of the RTD to the AWM updates.

RTD vintages	RTD common start date	AWM update	AWM start date	AWM end date	RTD euro area concept	AWM euro area concept
2001Q1–2001Q4	1994Q1	2	1970Q1	1999Q4	12	12
2002Q1–2003Q2	1994Q1	3	1970Q1	2000Q4	12	12
2003Q3–2004Q2	1994Q1	4	1970Q1	2002Q4	12	12
2004Q3–2005Q3	1994Q1	5	1970Q1	2003Q4	12	12
2005Q4–2006Q2	1995Q1	5	1970Q1	2003Q4	12	12
2006Q3–2006Q4	1995Q1	6	1970Q1	2005Q4	12	12
2007Q1–2007Q2	1996Q1	6	1970Q1	2005Q4	12,13	12
2007Q3	1996Q1	7	1970Q1	2006Q4	12,13	13
2007Q4–2008Q1	1996Q1	7	1970Q1	2006Q4	13	13
2008Q2	1995Q1	7	1970Q1	2006Q4	13,15	13
2008Q3–2008Q4	1995Q1	8	1970Q1	2007Q4	15	15
2009Q1–2009Q2	1995Q1	8	1970Q1	2007Q4	16	15
2009Q3–2010Q2	1995Q1	9	1970Q1	2008Q4	16	16
2010Q3–2010Q4	1995Q1	10	1970Q1	2009Q4	16	16
2011Q1	1995Q1	10	1970Q1	2009Q4	16,17	16
2011Q2	1995Q1	10	1970Q1	2009Q4	17	16
2011Q3–2012Q2	1995Q1	11	1970Q1	2010Q4	17	17
2012Q3–2013Q2	1995Q1	12	1970Q1	2011Q4	17	17
2013Q3–2013Q4	1995Q1	13	1970Q1	2012Q4	17	17
2014Q1–2014Q2	2000Q1	13	1970Q1	2012Q4	18	17
2014Q3–2014Q4	2000Q1	14	1970Q1	2013Q4	18	18
2015Q1–2015Q2	2000Q1	14	1970Q1	2013Q4	19	18
2015Q3–2015Q4	2000Q1	15	1970Q1	2014Q4	19	19

Notes: Data from the AWM is always taken from 1980Q1 until the quarter prior to the RTD common start date. When two RTD euro area concepts are indicated it means that some variables are based on one of them, while others are based on the other. In all cases, the lower euro area country concept concerns unit labor cost, while the higher concept is used for the aggregation of the other variables except in 2011Q1 when also the GDP deflator, total employment and the unemployment rate is based on euro area 16. Unit labor cost is the measure underlying the calculation of nominal wages as unit labor cost times real GDP divided by total employment. Unemployment has undergone gradual changes in the definition in December 2000, March and June 2002; see Box 4 “Changes in the definition of unemployment in EU Member States” in the March 2001 issue of the Monthly Bulletin for details.

Table 8. The ragged edge of the euro area RTD: Vintages with missing data for the variables.

Date	y	c	i	π	e	w	r	u
Backcast	2001Q1–2001Q2	2001Q1–2001Q2	2001Q1–2001Q2	2001Q1–2003Q3	2001Q1–2014Q4	2001Q1–2014Q4	–	–
	2002Q1	2002Q1	2002Q1					
		2003Q3	2003Q3					
		2004Q3	2004Q3	2004Q3				
		2006Q1	2006Q1	2006Q1				
		2006Q3	2006Q3	2006Q3				
		2014Q3–2014Q4	2014Q3–2014Q4	2014Q3–2014Q4				
Total	3 of 56	9 of 56	9 of 56	16 of 56	56 of 56	56 of 56	0 of 56	0 of 56
Nowcast	2001Q1–2014Q4	2001Q1–2014Q4	2001Q1–2014Q4	2001Q1–2014Q4	2001Q1–2014Q4	2001Q1–2014Q4	–	2005Q1
								2005Q3–2005Q4
								2006Q3
								2008Q1
Total	56 of 56	56 of 56	56 of 56	56 of 56	56 of 56	56 of 56	0 of 56	5 of 56

Table 9. Mean errors based on predictive mean as point forecast for the sample 2001Q1–2014Q4.

	Δy			π		
h	SW	SWFF	SWU	SW	SWFF	SWU
-1	0.045	−0.337	−0.218	0.154	0.227	0.136
0	−0.168	−0.388	−0.279	0.067	0.228	0.039
1	−0.382	−0.595	−0.439	0.028	0.278	0.004
2	−0.472	−0.655	−0.490	−0.018	0.284	−0.040
3	−0.502	−0.666	−0.486	−0.064	0.270	−0.084
4	−0.488	−0.650	−0.443	−0.113	0.241	−0.132
5	−0.467	−0.637	−0.400	−0.155	0.211	−0.172
6	−0.440	−0.622	−0.358	−0.186	0.185	−0.202
7	−0.411	−0.607	−0.318	−0.215	0.155	−0.231
8	−0.380	−0.587	−0.279	−0.238	0.129	−0.252

Table 10. Mean errors based on predictive mean as point forecast for the sample 2001Q1–2008Q3.

	Δy			π		
h	SW	SWFF	SWU	SW	SWFF	SWU
-1	0.045	−0.337	−0.218	0.177	0.238	0.172
0	−0.017	−0.298	−0.140	0.104	0.232	0.085
1	−0.227	−0.513	−0.253	0.070	0.269	0.047
2	−0.366	−0.631	−0.338	0.018	0.259	−0.007
3	−0.389	−0.638	−0.328	−0.037	0.230	−0.063
4	−0.360	−0.600	−0.280	−0.092	0.190	−0.119
5	−0.351	−0.586	−0.260	−0.140	0.152	−0.166
6	−0.332	−0.566	−0.237	−0.170	0.126	−0.195
7	−0.295	−0.526	−0.199	−0.198	0.100	−0.221
8	−0.268	−0.497	−0.176	−0.215	0.082	−0.235

TABLE 11. Mean errors based on predictive mean as point forecast for the sample 2008Q4–2014Q4.

	Δy			π		
h	SW	SWFF	SWU	SW	SWFF	SWU
-1	–	–	–	−0.008	0.155	−0.112
0	−0.356	−0.500	−0.453	0.021	0.222	−0.019
1	−0.581	−0.701	−0.680	0.027	0.290	−0.053
2	−0.616	−0.687	−0.696	−0.067	0.318	−0.085
3	−0.661	−0.706	−0.707	−0.102	0.327	−0.114
4	−0.676	−0.723	−0.684	−0.142	0.316	−0.151
5	−0.647	−0.716	−0.618	−0.178	0.302	−0.181
6	−0.615	−0.715	−0.554	−0.211	0.279	−0.214
7	−0.612	−0.746	−0.522	−0.245	0.250	−0.248
8	−0.584	−0.751	−0.469	−0.282	0.216	−0.284

TABLE 12. Point forecasts of real GDP growth of the SWU model for the 2008Q4 and 2009Q1 vintages.

	Forecast horizon								
Vintage	0	1	2	3	4	5	6	7	8
2008Q4	0.144	0.276	0.350	0.385	0.399	0.400	0.395	0.389	0.382
2009Q1	0.214	0.596	0.710	0.721	0.687	0.634	0.576	0.521	0.472
	Point forecasts from the 2009Q1 vintage subject to input change								
Source	0	1	2	3	4	5	6	7	8
Parameters	0.289	0.665	0.764	0.766	0.724	0.665	0.602	0.543	0.491
Revisions	−0.001	0.394	0.401	0.364	0.331	0.310	0.301	0.299	0.303
New data	0.349	0.406	0.421	0.417	0.405	0.391	0.378	0.367	0.358
Old data	0.240	0.308	0.342	0.358	0.362	0.360	0.357	0.354	0.351
Actuals	−2.529	−0.110	0.421	0.199	0.388	0.953	0.382	0.268	0.751

NOTES: The three cases for changes to the construction of the 2009Q1 vintage point forecasts are defined as follows: *Parameters* refers to changing the posterior parameter draws from the 2009Q1 vintage to the 2008Q4 vintage draws; *Revisions* means that all the historical data from the 2008Q4 vintage are used instead of the data for the same time periods from the 2009Q1 vintage, while the latest data points from the 2009Q1 vintage are still taken into account; *New data* refers to the case when the latest data points for each variable from the 2009Q1 vintage are excluded from the dataset and where the data prior to these dates are included; while *Old data* means that only data from the 2008Q4 vintage are used and where the latest data points from the 2009Q1 vintage are therefore treated as unobserved.

Table 13. Log predictive score using the MC estimator for the sample 2001Q1–2014Q4.

	Δy			π			Δy & π		
h	SW	SWFF	SWU	SW	SWFF	SWU	SW	SWFF	SWU
-1	−2.323	−2.523	−2.376	−4.487	−9.097	−3.953	−9.199	−10.368	−8.993
0	−46.182	−51.644	−52.235	9.430	−10.186	13.302	−36.769	−62.343	−39.172
1	−56.529	−65.860	−60.004	7.853	−16.356	11.210	−48.457	−87.512	−48.392
2	−59.566	−68.632	−61.202	4.434	−17.831	7.104	−54.739	−92.630	−53.752
3	−60.681	−68.704	−60.770	0.567	−17.545	2.933	−59.736	−91.769	−57.716
4	−59.509	−67.227	−58.488	−2.385	−14.858	−0.902	−61.650	−86.362	−59.662
5	−57.894	−65.743	−55.639	−4.496	−12.836	−3.711	−62.402	−81.849	−60.013
6	−56.601	−64.368	−53.813	−6.926	−11.185	−6.517	−63.764	−78.273	−61.196
7	−55.137	−63.089	−52.376	−9.013	−8.756	−9.345	−64.577	−74.024	−62.735
8	−53.212	−61.728	−51.169	−11.317	−8.153	−11.960	−65.108	−71.623	−64.204

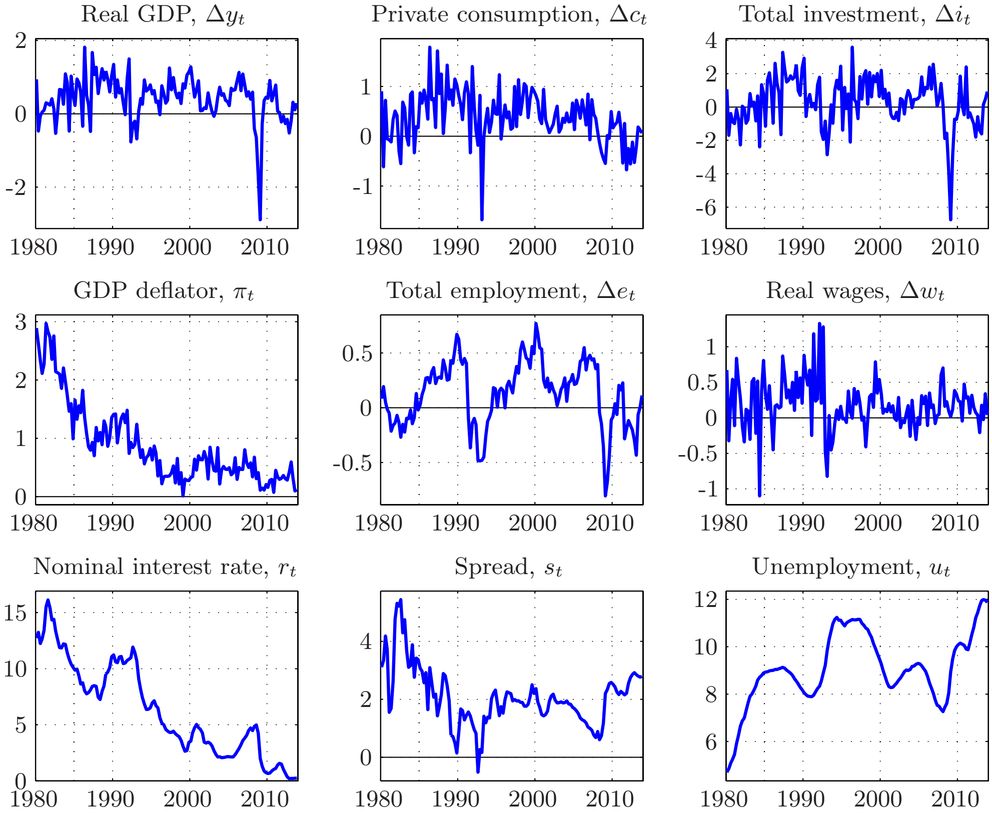
Table 14. Log predictive score using the MC estimator for the sample 2008Q4–2014Q4.

	Δy			π			Δy & π		
h	SW	SWFF	SWU	SW	SWFF	SWU	SW	SWFF	SWU
-1	–	–	–	0.822	0.346	0.755	–	–	–
0	−24.573	−27.523	−30.256	5.394	−4.113	7.236	−19.191	−31.729	−23.166
1	−29.061	−32.548	−31.405	2.486	−9.194	4.462	−26.307	−44.877	−26.544
2	−25.013	−26.474	−26.845	0.865	−10.300	2.776	−23.977	−40.914	−23.864
3	−25.027	−25.889	−26.018	−0.012	−9.065	1.758	−24.834	−38.864	−24.121
4	−23.867	−24.993	−24.053	−1.914	−7.906	−0.118	−25.630	−36.144	−24.234
5	−22.311	−23.783	−21.873	−1.842	−6.031	−0.259	−24.149	−32.430	−22.409
6	−20.835	−22.622	−19.935	−2.921	−4.837	−1.686	−23.875	−29.544	−22.035
7	−18.998	−21.490	−17.832	−3.797	−3.278	−3.206	−23.030	−26.410	−21.561
8	−17.666	−20.516	−16.506	−5.214	−2.990	−4.875	−23.200	−24.767	−21.960

TABLE 15. Differences in log predictive score between the MC estimator and the normal approximation for the sample 2001Q1–2014Q4.

	Δy			π			Δy & π		
h	SW	SWFF	SWU	SW	SWFF	SWU	SW	SWFF	SWU
-1	0.012	0.015	0.008	0.060	−0.042	0.025	0.030	−0.024	0.055
0	0.292	0.195	0.617	0.094	0.028	0.073	0.375	0.162	0.688
1	0.498	0.444	0.485	0.358	0.153	0.265	0.885	0.500	0.852
2	0.474	0.442	0.444	0.486	0.329	0.320	1.004	0.668	0.900
3	0.502	0.458	0.474	0.473	0.414	0.297	1.002	0.790	0.890
4	0.529	0.481	0.496	0.444	0.508	0.253	0.974	0.899	0.846
5	0.497	0.471	0.467	0.394	0.559	0.218	0.908	1.005	0.804
6	0.513	0.489	0.468	0.349	0.597	0.187	0.906	1.080	0.792
7	0.513	0.493	0.472	0.272	0.644	0.122	0.827	1.151	0.719
8	0.477	0.512	0.451	0.227	0.664	0.082	0.752	1.213	0.669

Figure 1. Quarterly euro area data on the observable variables of the SW, SWFF, and SWU models using Update 14 of the AWM database for the sample 1980Q1–2013Q4.



Notes: The seven observed variables of the SW model are given by all variable except for the spread and the unemployment rate. The SWFF models adds the spread to the set of observables, while the SWU model instead includes the unemployment rate.

FIGURE 2. Marginal posterior densities of the parameters of the SW model based on the sample 1985Q1–2013Q4 for the euro area.

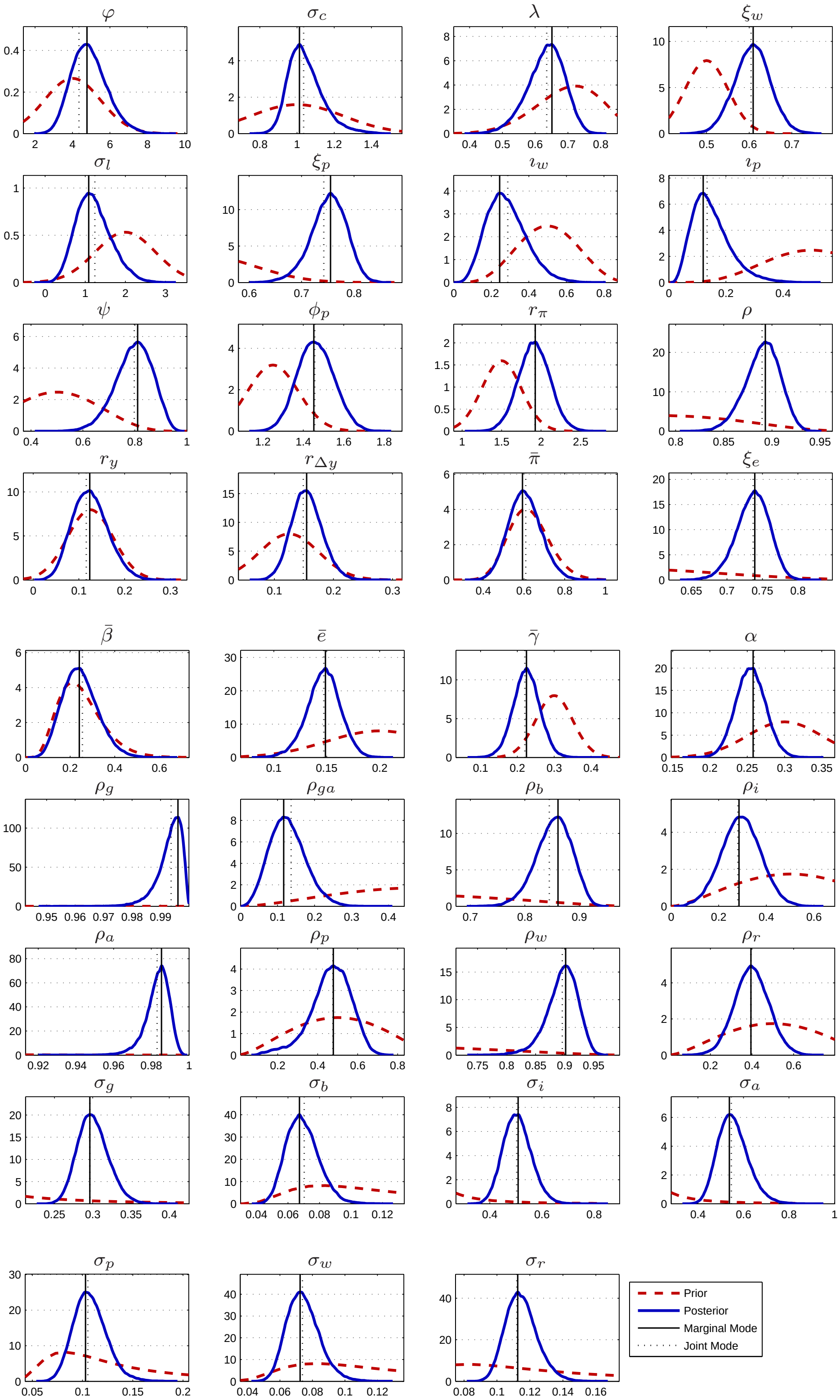


FIGURE 3. Marginal posterior densities of the parameters of the SWFF model based on the sample 1985Q1–2013Q4 for the euro area.

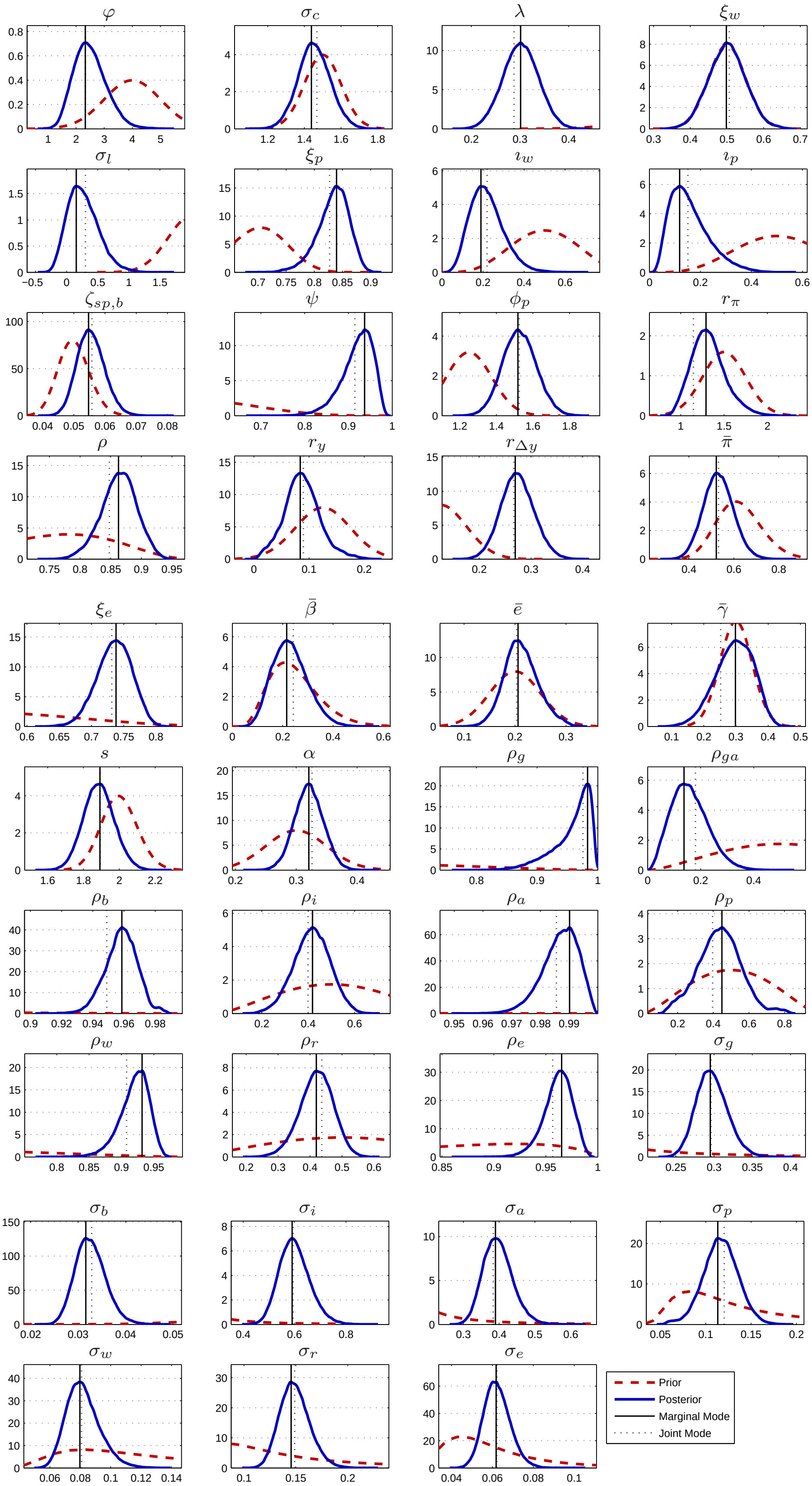


Figure 4. Marginal posterior densities of the parameters of the SWU model based on the sample 1985Q1–2013Q4 for the euro area.

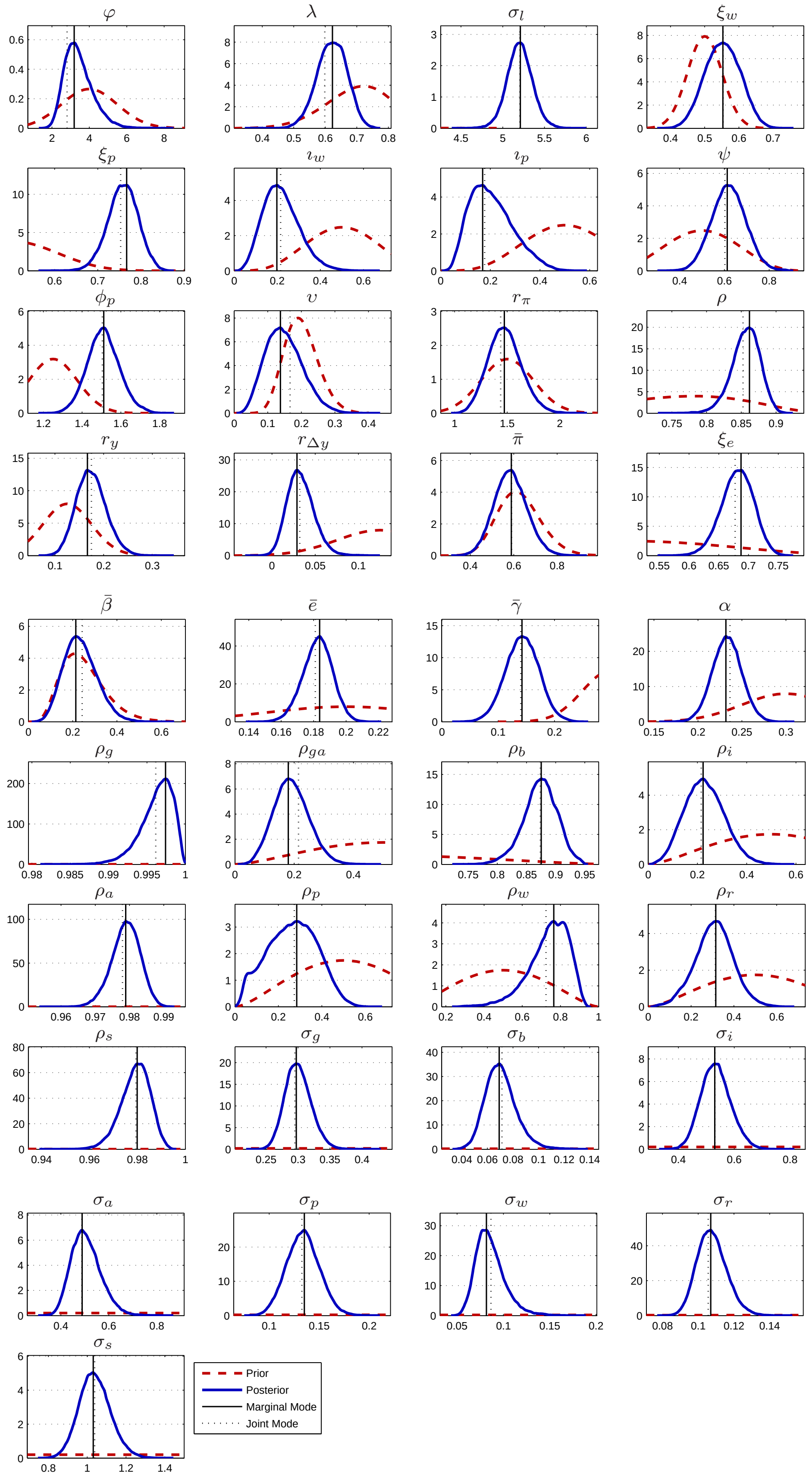


FIGURE 5. Scaled log-likelihood and log-posterior of the parameters of the SW model around the mode of the joint log-posterior based on the sample 1985Q1–2013Q4 for the euro area.

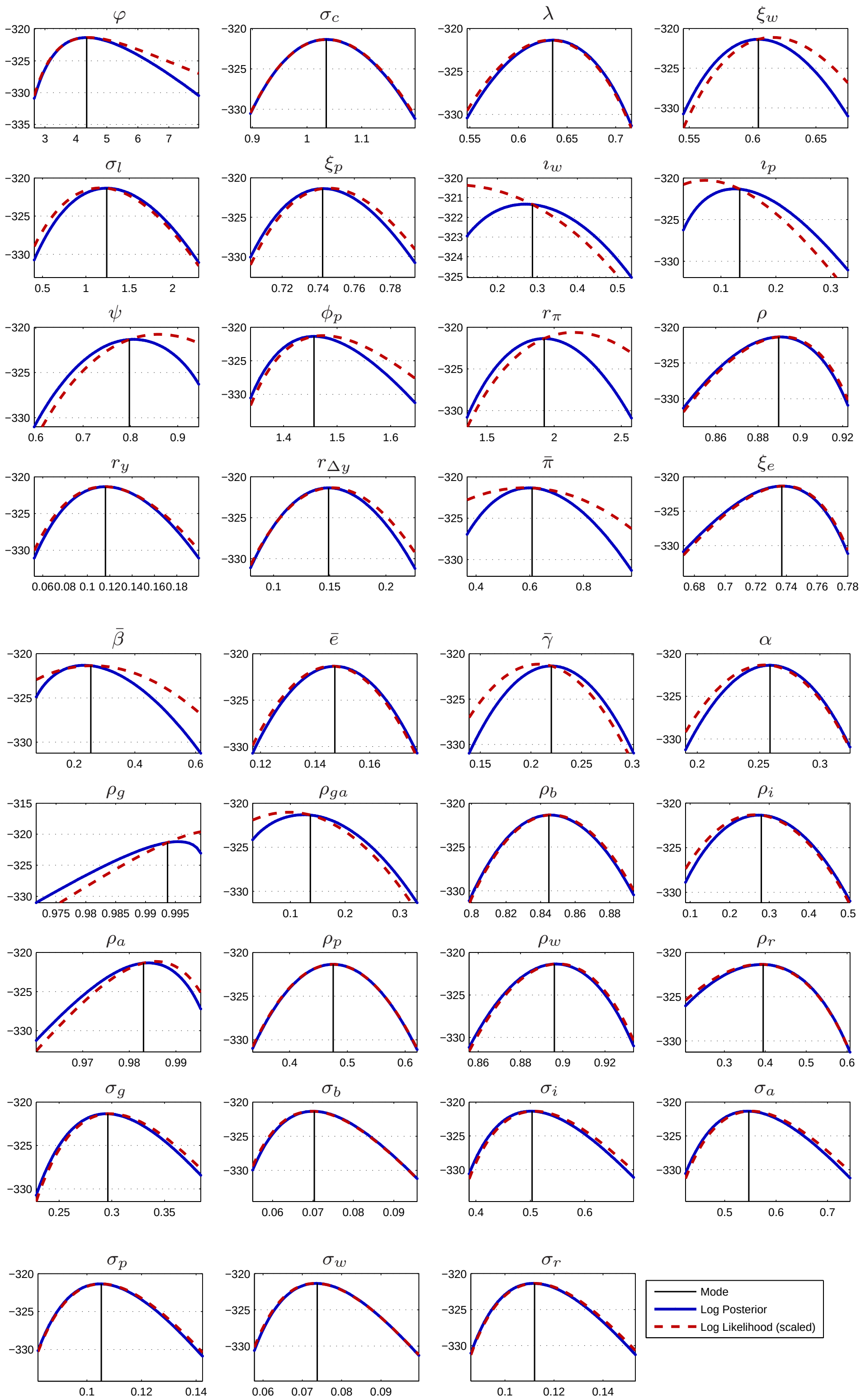


FIGURE 6. Scaled log-likelihood and log-posterior of the parameters of the SWFF model around the mode of the joint log-posterior based on the sample 1985Q1–2013Q4 for the euro area.

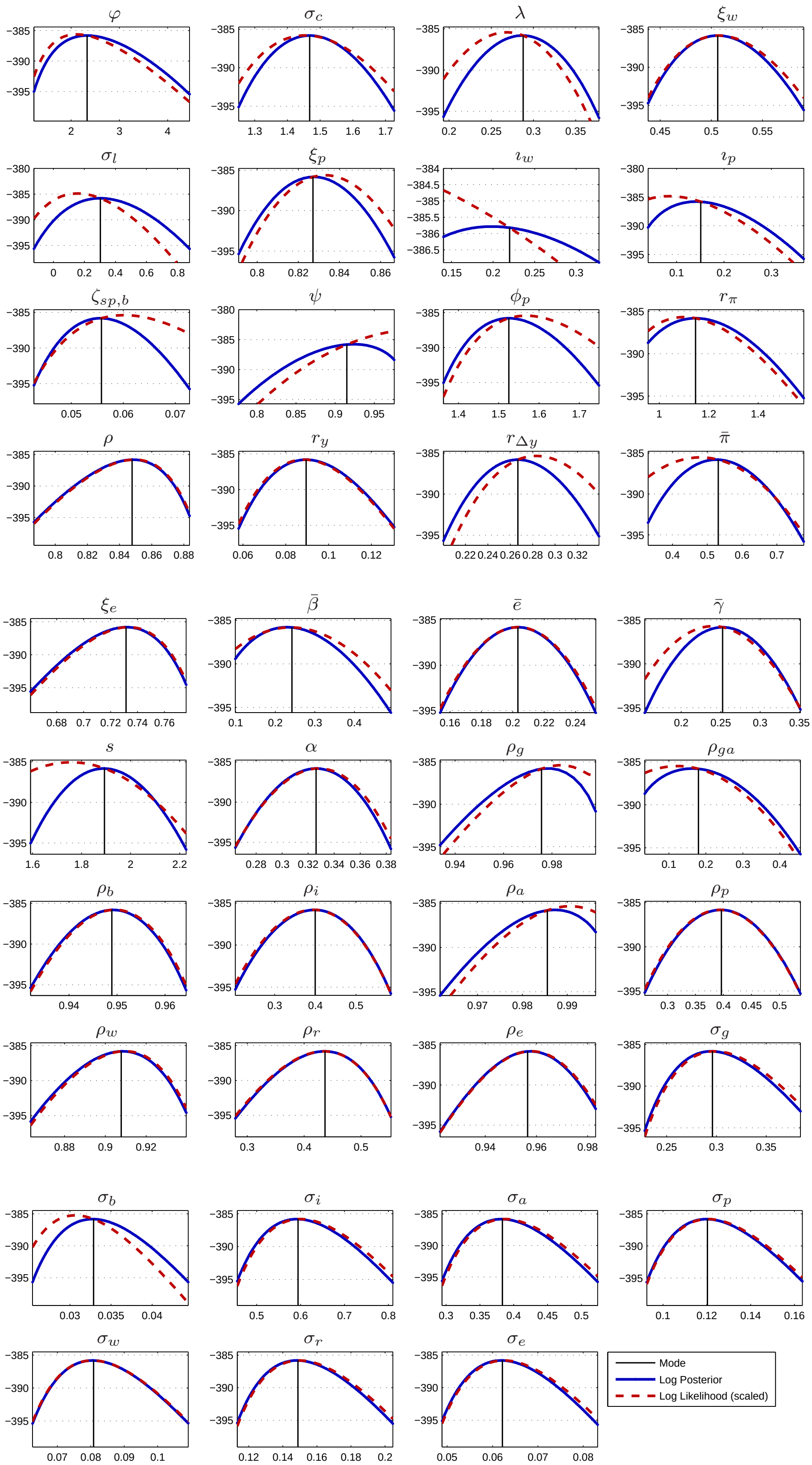


FIGURE 7. Scaled log-likelihood and log-posterior of the parameters of the SWU model around the mode of the joint log-posterior based on the sample 1985Q1–2013Q4 for the euro area.

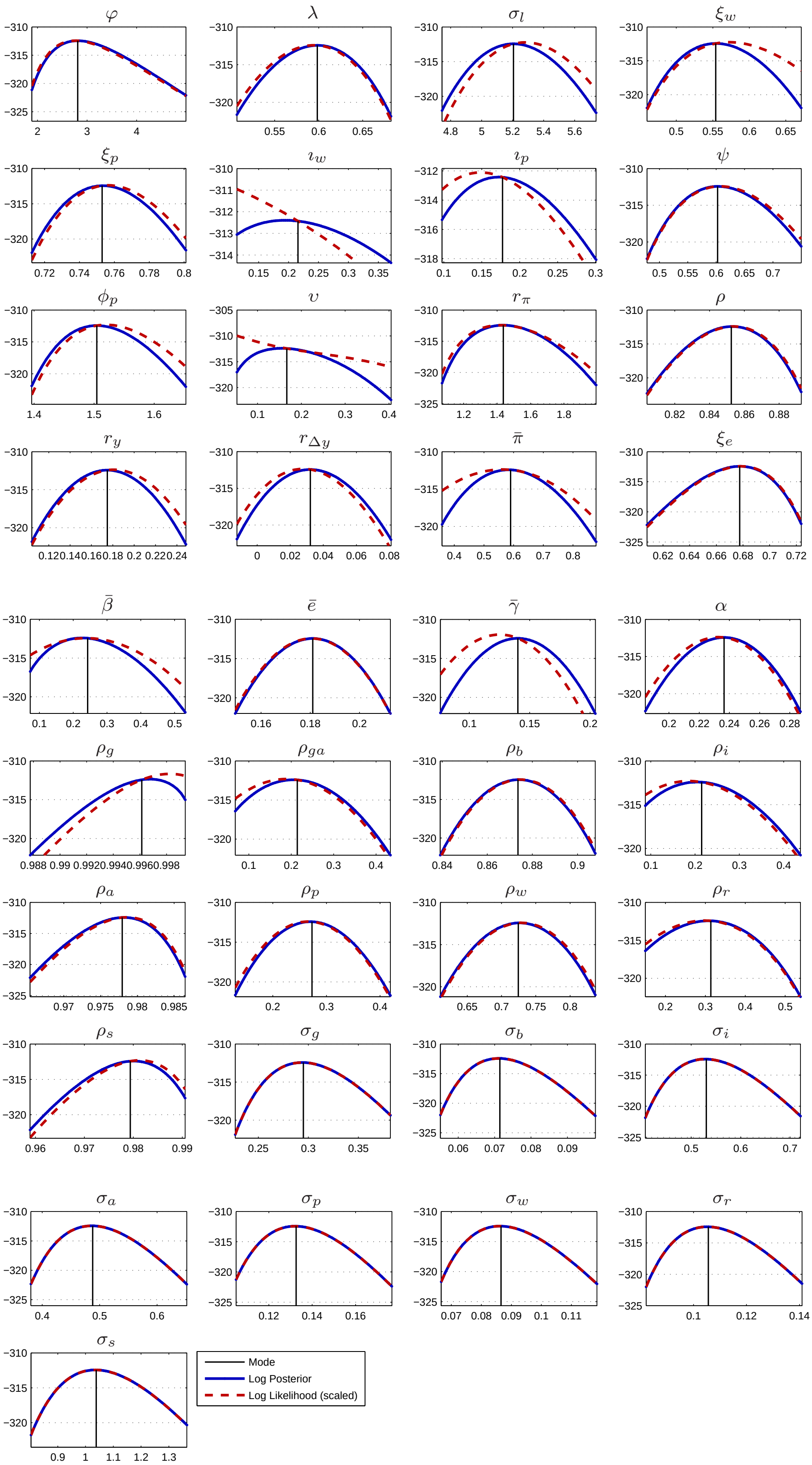


FIGURE 8. Impulse response functions in levels of the observables from an exogenous spending shock with 90 percent equal tails credible bands.

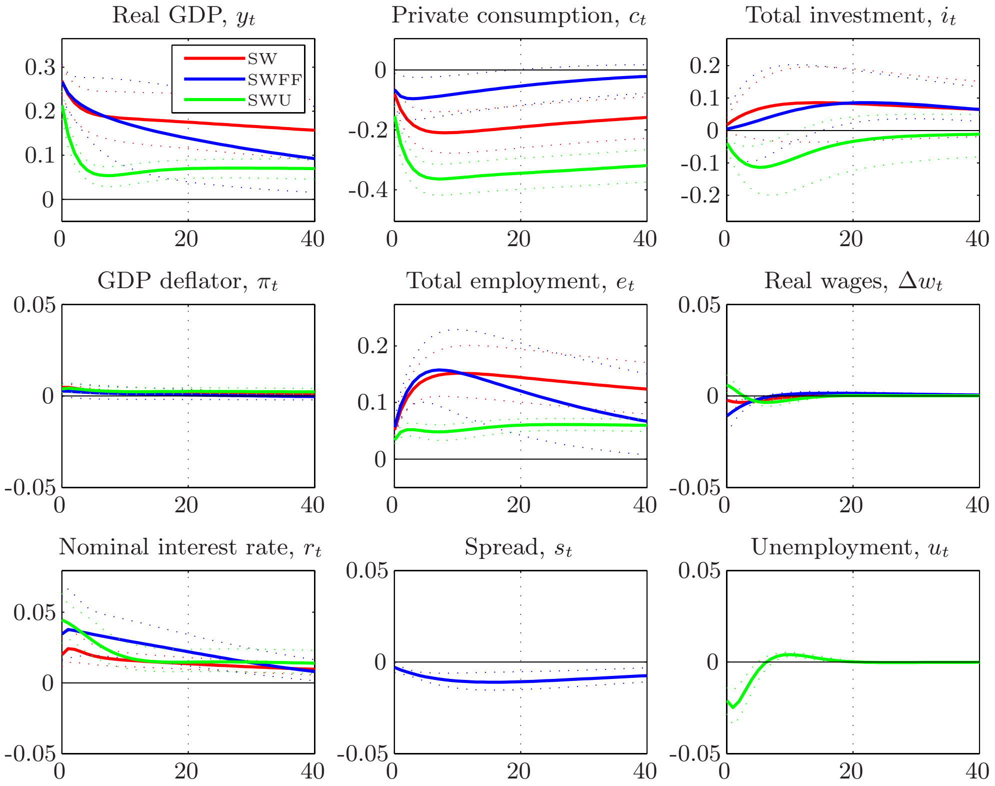


FIGURE 9. Impulse response functions in levels of the observables from a preference shock with 90 percent equal tails credible bands.

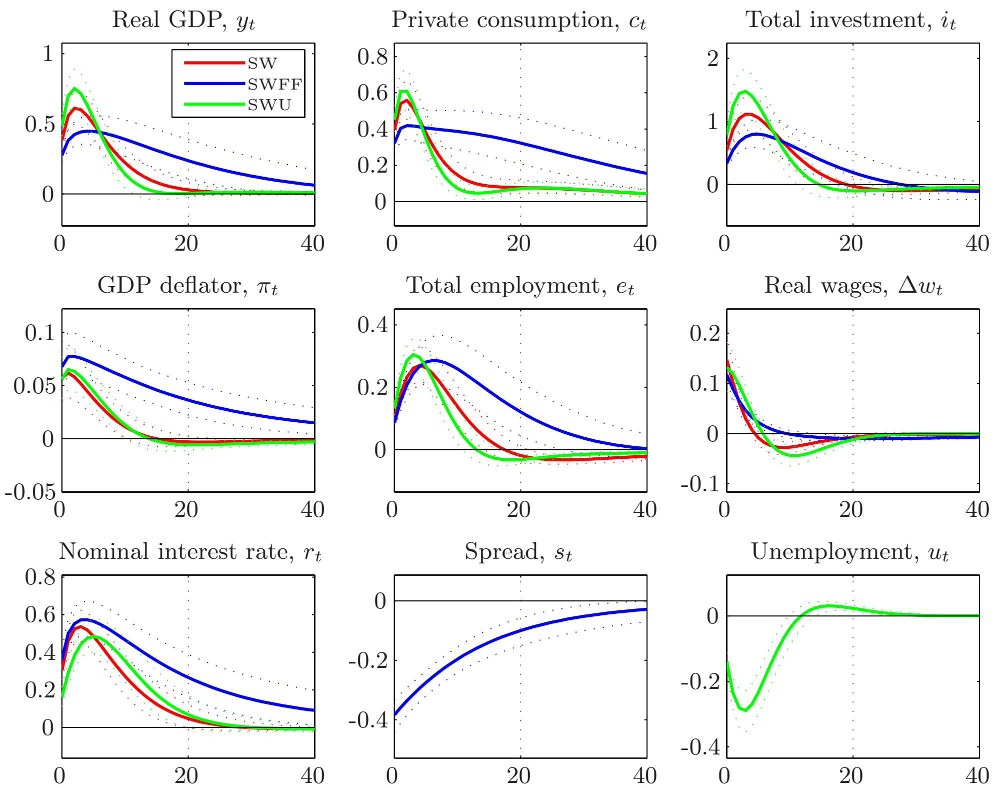


Figure 10. Impulse response functions in levels of the observables from an investment-specific technology shock with 90 percent equal tails credible bands.

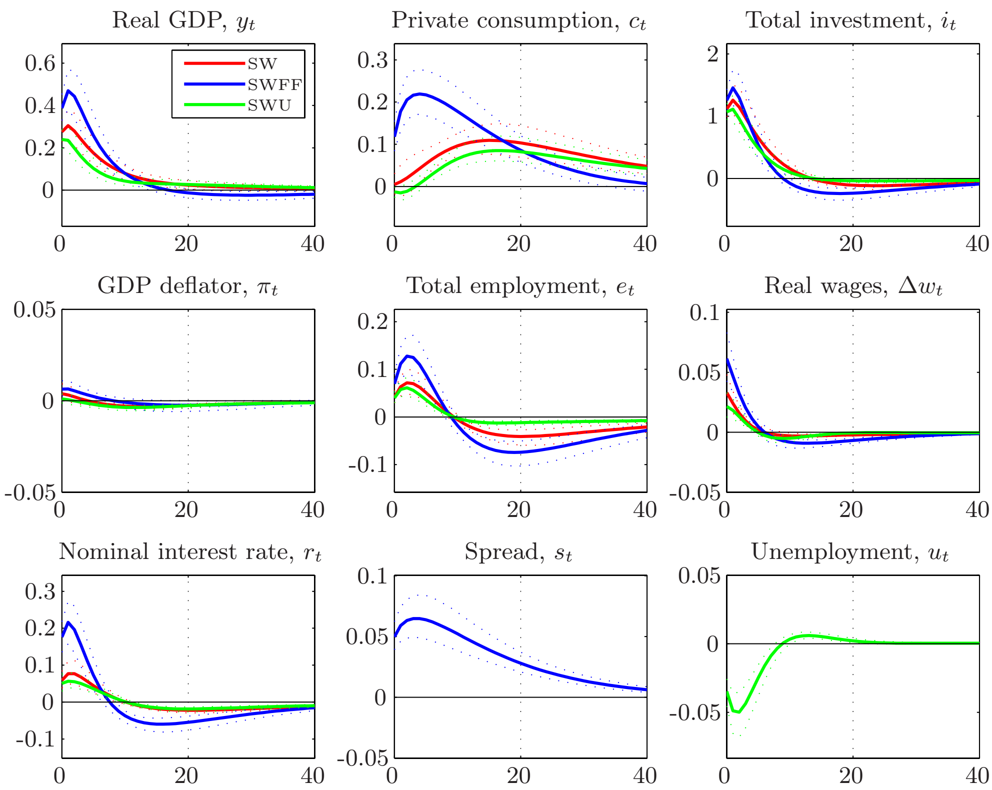


Figure 11. Impulse response functions in levels of the observables from a monetary policy shock with 90 percent equal tails credible bands.

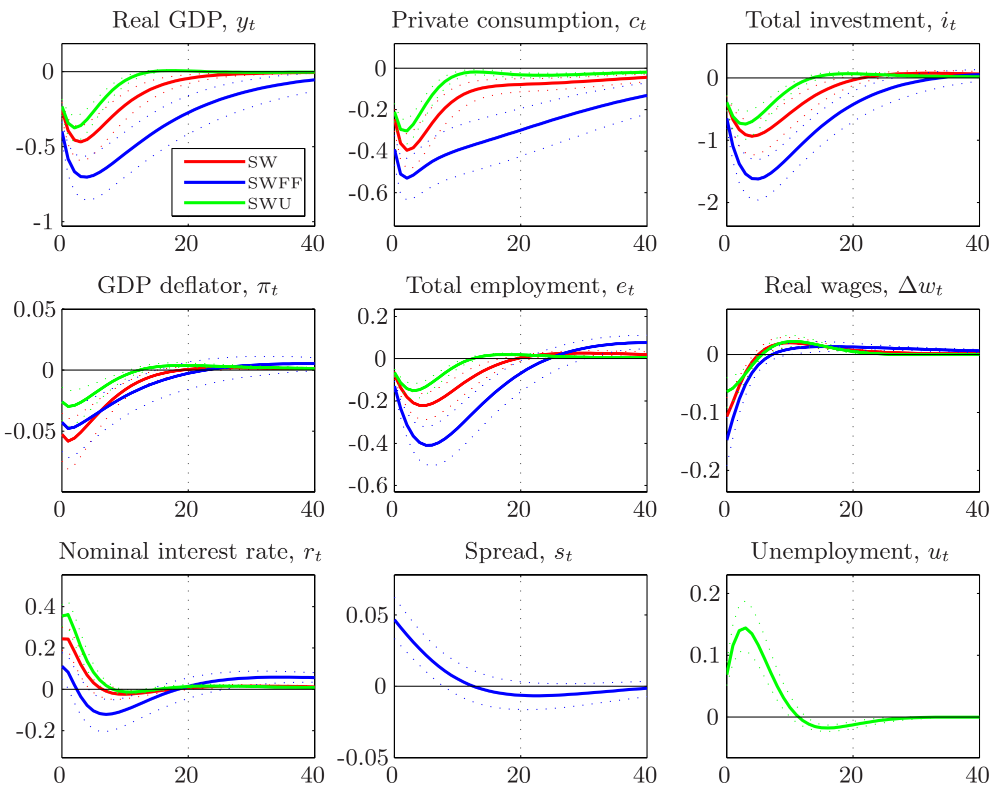


FIGURE 12. Impulse response functions in levels of the observables from a spread shock with 90 percent equal tails credible bands.

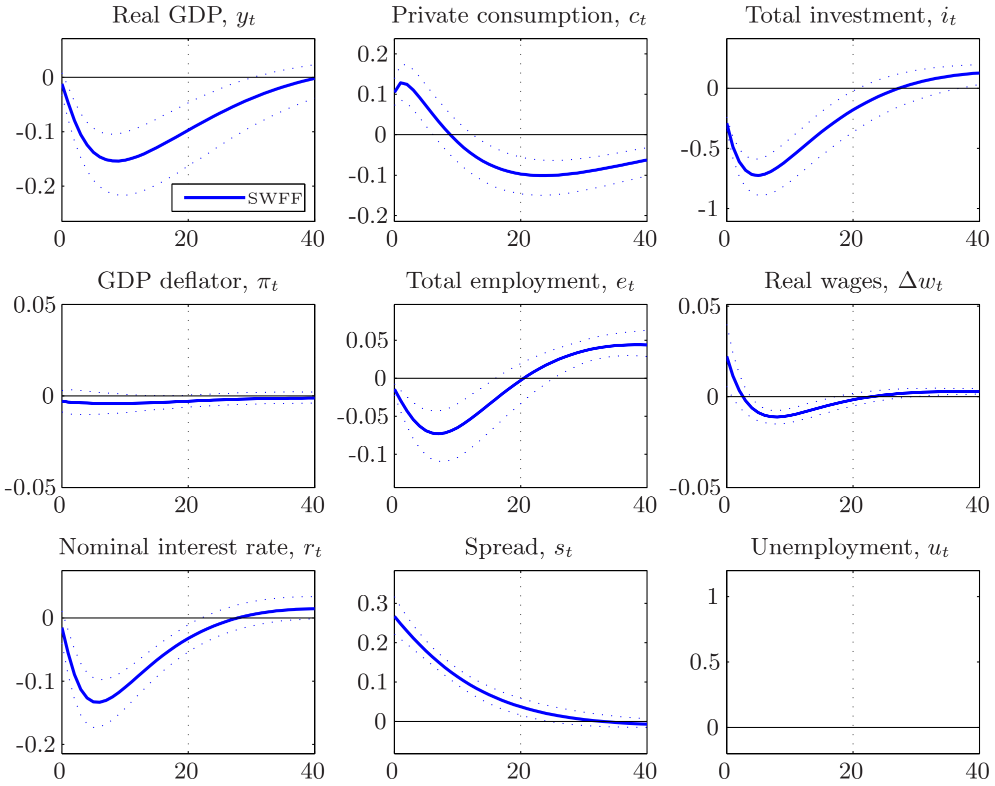


FIGURE 13. Impulse response functions in levels of the observables from a TFP shock with 90 percent equal tails credible bands.

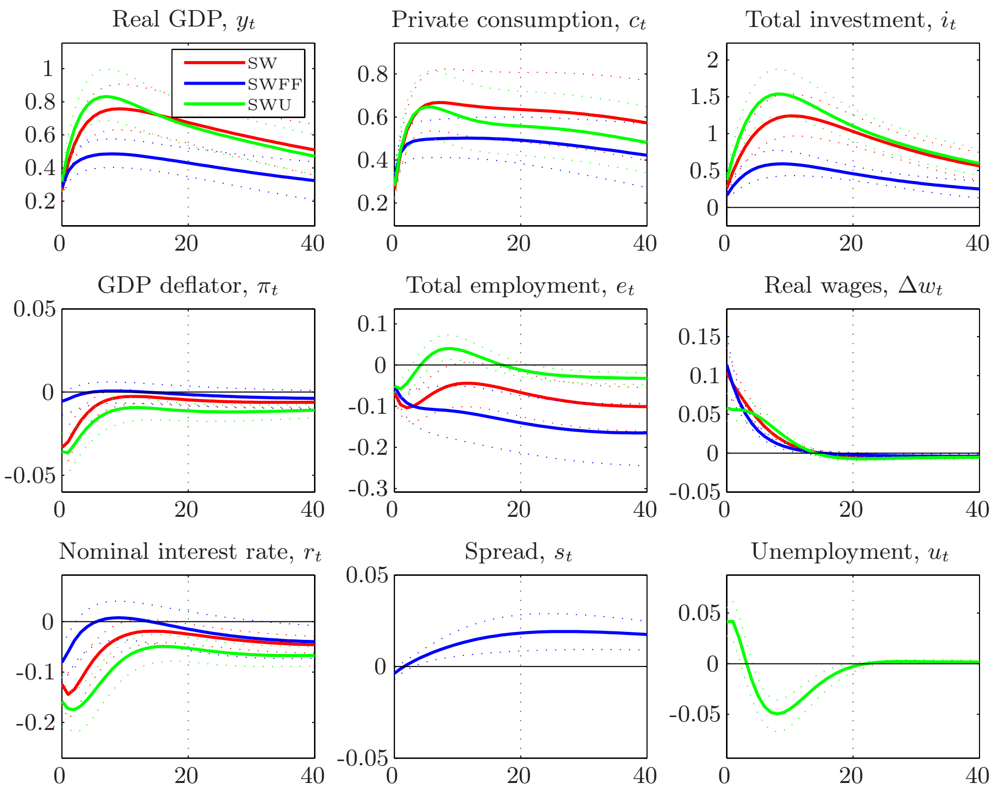


FIGURE 14. Impulse response functions in levels of the observables from a wage markup shock with 90 percent equal tails credible bands.

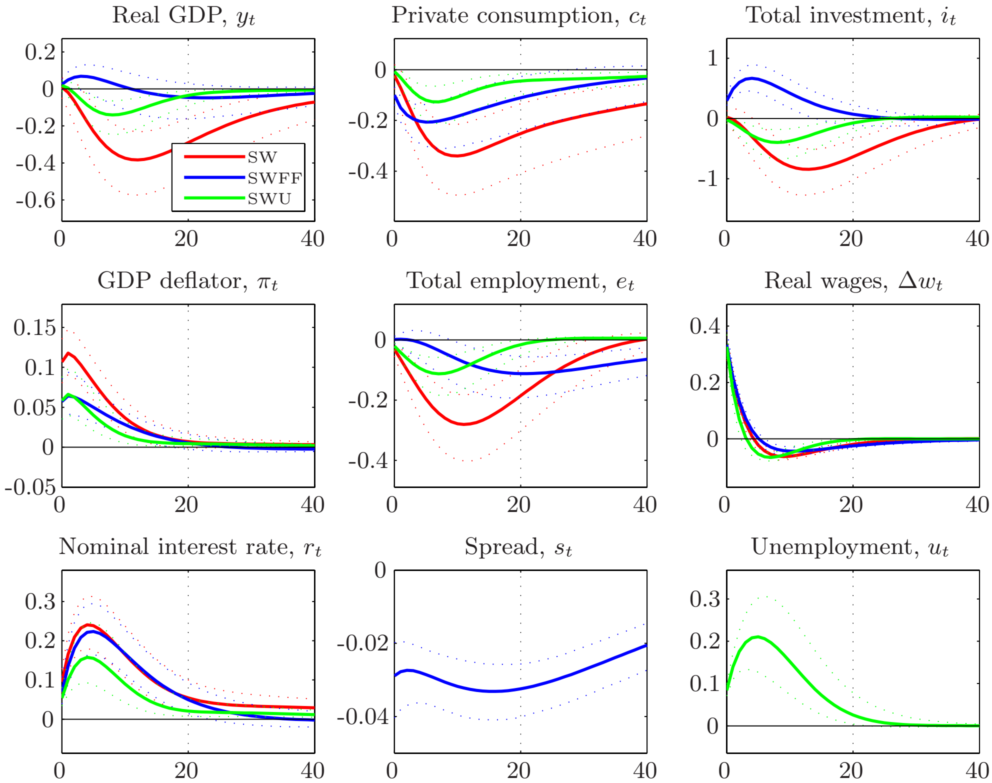


FIGURE 15. Impulse response functions in levels of the observables from a price markup shock with 90 percent equal tails credible bands.

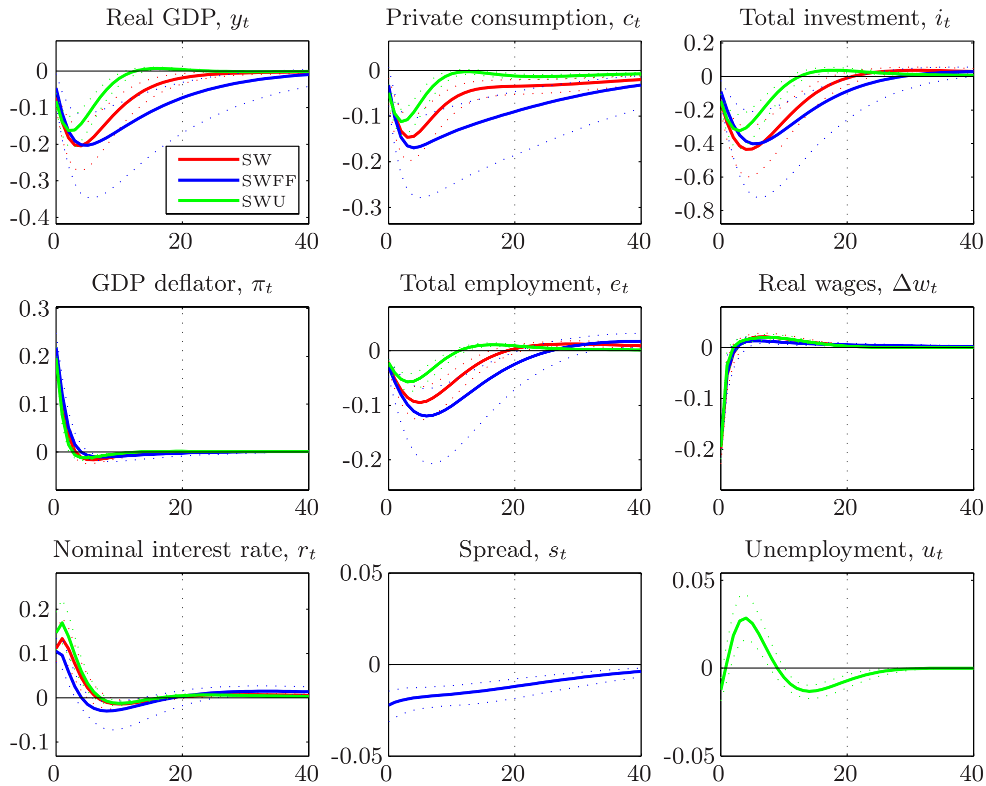


Figure 16. Impulse response functions in levels of the observables from a labor supply shock with 90 percent equal tails credible bands.

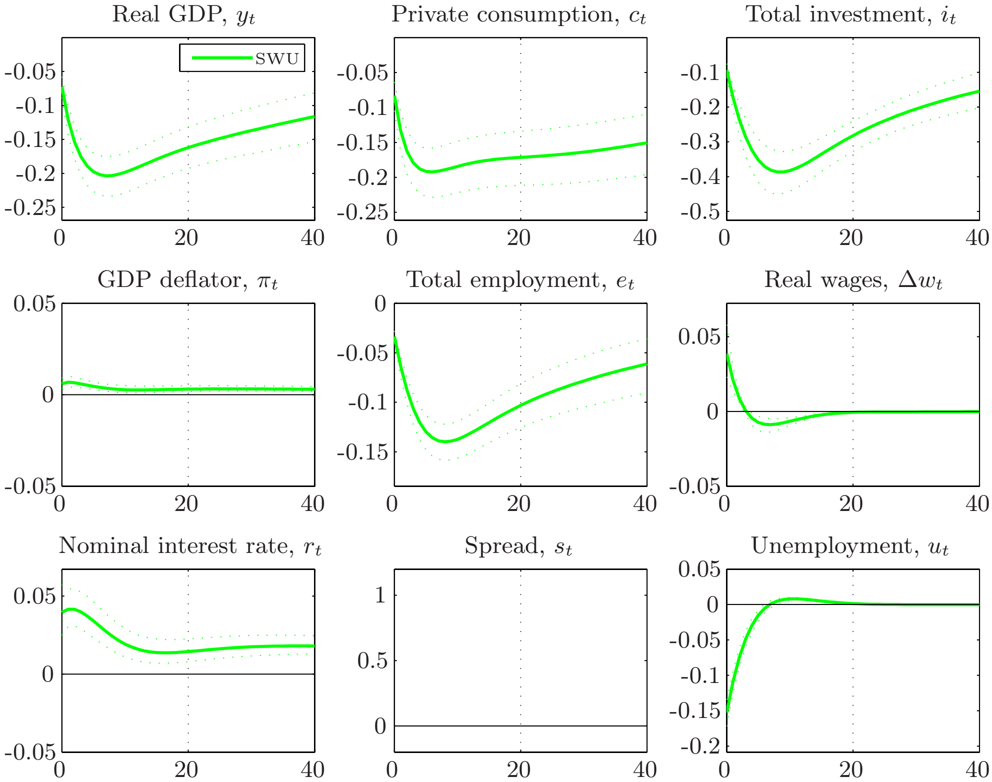


Figure 17. Euro area RTD vintages 2001Q1–2015Q4 extended back in time using AWM database annual vintages.

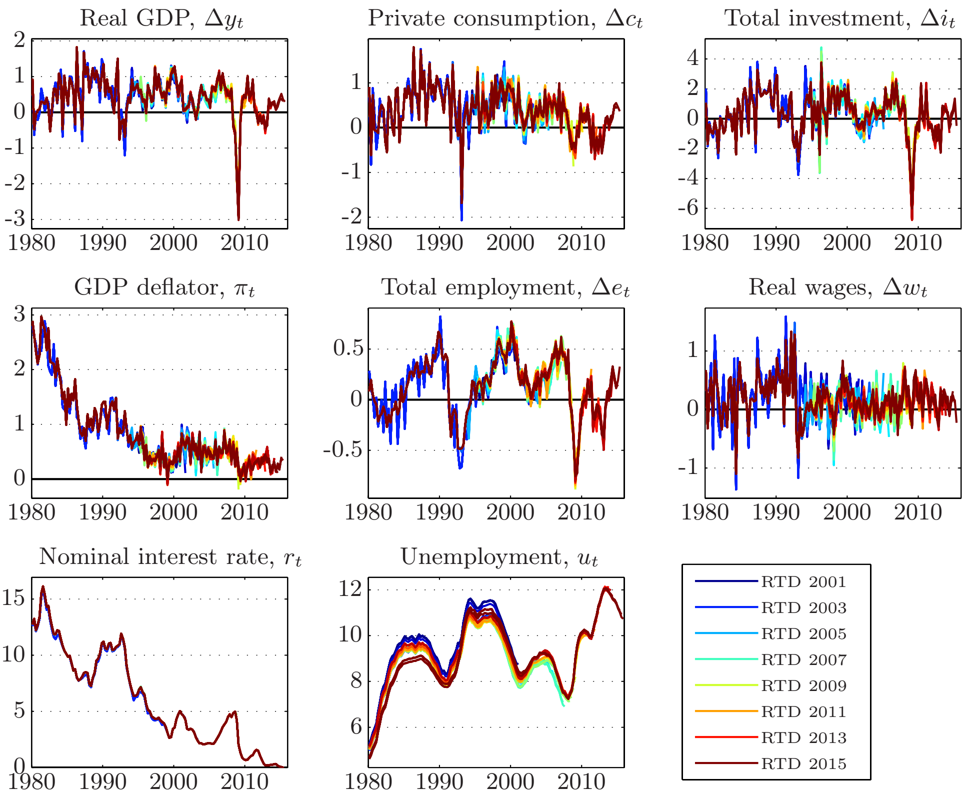
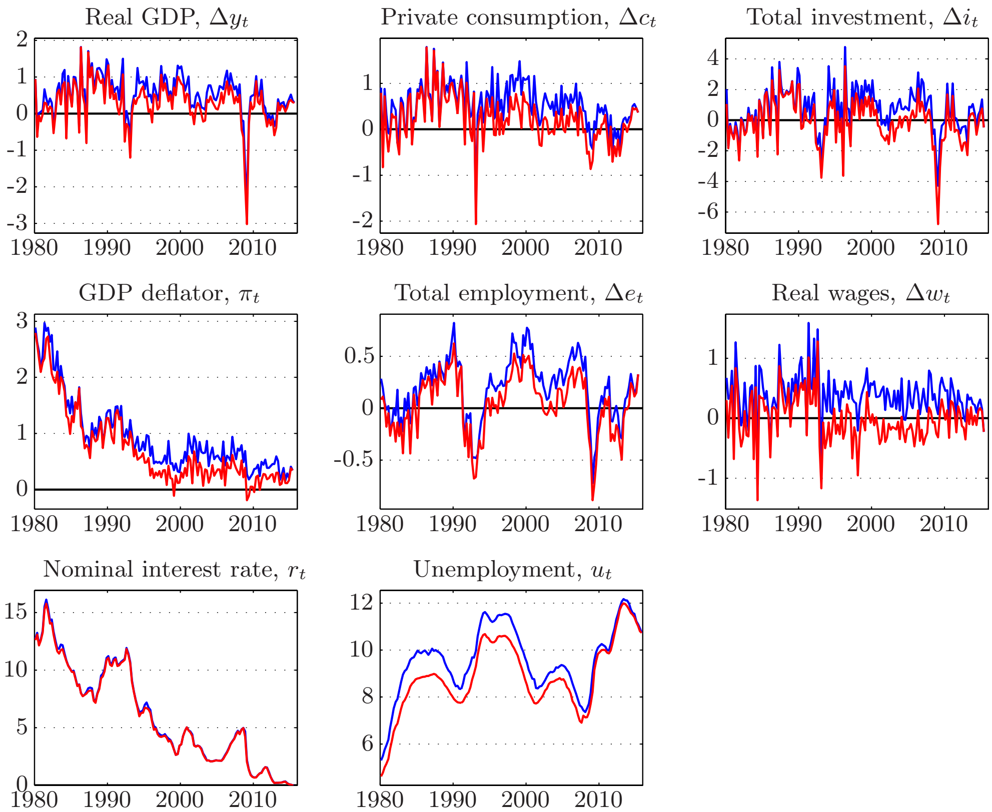


FIGURE 18. Maximum and minimum values and their difference of euro area RTD vintages 2001Q1–2015Q4 extended back in time using AWM database annual vintages.

A. MAXIMUM AND MINIMUM VALUES



B. DIFFERENCE BETWEEN MAXIMUM AND MINIMUM VALUES

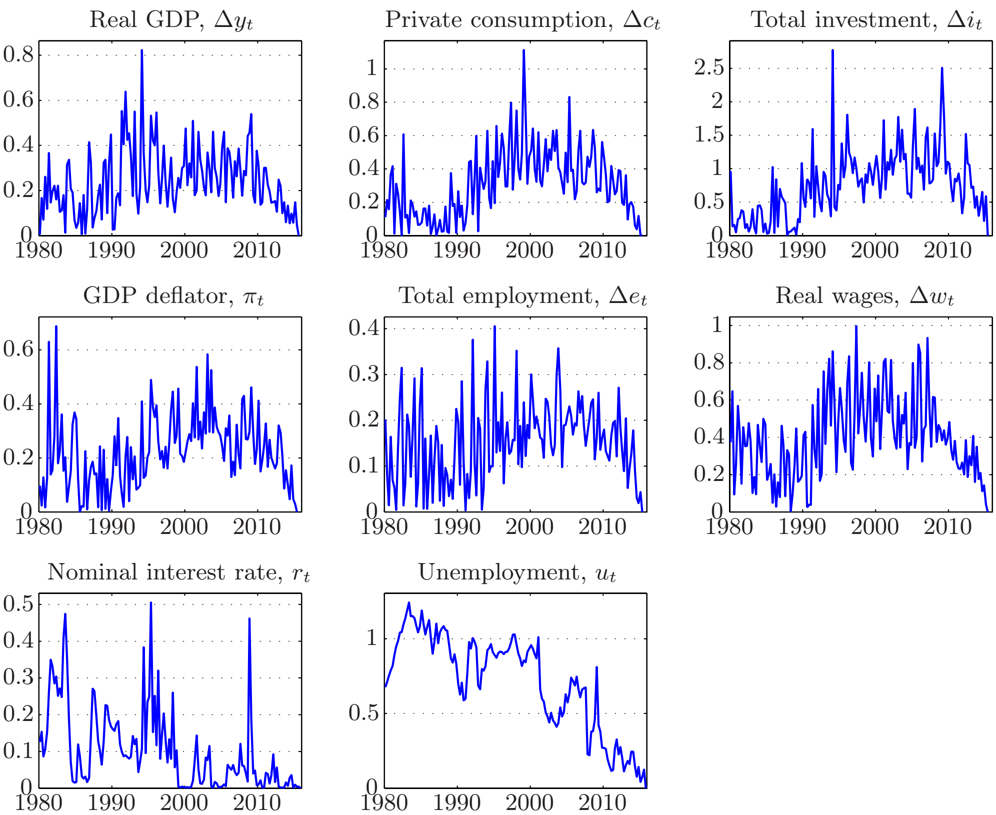
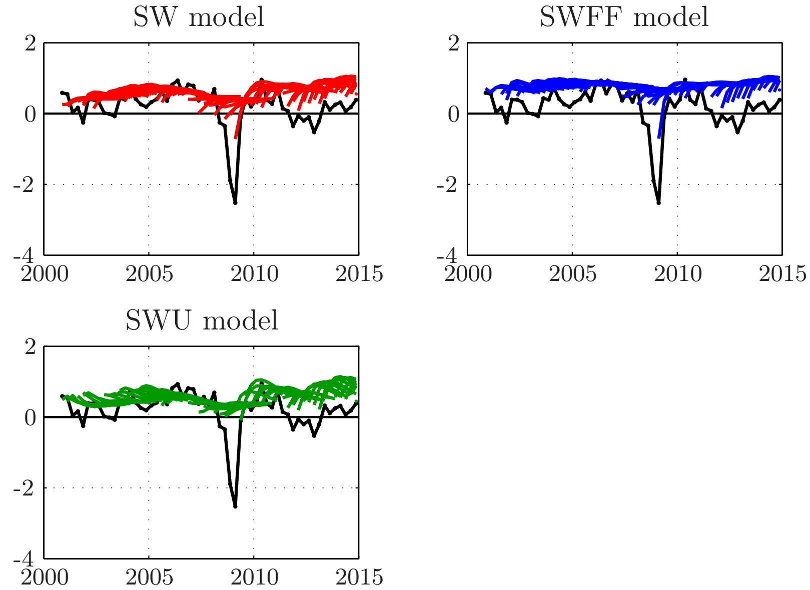


Figure 19. Recursive point forecasts of real GDP growth and GDP deflator inflation using the RTD vintages 2001Q1–2014Q4 along with the actual values.

A. Real GDP Growth



B. GDP Deflator Inflation

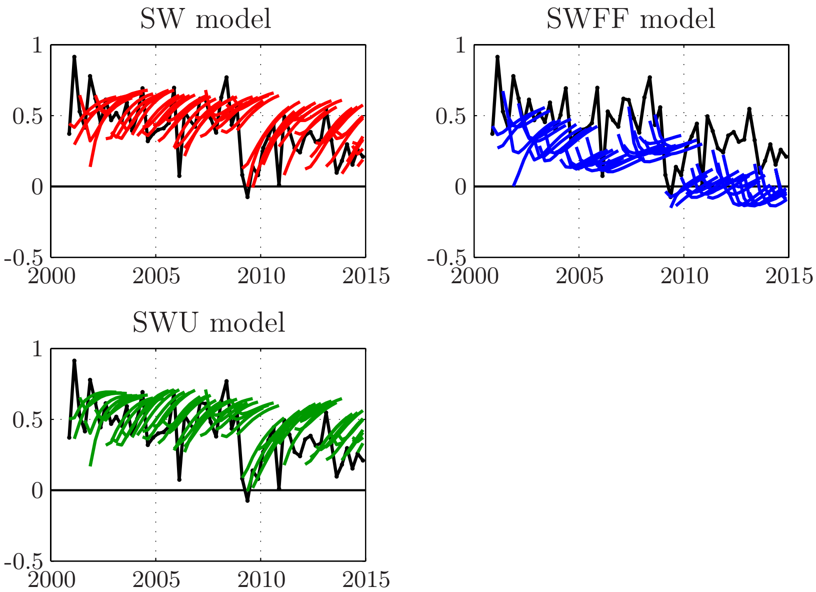
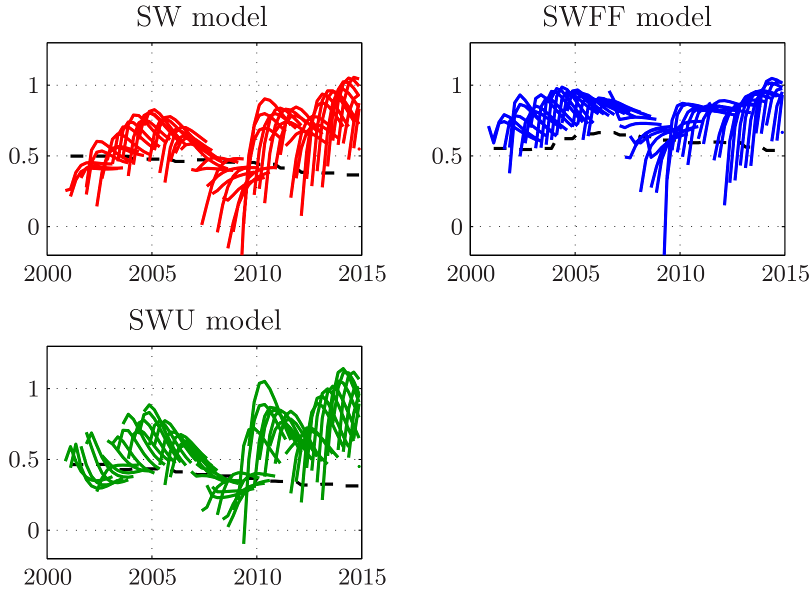


Figure 20. Recursive point forecasts of real GDP growth and GDP deflator inflation using the RTD vintages 2001Q1–2014Q4 along with the recursively estimated posterior mean values.

A. Real GDP Growth



B. GDP Deflator Inflation

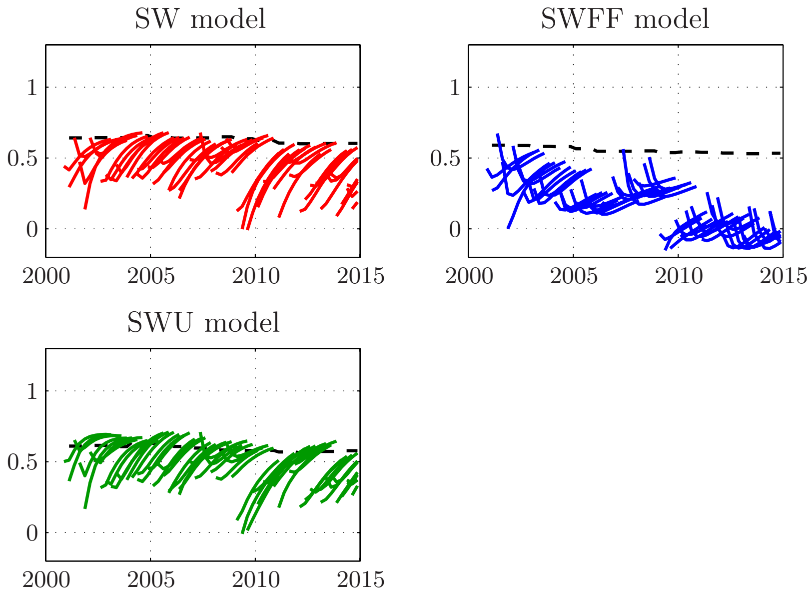
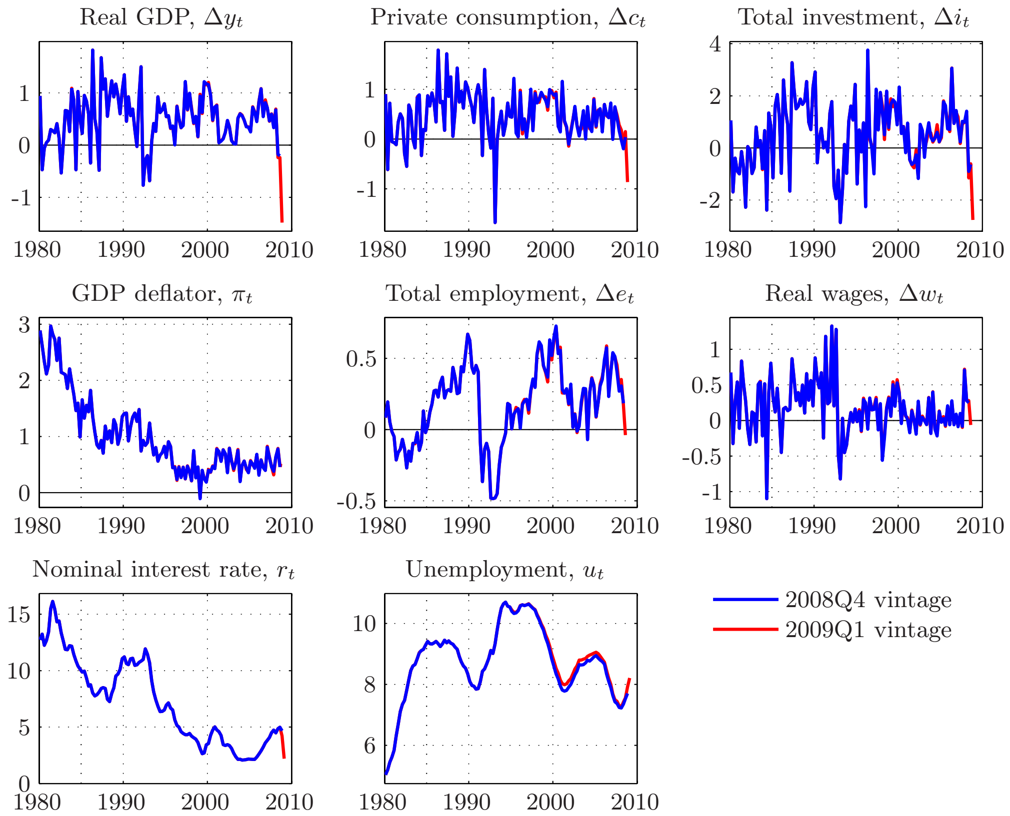


Figure 21. Quarterly euro area data on the observable variables of the SW and SWU models for two vintages and their revisions.

A. Real-time Data 2008Q4 and 2009Q1



B. Revisions in 2009Q1

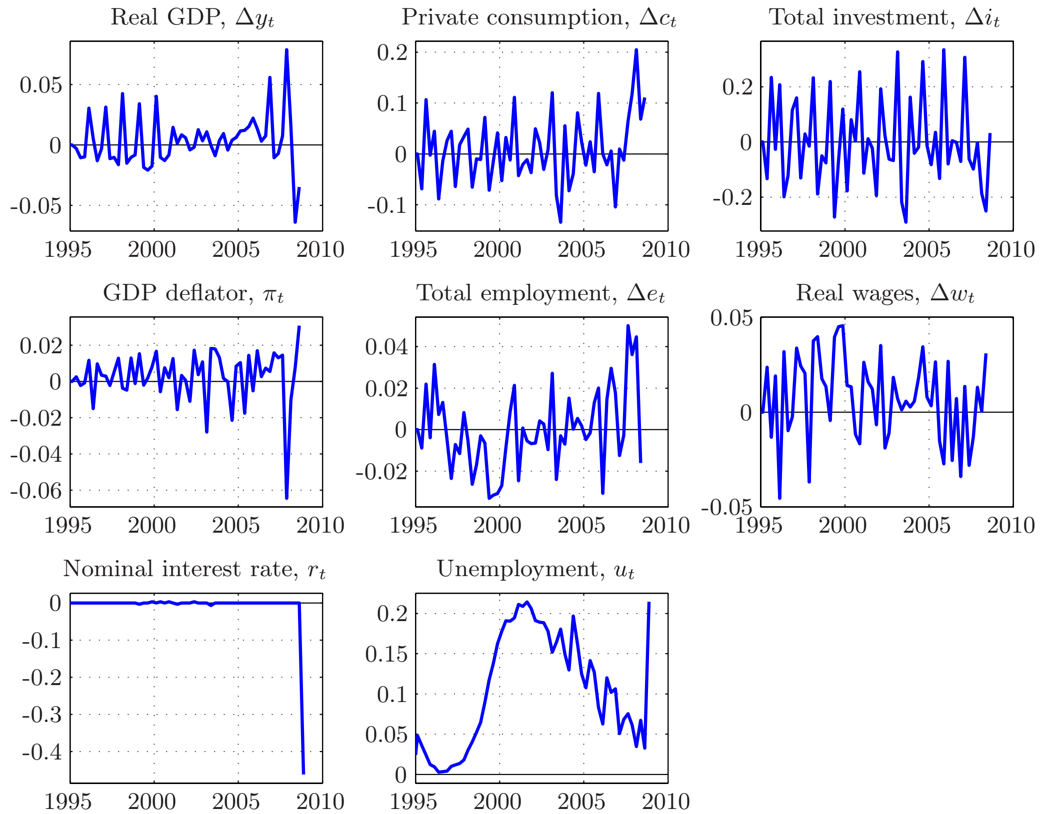
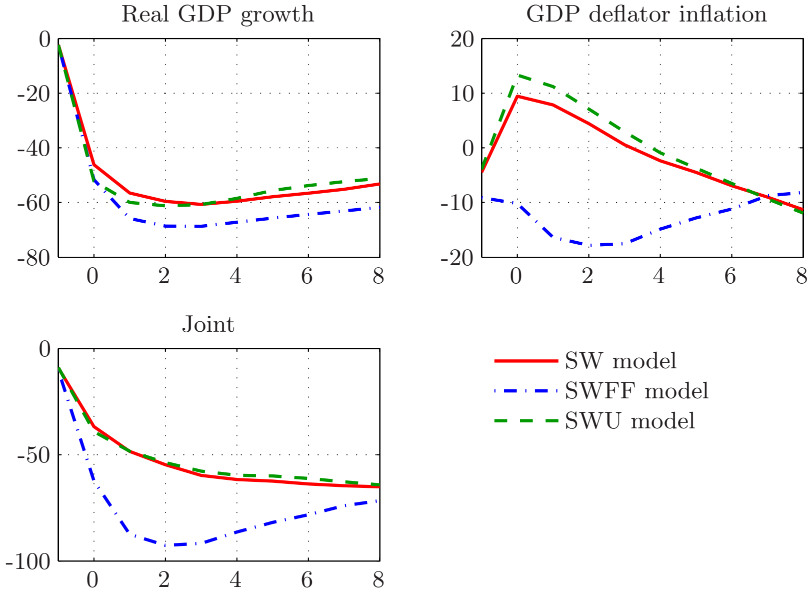
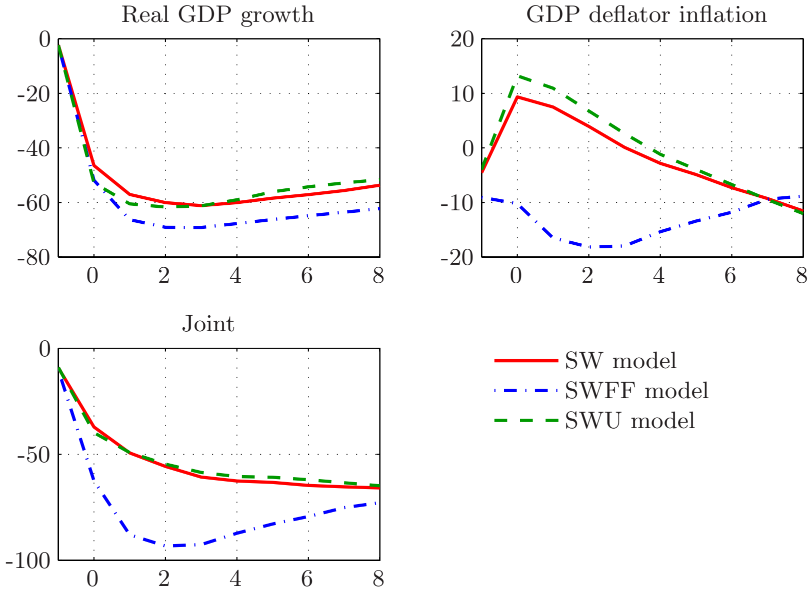


FIGURE 22. Log predictive score for real GDP growth and inflation based on backcasts, nowcasts, and one-to eight-quarter-head forecasts over the vintages 2001Q1–2014Q4.

A. MONTE CARLO INTEGRATION



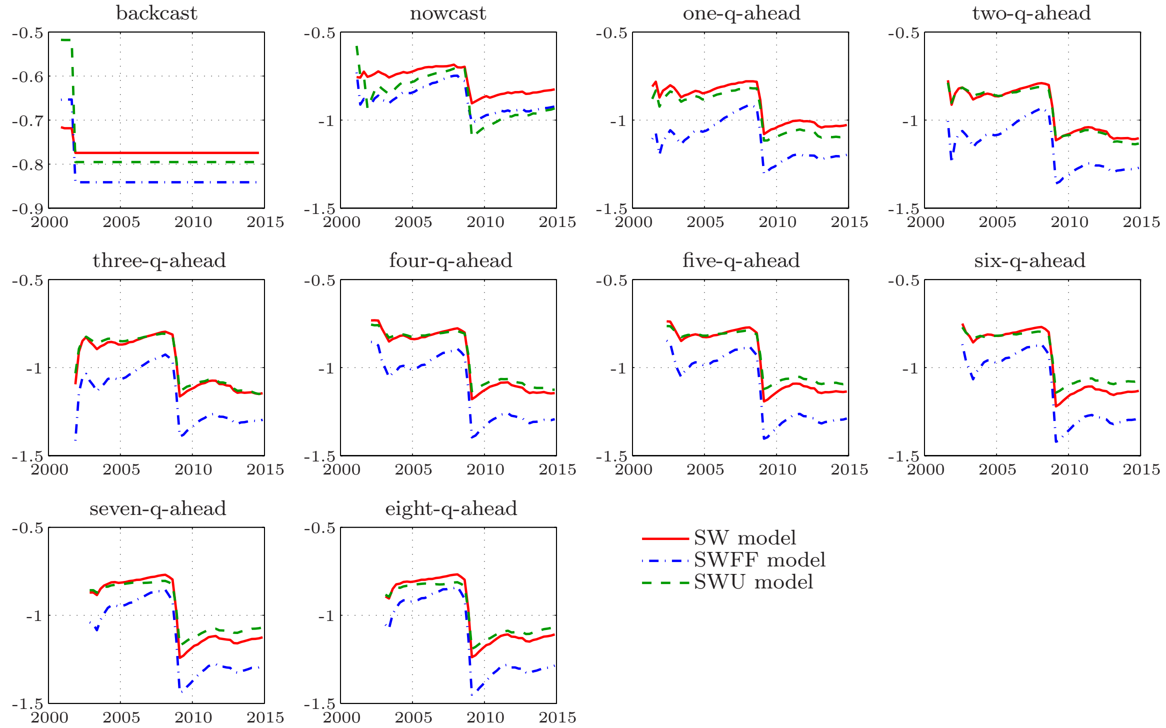
B. NORMAL APPROXIMATION



NOTES: Forecast horizons are displayed on the horizontal axis with $h = -1$ being the backcast horizon and $h = 0$ the nowcast horizon.

FIGURE 23. Recursive estimates of the average log predictive score of real GDP growth and inflation using the MC estimator for the vintages 2001Q1–2014Q4.

A. REAL GDP GROWTH



B. GDP DEFLATOR INFLATION

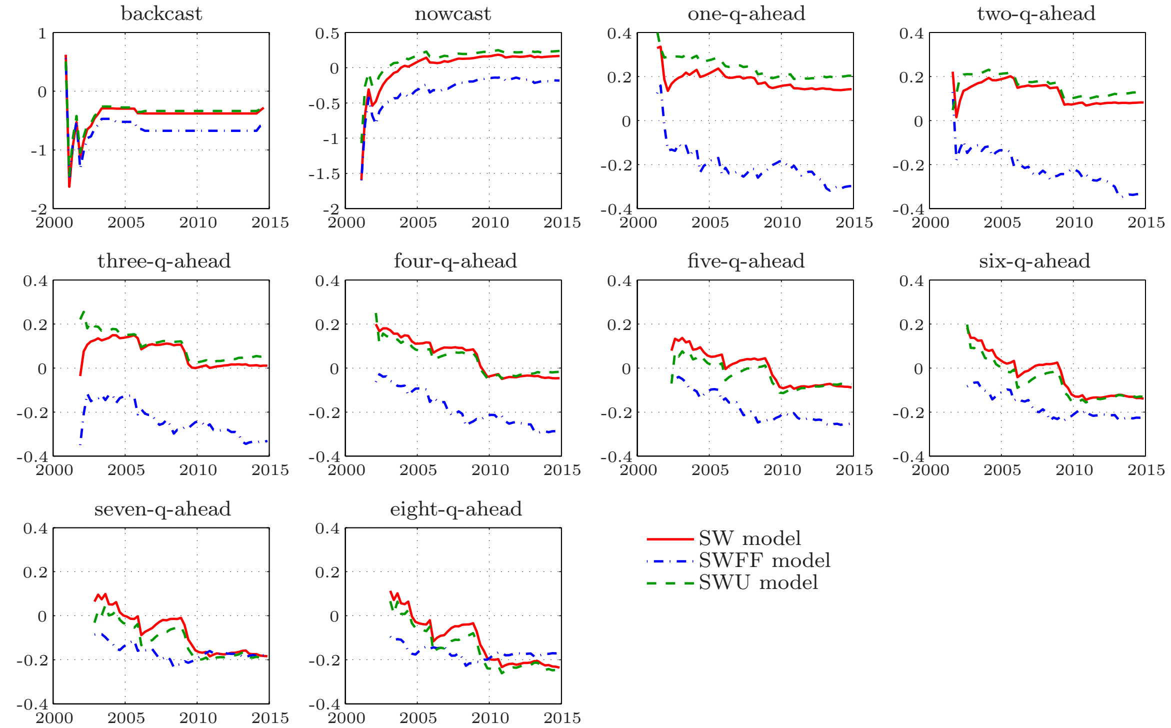


Figure 23. Recursive estimates of the average log predictive score of real GDP growth and inflation using the MC estimator for the vintages 2001Q1–2014Q4 (continued).

C. Real GDP growth & Inflation

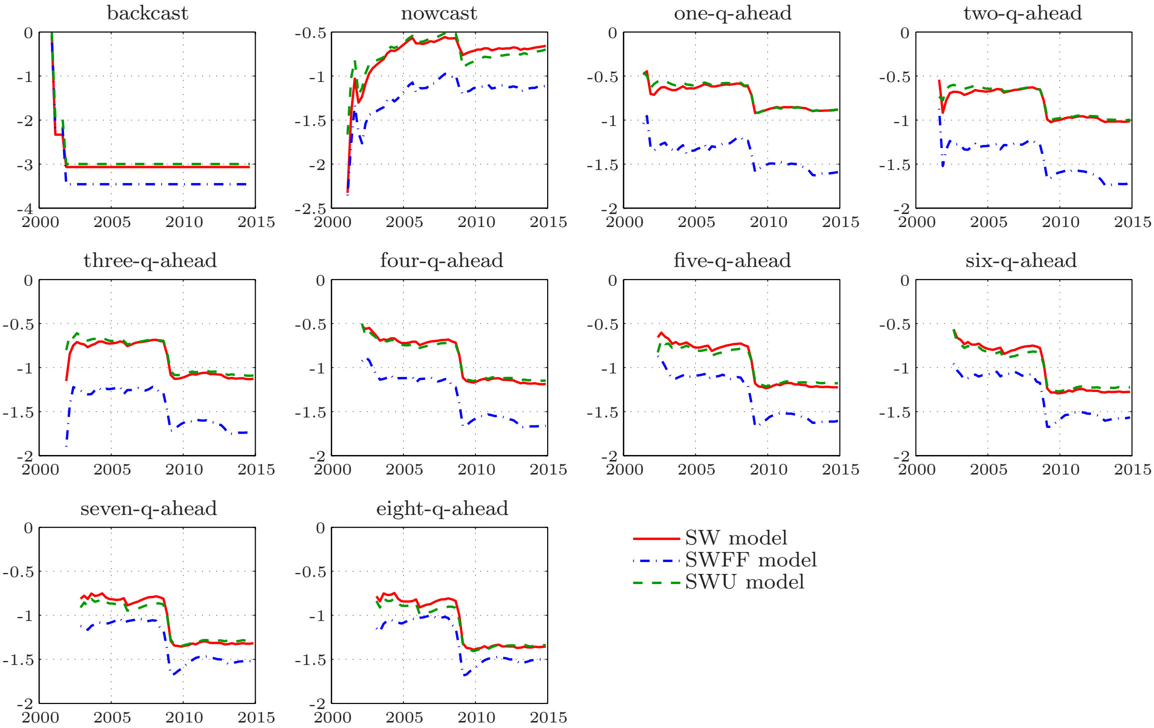


Figure 24. Recursive estimates of the average log predictive score of real GDP growth and inflation using the MC estimator for the vintages 2008Q4–2014Q4.

A. Real GDP growth

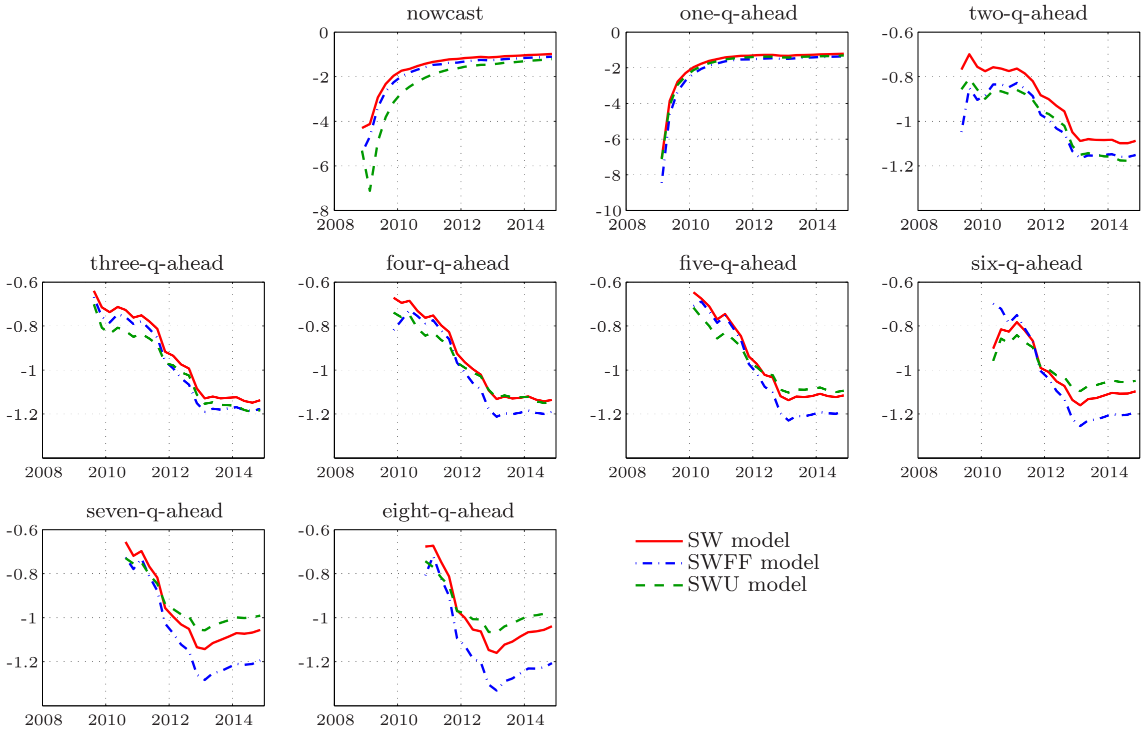
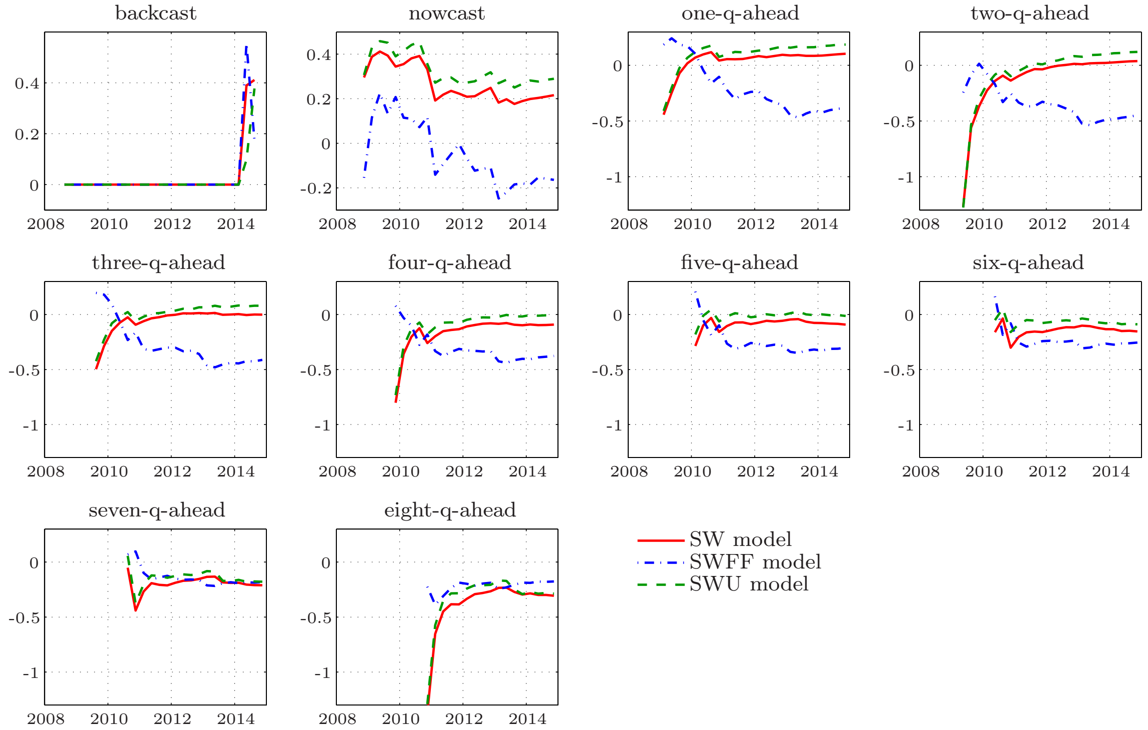


FIGURE 24. Recursive estimates of the average log predictive score of real GDP growth and inflation using the MC estimator for the vintages 2008Q4–2014Q4 (continued).

B. GDP Deflator Inflation



C. Real GDP growth & Inflation

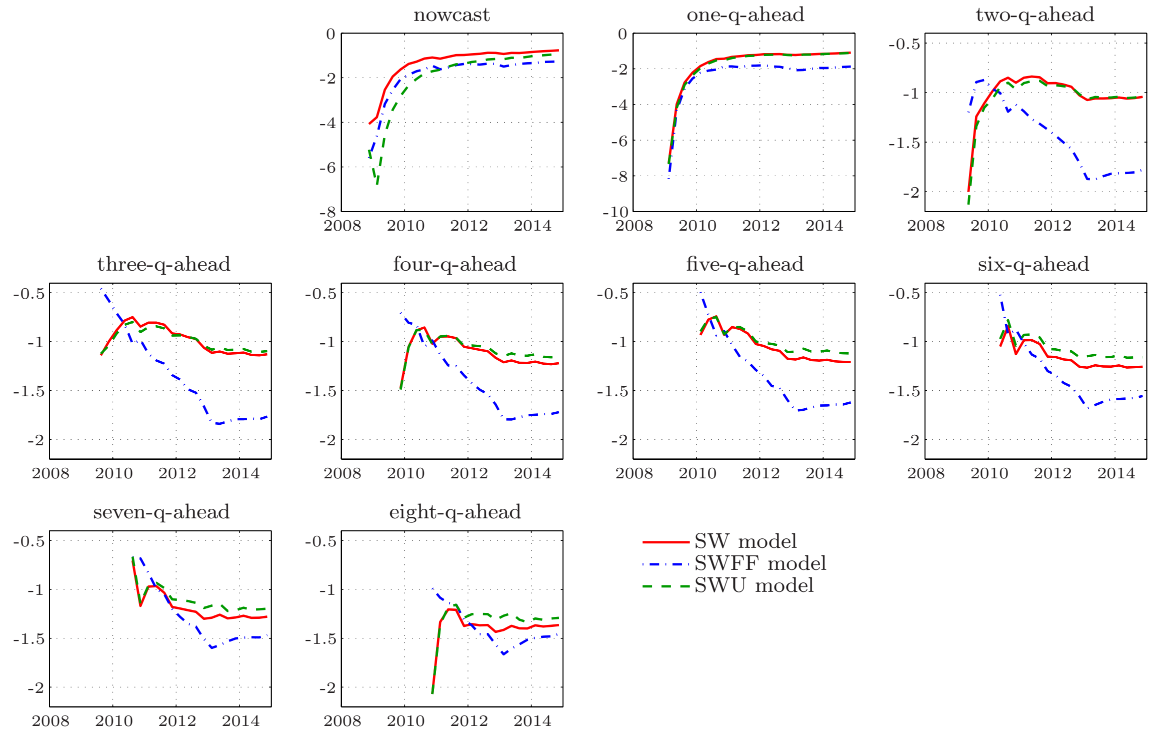
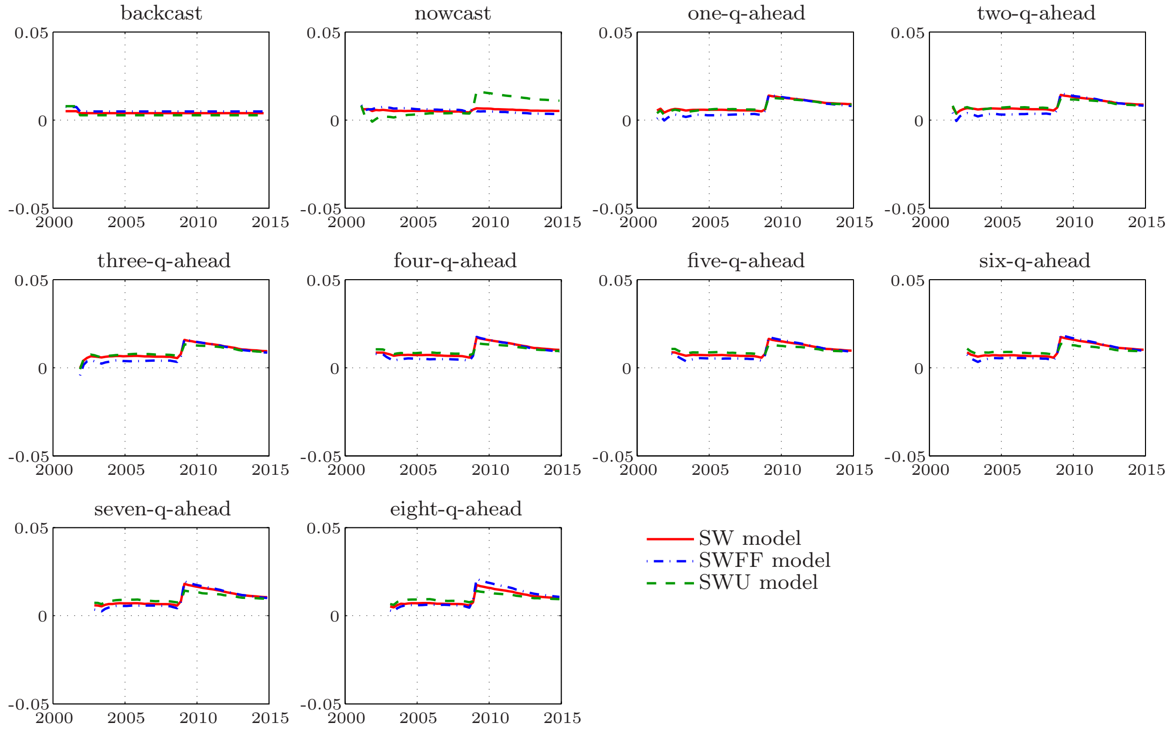


Figure 25. Differences between the MC estimator and the normal approximation of the recursive estimates in average log score of real GDP growth and inflation for the vintages 2001Q1–2014Q4.

A. Real GDP growth



B. GDP Deflator Inflation

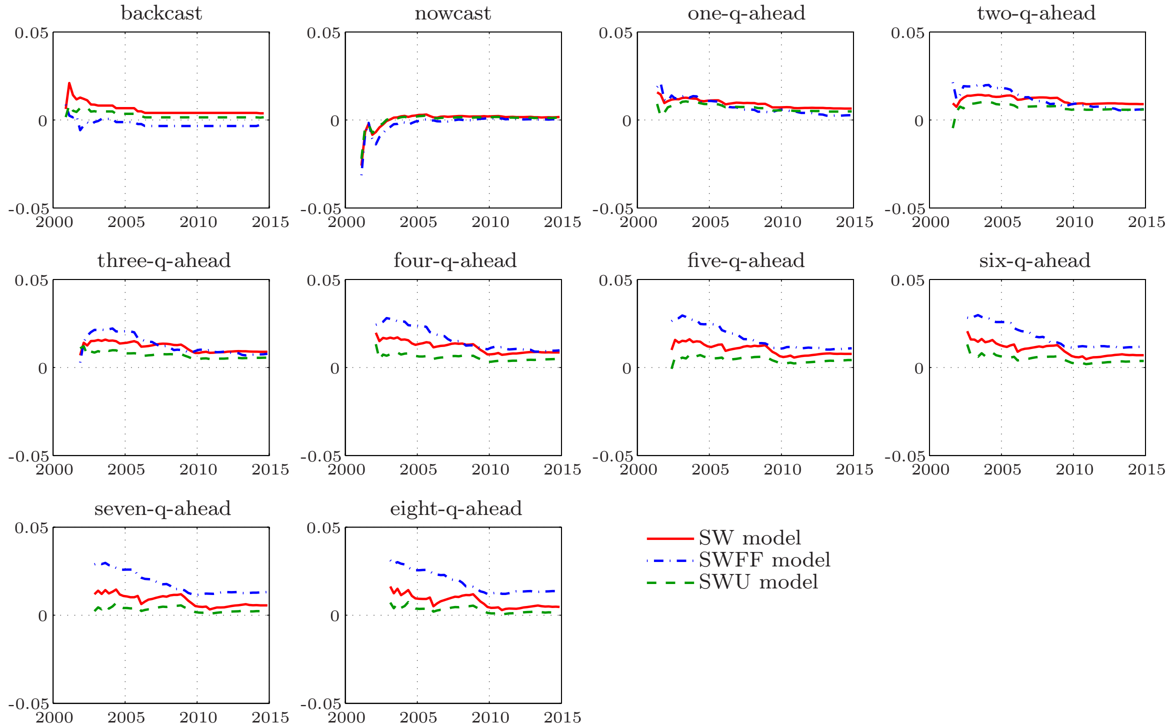


FIGURE 25. Differences between the MC estimator and the normal approximation of the recursive estimates in average log score of real GDP growth and inflation for the vintages 2001Q1–2014Q4 (continued).

C. REAL GDP GROWTH & INFLATION

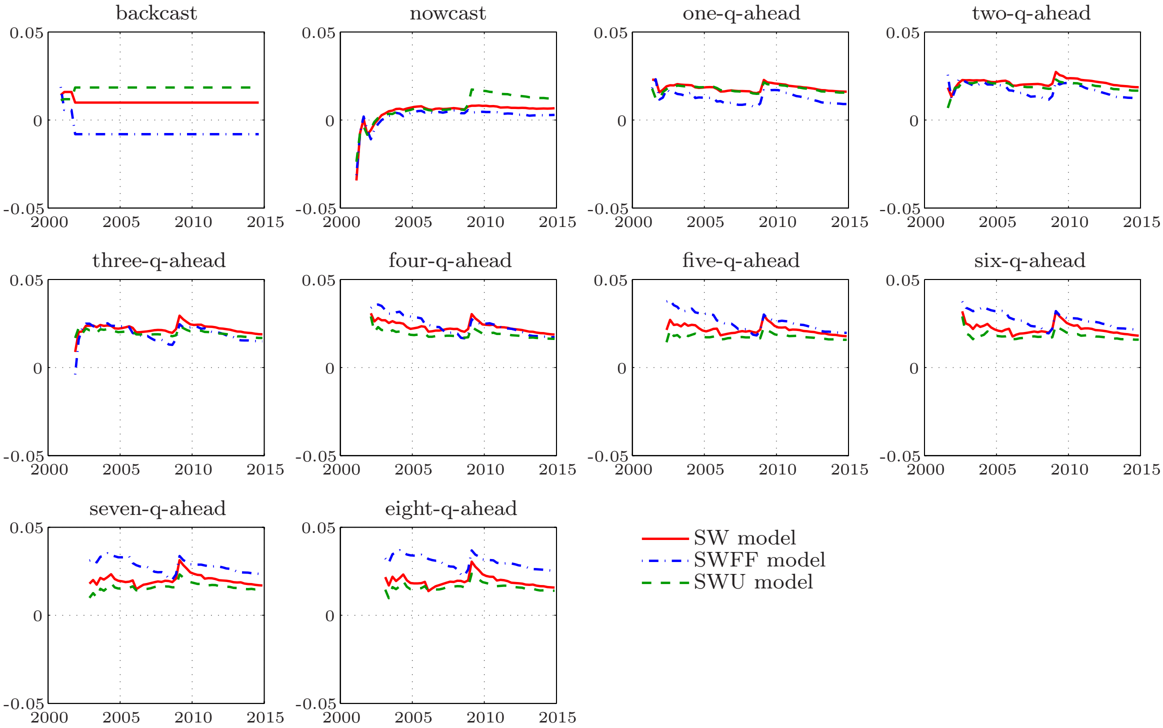
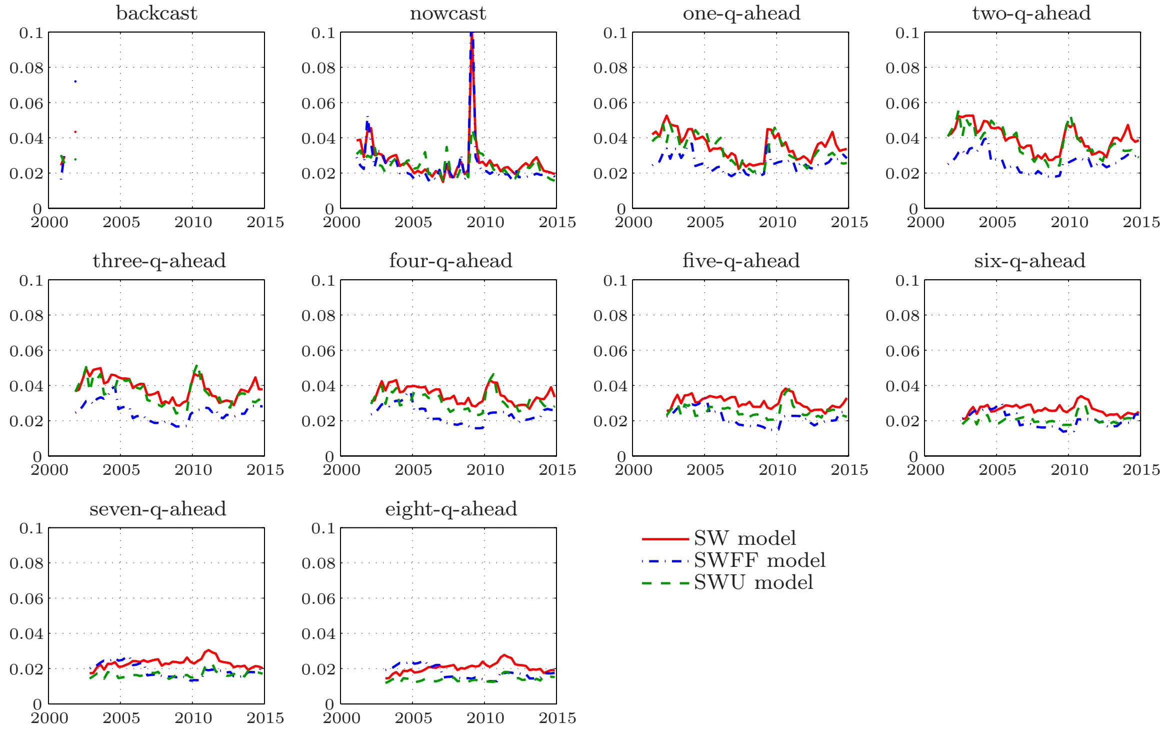


FIGURE 26. Recursive estimates of the share of parameter uncertainty of the predictive variances of real GDP growth and inflation for the vintages 2001Q1–2014Q4.

A. REAL GDP GROWTH



B. GDP DEFLATOR INFLATION

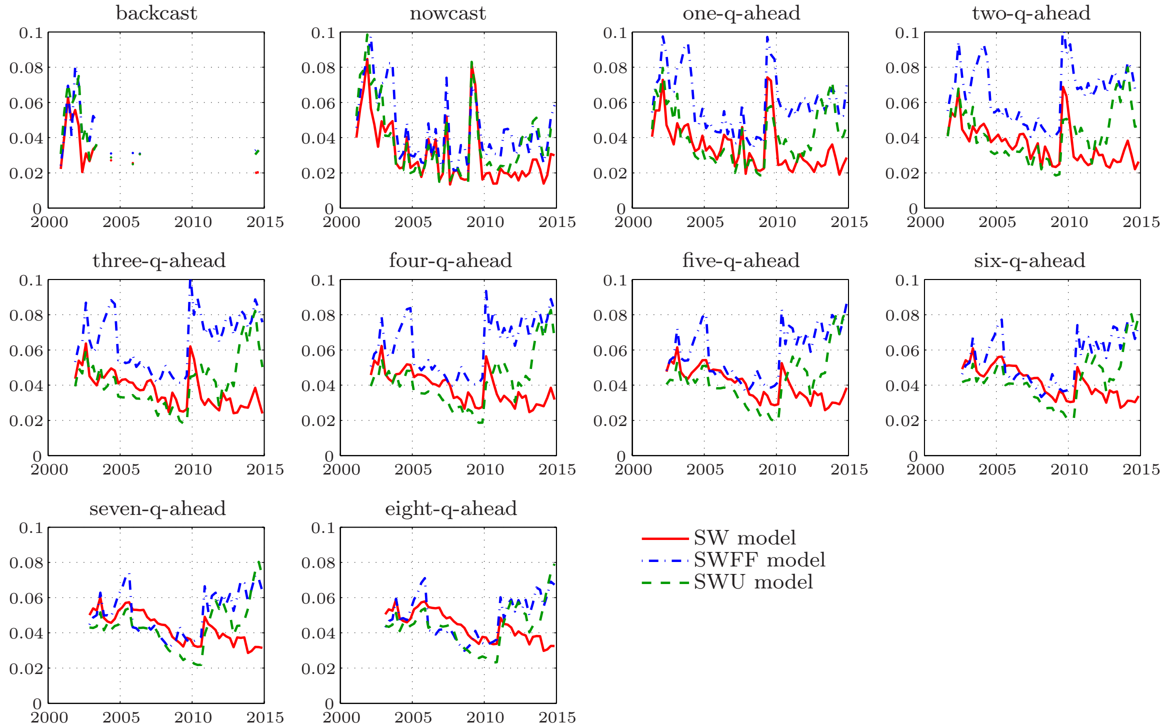
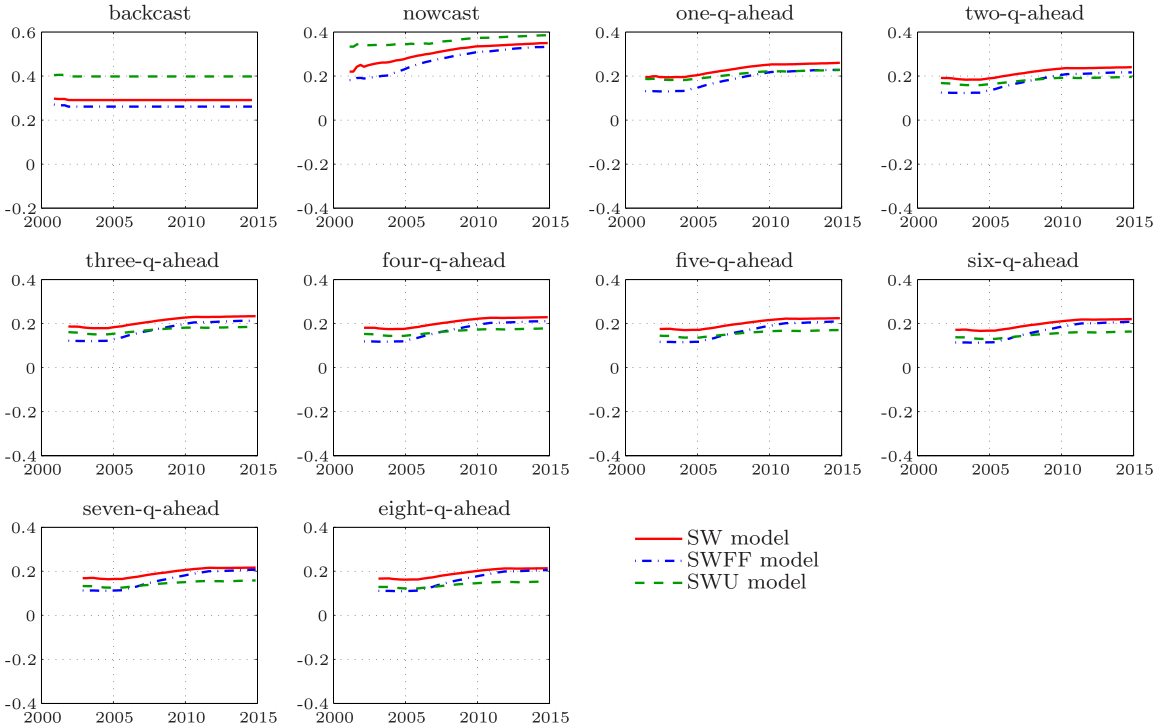


Figure 27. Decomposition of recursive estimates of the average log predictive score of real GDP growth using the normal approximation for the vintages 2001Q1–2014Q4.

A. Forecast uncertainty



B. Quadratic standardized forecast error

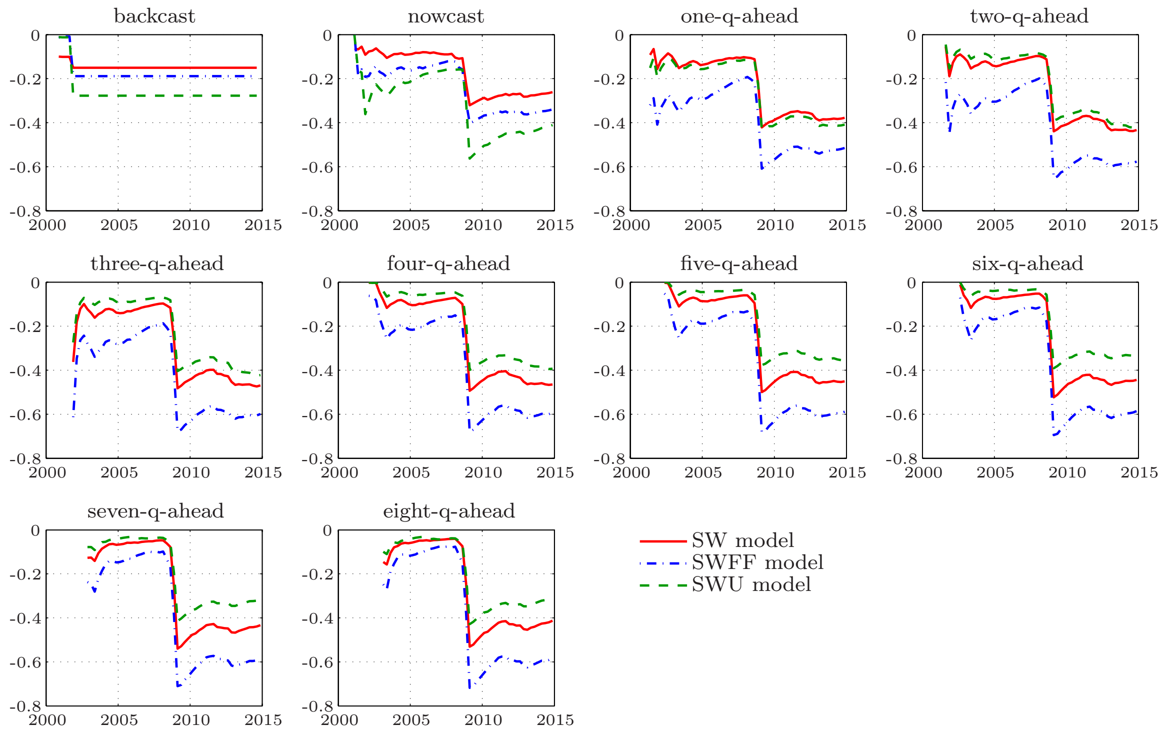
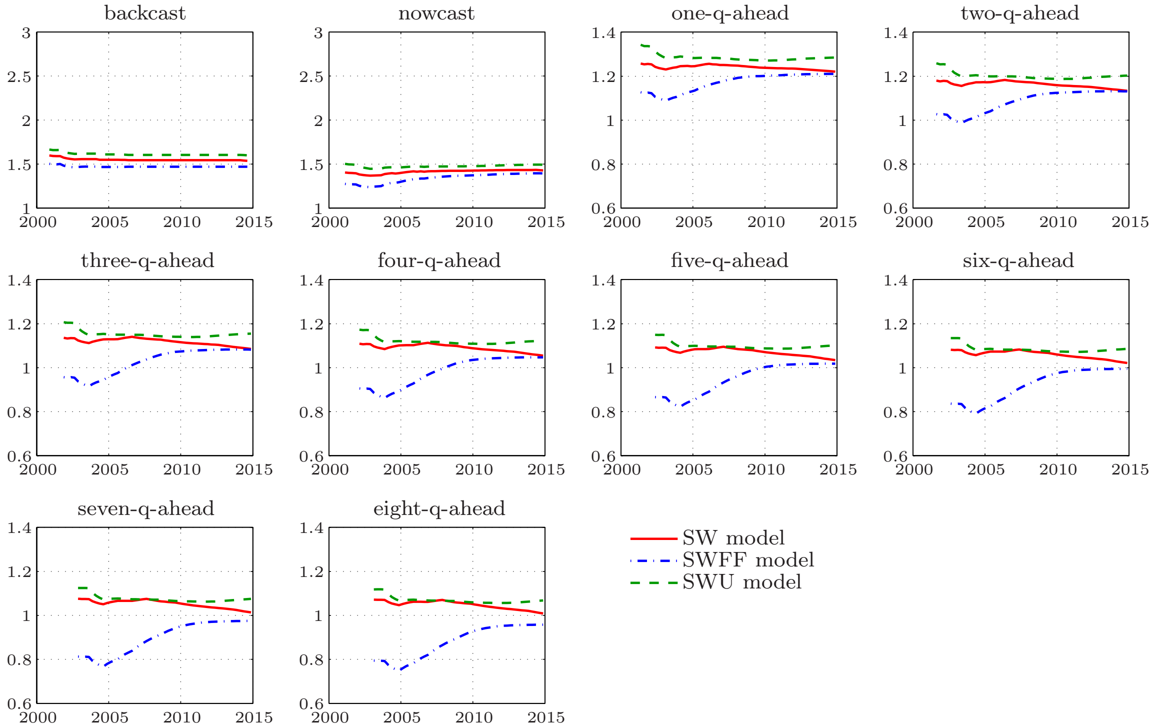


Figure 28. Decomposition of recursive estimates of the average log predictive score of inflation using the normal approximation for the vintages 2001Q1–2014Q4.

A. Forecast uncertainty



B. Quadratic standardized forecast error

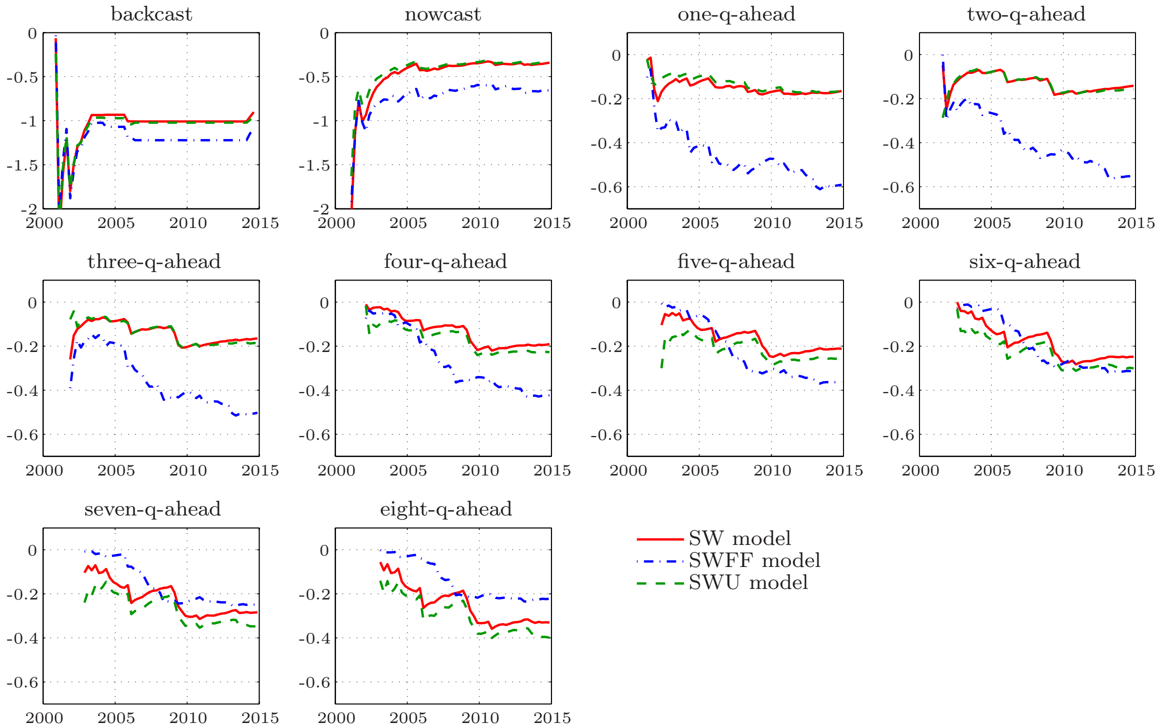
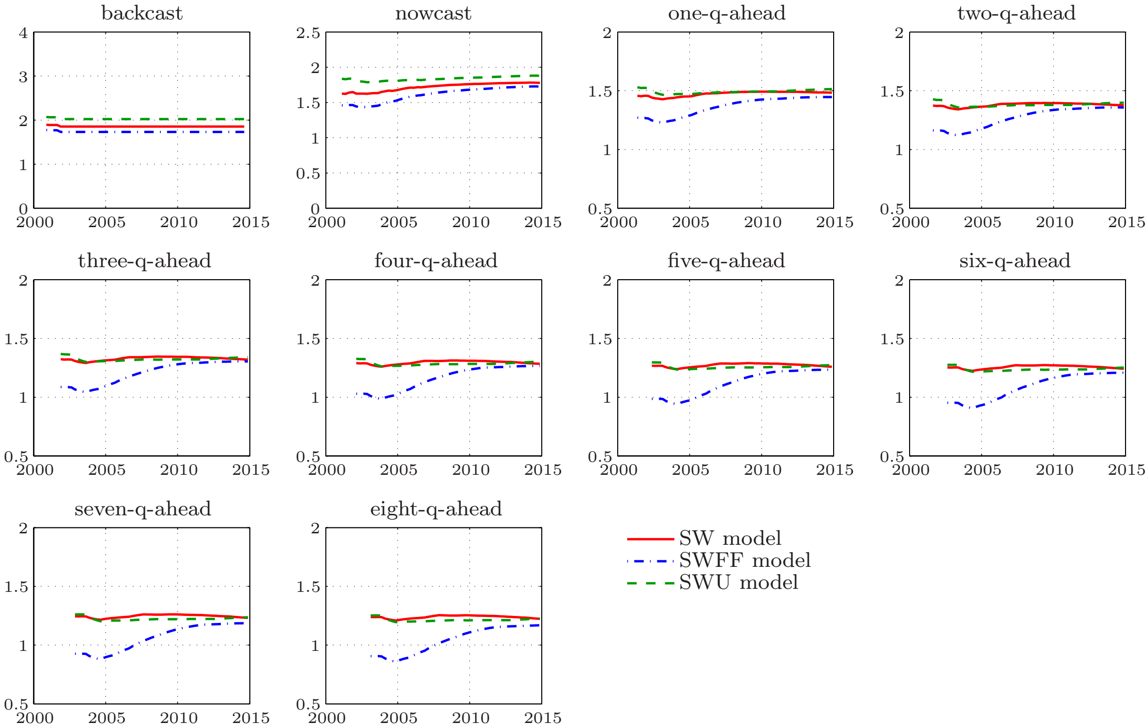
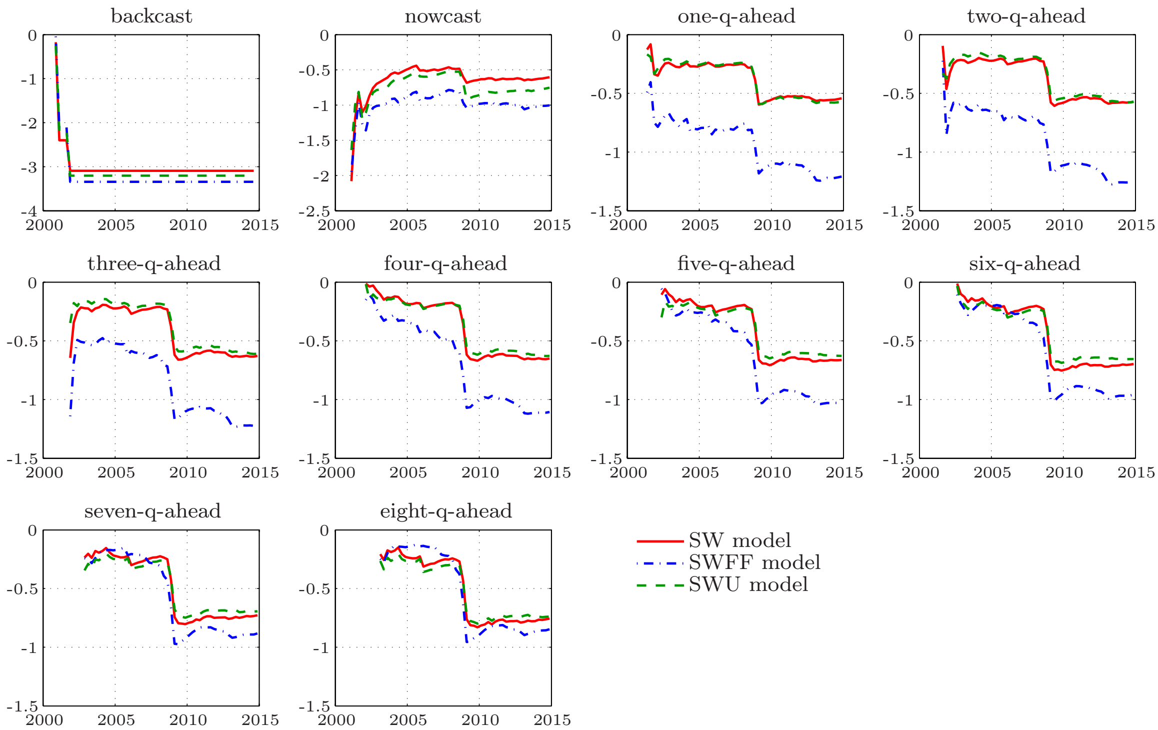


Figure 29. Decomposition of recursive estimates of the average log predictive score of inflation and real GDP growth using the normal approximation for the vintages 2001Q1–2014Q4.

A. Forecast uncertainty



B. Quadratic standardized forecast error



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Appendix: A Detailed Exposition of the DSGE Models

A.1. The Smets and Wouters Model Adapted to the Euro Area

A well known example of a medium-sized DSGE model is Smets and Wouters (2007), where the authors study shocks and frictions in US business cycles. The equations of the model are presented below, while a detailed discussion of the model is found in Smets and Wouters (2007); see also Smets and Wouters (2003, 2005). It should be emphasized that since the model uses a flexible-price based output gap measure in the monetary policy rule, the discussion will first consider the sticky price and wage system, followed by the flexible price and wage system. The equations for the seven exogenous variables are introduced thereafter, while the steady-state of the system closes the theoretical part of the empirical model. Finally, the model variables are linked to the observed variables via the measurement equations.

A.2. The Sticky Price and Wage Equations

The log-linearized aggregate resource constraint of this closed economy model is given by

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{i}_t + z_y \hat{z}_t + \varepsilon_t^g, \tag{A.1}$$

where $\hat{y}_t$ is (detrended) real GDP. It is absorbed by real private consumption ($\hat{c}_t$), real private investments ($\hat{i}_t$), the capital utilization rate ($\hat{z}_t$), and exogenous spending (ε_t^g). The parameter c_y is the steady-state consumption-output ratio and i_y is the steady-state investment-output ratio, where

$$c_y = 1 - i_y - g_y,$$

and g_y is the steady-state exogenous spending-output ratio. The steady-state investment-output ratio is determined by

$$i_y = (\gamma + \delta - 1)\, k_y,$$

where k_y is the steady-state capital-output ratio, γ is the steady-state growth rate, and δ is the depreciation rate of capital. Finally,

$$z_y = r^k k_y,$$

where r^k is the steady-state rental rate of capital. The steady-state parameters are shown in Section A.5, but it is noteworthy already at this stage that $z_y = \alpha$, the share of capital in production.

The dynamics of consumption follows from the consumption Euler equation and is equal to

$$\hat{c}_t = c_1 \hat{c}_{t-1} + (1 - c_1)\, E_t \hat{c}_{t+1} + c_2 \left(\hat{l}_t - E_t \hat{l}_{t+1}\right) - c_3 \left(\hat{r}_t - E_t \hat{\pi}_{t+1}\right) + \varepsilon_t^b, \tag{A.2}$$

where $\hat{l}_t$ is hours worked, $\hat{r}_t$ is the policy controlled nominal interest rate, and ε_t^b is proportional to the exogenous risk premium, i.e., a wedge between the interest rate controlled by the central bank and the return on assets held by households. It should be noted that in contrast to Smets and Wouters (2007), but identical to Smets and Wouters (2005) and Lindé et al. (2016), we have moved the risk premium variable outside the expression for the ex ante real interest rate. This means that $\varepsilon_t^b = -c_3 \epsilon_t^b$, where ϵ_t^b is the risk premium variable in Smets and Wouters (2007). Building on the work by Krishnamurthy and Vissing-Jorgensen (2012), Fisher (2015) shows that this shock can be given a structural interpretation, namely, as a shock to the demand for safe and liquid assets or, alternatively, as a liquidity preference shock.

The parameters of the consumption Euler equation are:

$$c_1 = \frac{\lambda/\gamma}{1+(\lambda/\gamma)}, \quad c_2 = \frac{(\sigma_c - 1)(w^h l/c)}{\sigma_c (1+(\lambda/\gamma))}, \quad c_3 = \frac{1-(\lambda/\gamma)}{\sigma_c (1+(\lambda/\gamma))},$$

where λ measures external habit formation, σ_c is the inverse of the elasticity of intertemporal substitution for constant labor, while $w^h l/c$ is the steady-state hourly real wage bill to consumption ratio. If $\sigma_c = 1$ (log-utility) and $\lambda = 0$ (no external habit) then the above equation reduces to the familiar purely forward looking consumption Euler equation.

The log-linearized investment Euler equation is given by

$$\hat{i}_t = i_1 \hat{i}_{t-1} + (1 - i_1) E_t \hat{i}_{t+1} + i_2 \hat{q}_t + \varepsilon_t^i, \tag{A.3}$$

where $\hat{q}_t$ is the real value of the existing capital stock, while ε_t^i is an exogenous investment-specific technology variable. The parameters of (A.3) are given by

$$i_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}, \quad i_2 = \frac{1}{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi},$$

where β is the discount factor used by households, and φ is the steady-state elasticity of the capital adjustment cost function.

The dynamic equation for the value of the capital stock is

$$\hat{q}_t = q_1 E_t \hat{q}_{t+1} + (1 - q_1) E_t \hat{r}_{t+1}^k - \left(\hat{r}_t - E_t \hat{\pi}_{t+1}\right) + c_3^{-1} \varepsilon_t^b, \tag{A.4}$$

where $\hat{r}_t^k$ is the rental rate of capital. The parameter q_1 is here given by

$$q_1 = \beta\gamma^{-\sigma_c}(1-\delta) = \frac{1-\delta}{r^k + 1 - \delta}.$$

Turning to the supply-side of the economy, the log-linearized aggregate production function can be expressed as

$$\hat{y}_t = \phi_p \left[\alpha \hat{k}_t^s + (1-\alpha)\hat{l}_t + \varepsilon_t^a\right], \tag{A.5}$$

where $\hat{k}_t^s$ is capital services used in production, and ε_t^a an exogenous total factor productivity variable. As mentioned above, the parameter α reflects the share of capital in production, while ϕ_p is equal to one plus the steady-state share of fixed costs in production.

The capital services variable is used to reflect that newly installed capital only becomes effective with a one period lag. This means that

$$\hat{k}_t^s = \hat{k}_{t-1} + \hat{z}_t, \tag{A.6}$$

where $\hat{k}_t$ is the installed capital. The degree of capital utilization is determined from cost minimization of the households that provide capital services and is therefore a positive function of the rental rate of capital. Specifically,

$$\hat{z}_t = z_1 \hat{r}_t^k, \tag{A.7}$$

where

$$z_1 = \frac{1-\psi}{\psi},$$

and ψ is a positive function of the elasticity of the capital adjustment cost function and normalized to lie between 0 and 1. The larger ψ is the costlier it is to change the utilization of capital.

The log-linearized equation that specifies the development of installed capital is

$$\hat{k}_t = k_1 \hat{k}_{t-1} + (1-k_1)\,\hat{i}_t + k_2 \varepsilon_t^i. \tag{A.8}$$

The two parameters are given by

$$k_1 = \frac{1-\delta}{\gamma}, \quad k_2 = (\gamma+\delta-1)\left(1+\beta\gamma^{1-\sigma_c}\right)\gamma\varphi.$$

From the monopolistically competitive goods market, the price markup ($\hat{\mu}_t^p$) is equal to minus the real marginal cost ($\hat{\mu}_t^c$) under cost minimization by firms. That is,

$$\hat{\mu}_t^p = \alpha\left(\hat{k}_t^s - \hat{l}_t\right) - \hat{w}_t + \varepsilon_t^a, \tag{A.9}$$

where the real wage is given by $\hat{w}_t$. Similarly, the real marginal cost is

$$\hat{\mu}_t^c = \alpha \hat{r}_t^k + (1-\alpha)\,\hat{w}_t - \varepsilon_t^a, \tag{A.10}$$

where (A.10) is obtained by substituting for the optimally determined capital-labor ratio in equation (A.12).

Due to price stickiness, and partial indexation to lagged inflation of those prices that cannot be re-optimized, prices adjust only sluggishly to their desired markups. Profit maximization by price-setting firms yields the log-linearized price Phillips curve

$$\begin{aligned} \hat{\pi}_t &= \pi_1 \hat{\pi}_{t-1} + \pi_2 E_t \hat{\pi}_{t+1} - \pi_3 \hat{\mu}_t^p + \varepsilon_t^p \\ &= \pi_1 \hat{\pi}_{t-1} + \pi_2 E_t \hat{\pi}_{t+1} + \pi_3 \hat{\mu}_t^c + \varepsilon_t^p, \end{aligned} \tag{A.11}$$

where ε_t^p is an exogenous price markup process. The parameters of the Phillips curve are given by

$$\pi_1 = \frac{\imath_p}{1+\beta\gamma^{1-\sigma_c}\imath_p}, \quad \pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\imath_p}, \quad \pi_3 = \frac{(1-\xi_p)\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)}{(1+\beta\gamma^{1-\sigma_c}\imath_p)\,\xi_p\left((\phi_p-1)\varepsilon_p+1\right)}.$$

The degree of indexation to past inflation is determined by the parameter $\imath_p$, ξ_p measures the degree of price stickiness such that $1-\xi_p$ is the probability that a firm can re-optimize its price, and ε_p is the curvature of the Kimball (1995) goods market aggregator.

Cost minimization of firms also implies that the rental rate of capital is related to the capital-labor ratio and the real wage according to.

$$\hat{r}_t^k = -\left(\hat{k}_t^s - \hat{l}_t\right) + \hat{w}_t. \tag{A.12}$$

In the monopolistically competitive labor market the wage markup is equal to the difference between the real wage and the marginal rate of substitution between labor and consumption

$$\hat{\mu}_t^w = \hat{w}_t - \left(\sigma_l \hat{l}_t + \frac{1}{1-(\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right]\right), \tag{A.13}$$

where σ_l is the elasticity of the labor input with respect to real wages.

Due to wage stickiness and partial wage indexation, real wages respond gradually to the desired wage markup

$$\hat{w}_t = w_1 \hat{w}_{t-1} + (1-w_1)\left[E_t \hat{w}_{t+1} + E_t \hat{\pi}_{t+1}\right] - w_2 \hat{\pi}_t + w_3 \hat{\pi}_{t-1} - w_4 \hat{\mu}_t^w + \varepsilon_t^w, \tag{A.14}$$

where ε_t^w is an exogenous wage markup process. The parameters of the wage equation are

$$w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}, \quad w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\imath_w}{1+\beta\gamma^{1-\sigma_c}},$$
$$w_3 = \frac{\imath_w}{1+\beta\gamma^{1-\sigma_c}}, \quad w_4 = \frac{(1-\xi_w)\left(1-\beta\gamma^{1-\sigma_c}\xi_w\right)}{\left(1+\beta\gamma^{1-\sigma_c}\right)\xi_w\left((\phi_w-1)\varepsilon_w+1\right)}.$$

The degree of wage indexation to past inflation is given by the parameter $\imath_w$, while ξ_w is the degree of wage stickiness. The steady-state labor market markup is equal to $\phi_w - 1$ and ε_w is the curvature of the Kimball labor market aggregator.

The sticky price and wage part of the model is closed by adding the monetary policy reaction function

$$\hat{r}_t = \rho \hat{r}_{t-1} + (1-\rho)\left[r_\pi \hat{\pi}_t + r_y\left(\hat{y}_t - \hat{y}_t^f\right)\right] + r_{\Delta y}\left[\Delta\hat{y}_t - \Delta\hat{y}_t^f\right] + \varepsilon_t^r, \tag{A.15}$$

where $\hat{y}_t^f$ is potential output measured as the level of output that would prevail under flexible prices and wages in the absence of the two exogenous markup processes, whereas ε_t^r is an exogenous monetary policy shock process.

As productivity is written in terms of hours worked, we introduce an auxiliary equation with Calvo-rigidity to link from observed total employment $(\widehat{e}_t)$ to unobserved hours worked:

$$\hat{e}_t - \hat{e}_{t-1} = \beta\left(E_t \hat{e}_{t+1} - \hat{e}_t\right) + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}\left(\hat{l}_t - \hat{e}_t\right), \tag{A.16}$$

where $1-\xi_e$ is the fraction of firms that are able to adjust employment to its desired total labor input; see Adolfson et al. (2005, 2007a).

A.3. The Flexible Price and Wage Equations

The flexible price equations are obtained by assuming that the two exogenous markup processes are zero, while $\xi_w = \xi_p = 0$, and $\imath_w = \imath_p = 0$. As a consequence, inflation is always equal to the steady-state inflation rate while real wages are equal to the marginal rate of substitution between labor and consumption as well as to the marginal product of labor. All other aspects of the economy are unaffected. Letting the superscript f denote the flexible price and wage economy versions of the variables we find that

$$
\begin{aligned}
\hat{y}_t^f &= c_y \hat{c}_t^f + i_y \hat{i}_t^f + z_y \hat{z}_t^f + \varepsilon_t^g, \\
\hat{c}_t^f &= c_1 \hat{c}_{t-1}^f + (1 - c_1) E_t \hat{c}_{t+1}^f + c_2 \left(\hat{l}_t^f - E_t \hat{l}_{t+1}^f \right) - c_3 \hat{r}_t^f + \varepsilon_t^b, \\
\hat{i}_t^f &= i_1 \hat{i}_{t-1}^f + (1 - i_1) E_t \hat{i}_{t+1}^f + i_2 \hat{q}_t^f + \varepsilon_t^i, \\
\hat{q}_t^f &= q_1 E_t \hat{q}_{t+1}^f + (1 - q_1) E_t \hat{r}_{t+1}^{k,f} - \hat{r}_t^f + c_3^{-1} \varepsilon_t^b, \\
\hat{y}_t^f &= \phi_p \left[\alpha \hat{k}_t^{s,f} + (1 - \alpha) \hat{l}_t^f + \varepsilon_t^a \right], \\
\hat{k}_t^{s,f} &= \hat{k}_{t-1}^f + \hat{z}_t^f, \\
\hat{z}_t^f &= z_1 \hat{r}_t^{k,f}, \\
\hat{k}_t^f &= k_1 \hat{k}_{t-1}^f + (1 - k_1) \hat{i}_t^f + k_2 \varepsilon_t^i, \\
\varepsilon_t^a &= \alpha \hat{r}_t^{k,f} + (1 - \alpha) \hat{w}_t^f, \\
\hat{r}_t^{k,f} &= - \left(\hat{k}_t^{s,f} - \hat{l}_t^f \right) + \hat{w}_t^f, \\
\hat{w}_t^f &= \sigma_l \hat{l}_t^f + \frac{1}{1 - (\lambda/\gamma)} \left[\hat{c}_t^f - \frac{\lambda}{\gamma} \hat{c}_{t-1}^f \right], \\
\hat{e}_t^f - \hat{e}_{t-1}^f &= \beta \left(E_t \hat{e}_{t+1}^f - \hat{e}_t^f \right) + \frac{(1 - \beta \xi_e)(1 - \xi_e)}{\xi_e} \left(\hat{l}_t^f - \hat{e}_t^f \right),
\end{aligned}
\tag{A.17}
$$

where $\hat{r}_t^f$ is the real interest rate of the flexible price and wage system.

A.4. The Exogenous Variables

There are seven exogenous processes in the Smets and Wouters (2007) model. These are generally modelled as AR(1) processes with the exception of the exogenous spending process (where the process depends on both the exogenous spending shock η_t^g and the total factor productivity shock η_t^a) and the exogenous price and wage markup processes, which are treated as $ARMA(1,1)$ processes. This means that

$$
\begin{aligned}
\varepsilon_t^g &= \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g + \rho_{ga} \sigma_a \eta_t^a, \\
\varepsilon_t^b &= \rho_b \varepsilon_{t-1}^b + \sigma_b \eta_t^b, \\
\varepsilon_t^i &= \rho_i \varepsilon_{t-1}^i + \sigma_i \eta_t^i, \\
\varepsilon_t^a &= \rho_a \varepsilon_{t-1}^a + \sigma_a \eta_t^a, \\
\varepsilon_t^p &= \rho_p \varepsilon_{t-1}^p + \sigma_p \eta_t^p - \mu_p \sigma_p \eta_{t-1}^p, \\
\varepsilon_t^w &= \rho_w \varepsilon_{t-1}^w + \sigma_w \eta_t^w - \mu_w \sigma_w \eta_{t-1}^w, \\
\varepsilon_t^r &= \rho_r \varepsilon_{t-1}^r + \sigma_r \eta_t^r.
\end{aligned}
\tag{A.18}
$$

The shocks η_t^j, $j = \{a, b, g, i, p, r, w\}$, are $N(0,1)$, where η_t^b is a preference shock (proportional to a risk premium shock), η_t^i is an investment-specific technology shock, η_t^p is a price markup shock, η_t^r is a monetary policy or interest rate shock, and η_t^w is a wage markup shock.

A.5. The Steady-State Equations

It remains to provide expressions for the steady-state values of the capital-output ratio, the rental rate of capital, and the hourly real wage bill to consumption ratio which relate them to the parameters of the model. The steady-state exogenous spending to output ratio g_y is a calibrated parameter and set to 0.18 by Smets and Wouters (2007). From the expressions for the q_1 parameter it follows that

$$r^k = \frac{1}{\beta\gamma^{-\sigma_c}} + \delta - 1.$$

The capital-output ratio is derived in a stepwise manner in the model appendix to Smets and Wouters (2007). The steady-state relations there are

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{(1-\alpha)}}{\phi_p (r^k)^\alpha}\right]^{1/(1-\alpha)}, \quad \frac{l}{k} = \frac{(1-\alpha) r^k}{\alpha w}, \quad \frac{k}{y} = \phi_p \left[\frac{l}{k}\right]^{\alpha-1},$$

where $k_y = k/y$. From these relationships it is straightforward, albeit tedious, to show that $z_y = r^k k_y = \alpha$.

The steady-state relation between real wages and hourly real wages is

$$w = \phi_w w^h,$$

so that the steady-state hourly real wage bill to consumption ratio is given by

$$\frac{w^h l}{c} = \frac{(1-\alpha) r^k k_y}{\phi_w \alpha c_y} = \frac{1-\alpha}{\phi_w c_y},$$

where the last equality follows from the relationship of $z_y = r^k k_y = \alpha$.

A.6. The Measurement Equations

The Smets and Wouters (2007) model is consistent with a balanced steady-state growth path driven by deterministic labor augmenting technological progress. The observed variables for the euro area are given by quarterly data of the log of real GDP for the euro area (y_t), the log of real private consumption (c_t), the log of real total investment (i_t), the log of total employment (e_t), the log of quarterly GDP deflator inflation (π_t), the log of real wages (w_t), and the short-term nominal interest rate (r_t) given by the 3-month EURIBOR rate. The measurement equations are given by

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ \Delta w_t \\ \Delta e_t \\ \pi_t \\ r_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} + \bar{e} \\ \bar{\gamma} + \bar{e} \\ \bar{\gamma} + \bar{e} \\ \bar{\gamma} \\ \bar{e} \\ \bar{\pi} \\ 4\bar{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{e}_t - \hat{e}_{t-1} \\ \hat{\pi}_t \\ 4\hat{r}_t \end{bmatrix}. \tag{A.19}$$

Since all observed variables except the short-term nominal interest rate (which is already reported in percent) are multiplied by 100, it follows that the steady-state values on the right hand side are given by

$$\bar{\gamma} = 100\,(\gamma - 1)\,, \quad \bar{\pi} = 100\,(\pi - 1)\,, \quad \bar{r} = 100\left(\frac{\pi}{\beta\gamma^{-\sigma_c}} - 1\right),$$

where π is steady-state inflation while $\bar{e}$ reflects steady-state labor force growth. The interest rate in the model, $\hat{r}_t$, is measured in quarterly terms in the model and is therefore multiplied by 4 in (A.19) to restore it to annual terms for the measurement.

Apart from the steady-state exogenous spending-output ratio only six additional parameters are calibrated. These are $\delta = 0.025$, $\phi_w = 1.5$, $\varepsilon_p = \varepsilon_w = 10$, and $\mu_p = \mu_w = 0$. The latter two parameters are estimated by Smets and Wouters (2007) on US data, but we have here opted to treat all exogenous processes in the model symmetrically. The remaining 20 structural and 15 shock process parameters are estimated. When estimating the parameters, we make us of the following transformation of the discount factor

$$\beta = \frac{1}{1 + (\bar{\beta}/100)}.$$

Following Smets and Wouters (2007), a prior distribution is assumed for the parameter $\bar{\beta}$, while β is determined from the above equation.

A.7. The Smets and Wouters Model with Financial Frictions

Changes to the Sticky Economy

Lombardo and McAdam (2012), Del Negro and Schorfheide (2013) and Del Negro et al. (2015) introduce financial frictions into variants of the Smets and Wouters model based on the financial accelerator approach of Bernanke et al. (1999); see also Christiano et al. (2003); Christiano, Motto, and Rostagno (2010) and De Graeve (2008). This amounts to replacing the value of the capital stock equation in (A.4) with

$$E_t\hat{r}^e_{t+1} - \hat{r}_t = \zeta_{sp,b}\big(\hat{q}_t + \hat{k}_t - \hat{n}_t\big) - c_3^{-1}\varepsilon^b_t + \varepsilon^e_t, \tag{A.20}$$

and

$$\hat{r}_t^e - \hat{\pi}_t = \left(1 - q_1\right)\hat{r}_t^k + q_1\hat{q}_t - \hat{q}_{t-1}, \tag{A.21}$$

where $\hat{r}_t^e$ is the gross return on capital for entrepreneurs, $\hat{n}_t$ is entrepreneurial equity (net worth), and ε_t^e captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs, a spread shock. The parameter q_1 is here given by

$$q_1 = \frac{1-\delta}{r^k + 1 - \delta},$$

where r^k is generally *not* equal to $\beta^{-1}\gamma^{\sigma_c} + \delta - 1$ since the spread between gross returns on capital for entrepreneurs and the nominal interest rate need not equal zero. The spread shock is assumed to follows the AR(1) process

$$\varepsilon_t^e = \rho_e\varepsilon_{t-1}^e + \sigma_e\eta_t^e. \tag{A.22}$$

The parameters $\zeta_{sp,b}$ is the steady-state elasticity of the spread with respect to leverage. It may be noted that if $\zeta_{sp,b} = \sigma_e = 0$, then the financial frictions are shut down and equations (A.20) and (A.21) yield the original value of the capital stock equation (A.4).

The log-linearized net worth of entrepreneurs equation is given by

$$\hat{n}_t = \zeta_{n,e}\left(\hat{r}_t^e - \hat{\pi}_t\right) - \zeta_{n,r}\left(\hat{r}_{t-1} - \hat{\pi}_t\right) + \zeta_{n,q}\left(\hat{q}_{t-1} + \hat{k}_{t-1}\right) + \zeta_{n,n}\hat{n}_{t-1} - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\varepsilon_{t-1}^e, \tag{A.23}$$

where $\zeta_{n,e}$, $\zeta_{n,r}$, $\zeta_{n,q}$, $\zeta_{n,n}$, and ζ_{n,σ_ω} are the steady-state elasticities of net worth with respect to the return on capital for entrepreneurs, the interest rate, the cost of capital, lagged net worth, and the volatility of the spread shock. Furthermore, ζ_{sp,σ_ω} is the steady-state elasticity of the spread with respect to the spread shock. Expressions for these elasticities are given in Section A.10.

A.8. Changes to the Flexible Economy

It is typically assumed for the flexible price and wage economy that there are no financial frictions; see, e.g., De Graeve (2008). Accordingly, net worth is zero, the gross real return on capital of entrepreneurs is equal to the real rate $\hat{r}_t^f$, while the value of the capital stock evolves according to

$$\hat{q}_t^f = q_1 E_t\hat{q}_{t+1}^f + \left(1 - q_1\right) E_t\hat{r}_{t+1}^{k,f} - \hat{r}_t^f + c_3^{-1}\varepsilon_t^b.$$

Since the steady-state real rental rate on capital in the flexible price and wage economy without financial frictions is equal to the real interest rate, an option regarding the q_1 parameter in this equation is to set it equal to $q_{1,f} = \beta\gamma^{-\sigma_c}(1-\delta)$. Formally, this appears to be the correct choice since the steady-state of the flexible price and wage and frictionless economy differs from the steady-state of the economic with sticky prices and wages and financial frictions. However, if the steady-state spread of the former economy, r^e/r, is close to unity, then q_1 and $q_{1,f}$ will be approximately equal.

A.9. Augmented Measurement Equations

The set of measurement equations is augmented in Del Negro and Schorfheide (2013) by

$$s_t = 4\bar{s} + 4E_t\left[\hat{r}^e_{t+1} - \hat{r}_t\right], \tag{A.24}$$

where $\bar{s}$ is equal to the natural logarithm of the steady-state spread measured in quarterly terms and in percent, $\bar{s} = 100\ln(r^e/r)$, while s_t is a suitable spread variable.[31] The parameter s is linked to the steady-state values of the model variable variables according to

$$\frac{r^e}{r} = (1 + s/100)^{1/4}, \quad \frac{r}{\pi} = \beta^{-1}\gamma^{\sigma_c}, \quad \frac{r^e}{\pi} = r^k + 1 - \delta.$$

If there are no financial frictions, then $r^e = r$, with the consequence that the steady-state real interest rate is equal to the steady-state real rental rate on capital plus one minus the depreciation rate of capital, i.e., the Smets and Wouters steady-state value; see Section A.5.

Del Negro and Schorfheide (2013) estimate s, $\zeta_{sp,b}$, ρ_e, and σ_e while the parameters $\bar{F}$ and κ^e are calibrated. The latter two parameters will appear in the next Section on the steady-state, but it is useful to know that they represent the steady-state default probability and survival rate of entrepreneurs, respectively, with $\bar{F}$ determined such that in annual terms the default probability is 0.03 (0.0075 in quarterly terms) and $\kappa^e = 0.99$. These values are also used by Del Negro et al. (2015) as well as in the DSGE model of the Federal Reserve Bank of New York; see Del Negro, Eusepi, Giannoni, Sbordone, Tambalotti, Cocci, Hasegawa, and Linder (2013). Finally, and in line with the SW model in Section A.1, the financial frictions extension also involves the following calibrated parameters: $\delta = 0.025$, $\phi_w = 1.5$, $\varepsilon_p = \varepsilon_w = 10$, and $\mu_p = \mu_w = 0$..

A.10. The Steady-State in the Model with Financial Frictions

A.10.1. Preliminaries: The Log-Normal Distribution

To determine the relationship between the elasticities in equations (A.20) and (A.23) and the steady-state of the model with financial frictions it is important to first sort out some details concerning an idiosyncratic shock which affects entrepreneurs' capital. The distribution of the shock in time t is known before it is realized and this distribution is the same for all entrepreneurs, while the shock is iid across entrepreneurs and over time. In the steady state, the natural logarithm of the shock, denoted by $\ln\omega$, is normal with mean m_ω and variance σ^2_ω, where it is assumed that the expected value of ω is unity. Since its distribution is log-normal[32] with

[31] It is measured as the annualized Moody's seasoned Baa corporate bond yield spread over the 10-year treasury note yield at constant maturity by Del Negro and Schorfheide (2013) and Del Negro et al. (2015). For the euro area we consider a spread over the short-term nominal interest rate based on lending rate data.

[32] The density function of the *log-normal distribution* is

$$p_{LN}(z|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}}z^{-1}\exp\left(-\frac{1}{2\sigma^2}\left(\ln z - \mu\right)^2\right), \qquad z > 0.$$

The mean of the log-normal distribution is $\mu_{LN} = \exp(\mu + (\sigma^2/2))$, the variance is $\sigma^2_{LN} = \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$, while the mode is $\tilde{\mu}_{LN} = \exp(\mu - \sigma^2)$; see Gelman, Carlin, Stern, and Rubin (2004).

$\mu_\omega = \exp(m_\omega + (1/2)\sigma_\omega^2) = 1$, it follows that $m_\omega = -(1/2)\sigma_\omega^2$. The probability density function (PDF) of ω is consequently

$$p(\omega) = \frac{1}{\sqrt{2\pi\sigma_\omega^2}}\omega^{-1}\exp\left(-\frac{1}{2\sigma_\omega^2}\big(\ln\omega + (1/2)\sigma_\omega^2\big)^2\right),$$

while the cumulative distribution function (CDF) is

$$F(\omega) = \Phi\left(\frac{\ln\omega + (1/2)\sigma_\omega^2}{\sigma_\omega}\right),$$

where $\Phi(z)$ is the CDF of the normal distribution. For values of ω below a threshold, $\bar{\omega}$, the entrepreneur defaults on its debt.[33]

The expected value of ω conditional on it being greater than $\bar{\omega}$ is given by

$$\begin{aligned} E\big[\omega\big|\omega > \bar{\omega}\big] &= \int_{\bar{\omega}}^{\infty}\omega p(\omega)d\omega = \Phi\left(\frac{(1/2)\sigma_\omega^2 - \ln\bar{\omega}}{\sigma_\omega}\right) \\ &= 1 - \Phi\left(\frac{\ln\bar{\omega} - (1/2)\sigma_\omega^2}{\sigma_\omega}\right), \end{aligned}$$

where the last equality follows from $\Phi(z) = 1 - \Phi(-z)$, i.e. that the normal distribution is symmetric. We then find that

$$\begin{aligned} G(\bar{\omega}) &= \int_0^{\bar{\omega}}\omega p(\omega)d\omega = \int_0^{\infty}\omega p(\omega)d\omega - \int_{\bar{\omega}}^{\infty}\omega p(\omega)d\omega \\ &= \Phi\left(\frac{\ln\bar{\omega} - (1/2)\sigma_\omega^2}{\sigma_\omega}\right). \end{aligned}$$

Furthermore, let

$$\begin{aligned} \Gamma(\bar{\omega}) &= \int_0^{\bar{\omega}}\omega p(\omega)d\omega + \bar{\omega}\int_{\bar{\omega}}^{\infty}p(\omega)d\omega \\ &= \bar{\omega}\left[1 - \Phi\left(\frac{\ln\bar{\omega} + (1/2)\sigma_\omega^2}{\sigma_\omega}\right)\right] + G(\bar{\omega}). \end{aligned}$$

Defining

$$z^\omega = \frac{\ln\bar{\omega} + (1/2)\sigma_\omega^2}{\sigma_\omega},$$

we obtain

$$G(\bar{\omega}) = \Phi\left(z^\omega - \sigma_\omega\right), \quad \Gamma(\bar{\omega}) = \bar{\omega}\left[1 - \Phi(z^\omega)\right] + \Phi\left(z^\omega - \sigma_\omega\right). \tag{A.25}$$

Let $\phi(z)$ be the standard normal PDF such that $\partial\Phi(z)/\partial z = \phi(z)$. From the expression for this density it follows that

$$\phi(z^\omega - \sigma_\omega) = \bar{\omega}\phi(z^\omega), \quad \phi'(z) = \frac{\partial\phi(z)}{\partial z} = -z\phi(z), \quad \frac{\partial z^\omega}{\partial\bar{\omega}} = -\frac{1}{\bar{\omega}\sigma_\omega}.$$

Making use of these two results it can be shown that

$$G'(\bar{\omega}) = -\frac{1}{\sigma_\omega}\phi(z^\omega), \quad G''(\bar{\omega}) = -\frac{z^\omega}{\bar{\omega}\sigma_\omega^2}\phi(z^\omega), \tag{A.26}$$

[33] The entrepreneurs are assumed to be gender free in this economy and supposedly multiply by splitting into two after having too much curry.

where $G'(\bar{\omega}) = \partial G(\bar{\omega})/\partial\bar{\omega}$ and $G''(\bar{\omega}) = \partial^2 G(\omega)/\partial\bar{\omega}^2$. In addition,

$$\Gamma'(\bar{\omega}) = \frac{\Gamma(\bar{\omega}) - G(\bar{\omega})}{\bar{\omega}} = 1 - \Phi(z^\omega), \quad \Gamma''(\bar{\omega}) = -\frac{1}{\bar{\omega}\sigma_\omega}\phi(z^\omega), \tag{A.27}$$

where $\Gamma'(\bar{\omega}) = \partial\Gamma(\bar{\omega})/\partial\bar{\omega}$ and $\Gamma''(\bar{\omega}) = \partial^2\Gamma(\omega)/\partial\bar{\omega}^2$. We will also have use for the following derivatives

$$G_{\sigma_\omega}(\bar{\omega}) = -\frac{z^\omega}{\sigma_\omega}\phi(z^\omega - \sigma_\omega), \quad \Gamma_{\sigma_\omega}(\bar{\omega}) = -\phi(z^\omega - \sigma_\omega), \tag{A.28}$$

where $G_{\sigma_\omega}(\bar{\omega}) = \partial G(\bar{\omega})/\partial\sigma_\omega$ and $\Gamma_{\sigma_\omega}(\bar{\omega}) = \partial\Gamma(\bar{\omega})/\partial\sigma_\omega$ and we have made use of

$$\frac{\partial z^\omega}{\partial\sigma_\omega} = 1 - \frac{z^\omega}{\sigma_\omega}.$$

Finally, it can be shown that

$$G'_{\sigma_\omega}(\bar{\omega}) = -\frac{\phi(z^\omega)}{\sigma_\omega^2}\Big[1 - z^\omega(z^\omega - \sigma_\omega)\Big], \quad \Gamma'_{\sigma_\omega}(\bar{\omega}) = \left(\frac{z^\omega}{\sigma_\omega} - 1\right)\phi(z^\omega), \tag{A.29}$$

where $G'_{\sigma_\omega}(\bar{\omega}) = \partial^2 G(\bar{\omega})/\partial\bar{\omega}\partial\sigma_\omega$ and $\Gamma'_{\sigma_\omega}(\bar{\omega}) = \partial^2\Gamma(\bar{\omega})/\partial\bar{\omega}\partial\sigma_\omega$.

A.10.2. Steady-State Elasticities

The steady-state default probability, $\bar{F}$, is assumed to be calibrated at 0.03 (0.0075) in annual (quarterly) terms by Del Negro and Schorfheide (2013). This means that

$$F(\bar{\omega}) = \bar{F}, \tag{A.30}$$

or

$$z^\omega = \frac{\ln\bar{\omega} + (1/2)\sigma_\omega^2}{\sigma_\omega} = \Phi^{-1}(\bar{F}), \tag{A.31}$$

where $\Phi^{-1}(\cdot)$ is the inverted normal. With $\bar{F}$ being known it follows that z^ω is known and that $\bar{\omega}$ may be treated as a function of σ_ω. That is,

$$\bar{\omega}(\sigma_\omega) = \exp\big(z^\omega\sigma_\omega - (1/2)\sigma_\omega^2\big). \tag{A.32}$$

Del Negro and Schorfheide (2013) suggest that we may solve for the steady-state value of σ_ω from

$$\zeta_{sp,b} = -\frac{\zeta_{b,\bar{\omega}}/\zeta_{z,\bar{\omega}}}{1 - \big(\zeta_{b,\bar{\omega}}/\zeta_{z,\bar{\omega}}\big)}\frac{n/k}{1 - (n/k)}, \tag{A.33}$$

where $\zeta_{b,\bar{\omega}}$ and $\zeta_{z,\bar{\omega}}$ are elasticities to be defined below, the parameter $\zeta_{sp,b}$ is estimated, while the steady-state ratio

$$\frac{n}{k}(\bar{\omega}) = 1 - \big[\Gamma(\bar{\omega}) - \mu^e G(\bar{\omega})\big]\frac{r^e}{r}, \tag{A.34}$$

and

$$\mu^e(\bar{\omega}) = \frac{1 - (r/r^e)}{\Big(G'(\bar{\omega})/\Gamma'(\bar{\omega})\Big)\big[1 - \Gamma(\bar{\omega})\big] + G(\bar{\omega})}. \tag{A.35}$$

It may be noted that $(n/k)/[1 - (n/k)]$ is equal to the inverse of steady-state leverage (where leverage is denoted by ϱ in the Technical Appendix to Del Negro and Schorfheide, 2013), while

μ^e is related to the steady-state bankruptcy costs.[34] Furthermore, the ratio r^e/r is determined from the estimate of s; see below equation (A.24).

In the Technical Appendix to Del Negro and Schorfheide (2013) it is shown that

$$\zeta_{b,\bar{\omega}} = \frac{\bar{\omega}\mu^e(n/k)\left[\Gamma''(\bar{\omega})G'(\bar{\omega}) - G''(\bar{\omega})\Gamma'(\bar{\omega})\right]}{\left[\Gamma'(\bar{\omega}) - \mu^e G'(\bar{\omega})\right]^2 \left[1 - \Gamma(\bar{\omega}) + \Gamma'(\bar{\omega})\frac{\Gamma(\bar{\omega}) - \mu^e G(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu^e G'(\bar{\omega})}\right](r^e/r)}, \tag{A.36}$$

while

$$\zeta_{z,\bar{\omega}} = \frac{\bar{\omega}\left[\Gamma'(\bar{\omega}) - \mu^e G'(\bar{\omega})\right]}{\Gamma(\bar{\omega}) - \mu^e G(\bar{\omega})}, \tag{A.37}$$

while n/k and μ^e are given by (A.34) and (A.35), respectively. Plugging the expression for $\zeta_{b,\bar{\omega}}$, $\zeta_{z,\bar{\omega}}$, μ^e, and n/k into equation (A.33) and making use of (A.32) it is possible to solve for σ_ω numerically, such as with the `fzero` or the `fsolve` function in Matlab.

The elasticities of the net worth equation can now be computed from the parameters of the model. The spread elasticity with respect to the spread shock is

$$\zeta_{sp,\sigma_\omega} = \frac{(\zeta_{b,\bar{\omega}}/\zeta_{z,\bar{\omega}})\zeta_{z,\sigma_\omega} - \zeta_{b,\sigma_\omega}}{1 - (\zeta_{b,\bar{\omega}}/\zeta_{z,\bar{\omega}})}, \tag{A.38}$$

where

$$\zeta_{b,\sigma_\omega} = \frac{\sigma_\omega\left[A(\bar{\omega}) + B(\bar{\omega})\right]}{C(\bar{\omega})}, \tag{A.39}$$

with

$$\begin{aligned}
A(\bar{\omega}) &= \left[\frac{1 - \mu^e(G_{\sigma_\omega}(\bar{\omega})/\Gamma_{\sigma_\omega}(\bar{\omega}))}{1 - \mu^e(G'(\bar{\omega})/\Gamma'(\bar{\omega}))} - 1\right]\Gamma_{\sigma_\omega}(\bar{\omega})(r^e/r), \\
B(\bar{\omega}) &= \frac{\mu^e(n/k)\left[G'(\bar{\omega})\Gamma'_{\sigma_\omega}(\bar{\omega}) - \Gamma'(\bar{\omega})G'_{\sigma_\omega}(\bar{\omega})\right]}{\left[\Gamma'(\bar{\omega}) - \mu^e G'(\bar{\omega})\right]^2}, \\
C(\bar{\omega}) &= \left[1 - \Gamma(\bar{\omega})\right](r^e/e) + \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu^e G'(\bar{\omega})}\left[1 - (n/k)\right],
\end{aligned}$$

while

$$\zeta_{z,\sigma_\omega} = \frac{\sigma_\omega\left[\Gamma_{\sigma_\omega}(\bar{\omega}) - \mu^e G_{\sigma_\omega}(\bar{\omega})\right]}{\Gamma(\bar{\omega}) - \mu^e G(\bar{\omega})}. \tag{A.40}$$

Turning to the remaining elasticities of the net worth equation, it is shown in the Technical Appendix of Del Negro and Schorfheide (2013) that

$$\zeta_{n,e} = \kappa^e \frac{r^e}{\pi\gamma}(n/k)^{-1}\Big(1 - \mu^e G(\bar{\omega})\left[1 - (\zeta_{G,\bar{\omega}}/\zeta_{z,\bar{\omega}})\right]\Big), \tag{A.41}$$

where

$$\zeta_{G,\bar{\omega}} = \frac{\bar{\omega}G'(\bar{\omega})}{G(\bar{\omega})}.$$

[34] The bank monitors the entrepreneurs and extracts a fraction $(1 - \mu^e)$ of its revenues given that the bank assumes the draw of ω is below the threshold $\bar{\omega}$. The remaining fraction μ^e may therefore be regarded as the bankruptcy costs.

Next,

$$\zeta_{n,r} = \big(\kappa^e/\beta\big)\big(n/k\big)^{-1}\Big(1 - \big(n/k\big) + \mu^e G\big(\bar{\omega}\big)\big(r^e/r\big)\big[1 - \big(\zeta_{G,\bar{\omega}}/\zeta_{z,\bar{\omega}}\big)\big]\Big), \tag{A.42}$$

$$\zeta_{n,q} = \kappa^e \frac{r^e}{\pi\gamma}\big(n/k\big)^{-1}\left[1 - \mu^e G\big(\bar{\omega}\big)\left(1 + \frac{\zeta_{G,\bar{\omega}}\big(n/k\big)}{\zeta_{z,\bar{\omega}}\big[1 - \big(n/k\big)\big]}\right)\right] - \big(\kappa^e/\beta\big)\big(n/k\big)^{-1}, \tag{A.43}$$

$$\zeta_{n,n} = \big(\kappa^e/\beta\big) + \kappa^e \frac{r^e}{\pi\gamma}\mu^e G\big(\bar{\omega}\big)\frac{\zeta_{G,\bar{\omega}}}{\zeta_{z,\bar{\omega}}\big[1 - \big(n/k\big)\big]}. \tag{A.44}$$

$$\tag{A.45}$$

Finally,

$$\zeta_{n,\sigma_\omega} = \kappa^e \mu^e G\big(\bar{\omega}\big)\frac{r^e}{\pi\gamma}\big(n/k\big)^{-1}\big[\zeta_{G,\sigma_\omega} - \big(\zeta_{G,\bar{\omega}}/\zeta_{z,\bar{\omega}}\big)\zeta_{z,\sigma_\omega}\big], \tag{A.46}$$

where

$$\zeta_{G,\sigma_\omega} = \frac{\sigma_\omega G_{\sigma_\omega}\big(\bar{\omega}\big)}{G\big(\bar{\omega}\big)}.$$

A.11. The Smets and Wouters Model with Unemployment

The Galí, Smets, and Wouters (2012, GSW) model is an extension of the standard Smets and Wouters model which explicitly provides a mechanism for explaining unemployment. This is accomplished by modelling the labor supply decisions on the extensive margin (whether to work or not) rather than on the intensive margin (how many hours to work). As a consequence, the unemployment rate is added as an observable variable, while labor supply shocks are admitted. This allows the authors to overcome the lack of identification of wage markup and labor supply shocks raised by Chari et al. (2009) in their critique of new Keynesian models. From a technical perspective the GSW model is also based on the assumption of log-utility, i.e. the parameter σ_c is assumed to be unity, but the equations presented below will instead be written as if this is a free parameter and therefore treat σ_c as in Section A.1 and A.7.

A.12. Extending the Sticky Economy

Smets, Warne, and Wouters (2014) present a variant of the GSW model aimed for the euro area and this version is presented below. The log-linearized aggregate resource constraint is given by equation (A.1). The consumption Euler equation may be expressed as

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \big(\hat{r}_t - E_t\hat{\pi}_{t+1}\big) - c_3^{-1}\varepsilon_t^b, \tag{A.47}$$

where $\hat{\lambda}_t$ is the log-linearized marginal utility of consumption, given by

$$\hat{\lambda}_t = -\frac{\sigma_c}{1 - (\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right] + \frac{(\sigma_c - 1)(w^h l/c)}{1 - (\lambda/\gamma)}\hat{l}_t, \tag{A.48}$$

while ε_t^b is the preference shock. Making use of (A.47) and (A.48), the consumption Euler equation can be written in the more familiar form in equation (A.2), and where c_3 is given by the expressions below this equation. However, below we have use for the expression for $\hat{\lambda}_t$.

Concerning the log-linearized price Phillips curve in the GSW model, it is similar to equation (A.11), but differs in the way that the price markup shock enters the model:

$$\hat{\pi}_t = \pi_1 \hat{\pi}_{t-1} + \pi_2 E_t \hat{\pi}_{t+1} - \pi_3 \big(\hat{\mu}_t^p - \hat{\mu}_t^{n,p}\big). \tag{A.49}$$

The expressions for π_i below equation (A.11) hold and the natural price markup shock $\hat{\mu}_t^{n,p} = 100\epsilon_t^p$. The average price markup and the real marginal cost variables are given by equation (A.9) and (A.10), respectively. Relative to equation (A.11), the price markup shock $\varepsilon_t^p = \pi_3 100\epsilon_t^p$. Smets, Warne, and Wouters (2014) uses the shock ϵ_t^p, while we wish to treat the price markup shock symmetrically in the three models and therefore keep equation (A.11) such that our price markup shock is given by ε_t^p.[35]

In the GSW model, the wage Phillips curve in equation (A.14) is replaced with the following expression for real wages

$$\hat{w}_t = w_1 \hat{w}_{t-1} + (1 - w_1)\big[E_t \hat{w}_{t+1} + E_t \hat{\pi}_{t+1}\big] - w_2 \hat{\pi}_t + w_3 \hat{\pi}_{t-1} - w_4 \big[\hat{\mu}_t^w - \hat{\mu}_t^{n,w}\big], \tag{A.50}$$

where $\hat{\mu}_t^w$ is the average wage markup and $\hat{\mu}_t^{n,w}$ is the natural wage markup. Notice that the w_i parameters are given by the expressions below equation (A.14) for $i = 1, 2, 3$, while

$$w_4 = \frac{\big(1 - \xi_w\big)\big(1 - \beta\gamma^{1-\sigma_c}\xi_w\big)}{\big(1 + \beta\gamma^{1-\sigma_c}\big)\xi_w\big(1 + \varepsilon_w \sigma_l\big)}.$$

In addition, GSW and Smets, Warne, and Wouters (2014) let the curvature of the Kimball labor market aggregator be given by

$$\varepsilon_w = \frac{\phi_w}{\phi_w - 1}.$$

The average wage markup is defined as the difference between the real wage and the marginal rate of substitution, which is a function of the adjusted smoothed trend in consumption, $\hat{x}_t$, the marginal utility of consumption $\hat{\lambda}_t$, total employment, $\hat{e}_t$, and the labor supply shock. This expression is equal to the elasticity of labor supply times the unemployment rate, i.e.

$$\begin{aligned} \hat{\mu}_t^w &= \sigma_l \hat{u}_t \\ &= \hat{w}_t - \big(\hat{x}_t - \hat{\lambda}_t + \varepsilon_t^s + \sigma_l \hat{e}_t\big), \end{aligned} \tag{A.51}$$

where unemployment is defined as labor supply minus total employment:

$$\hat{u}_t = \hat{l}_t^s - \hat{e}_t. \tag{A.52}$$

The labor supply shock is assumed to follow and AR(1) process such that

$$\varepsilon_t^s = \rho_s \varepsilon_{t-1}^s + \sigma_s \eta_t^s. \tag{A.53}$$

[35]This has the advantage of separating the parameters describing the price markup shock process from the parameters of the price Phillips curve as π_3 is no longer multiplied by the shock process. This separation of parameters may be also useful when estimating the parameters of the model.

The natural wage markup shock is expressed as $100\epsilon_t^w$ and is, in addition, equal to the elasticity of labor supply times the natural rate of unemployment. Accordingly,

$$\hat{\mu}_t^{n,w} = 100\epsilon_t^w = \sigma_l \hat{u}_t^n. \tag{A.54}$$

The natural rate of unemployment, $\hat{u}_t^n$, is defined as the unemployment rate that would prevail in the absence of nominal wage rigidities, and is here proportional to the natural wage markup. Finally, we here let the wage markup shock, ε_t^w, be defined such that:

$$\varepsilon_t^w = w_4 \hat{\mu}_t^{n,w} = w_4 100\epsilon_t^w. \tag{A.55}$$

Hence, the wage markup shock ε_t^w enters equation (A.50) suitably re-scaled and, apart from the definition of the w_4 parameter, the wage Phillips curve is identical to equation (A.14). This has the advantage of allowing us to treat the wage markup shock as symmetrically as possible in the three models.[36]

The adjusted smoothed trend in consumption is given by

$$\hat{x}_t = \hat{\kappa}_t - \frac{1}{1-(\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right], \tag{A.56}$$

where the second term on the right hand side is the adjustment, while the smoothed trend in consumption is given by

$$\hat{\kappa}_t = \left(1-\upsilon\right)\hat{\kappa}_{t-1} + \frac{\upsilon}{1-(\lambda/\gamma)}\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right]. \tag{A.57}$$

Making use of equation (A.48), we find that $\hat{x}_t - \hat{\lambda}_t = \hat{\kappa}_t$ when $\sigma_c = 1$, thereby simplifying the expression of the average markup in (A.51). Provided that $\sigma_c = 1$, the parameter υ measures the weight on the marginal utility of consumption of the smooth trend in consumption. Notice that if $\sigma_c = \upsilon = 1$ and $\varepsilon_t^s = 0$, then the average wage markup in (A.51) is very similar to the wage markup in equation (A.13) of the Smets and Wouters model, with the only difference being that $\hat{e}_t$ replaces $\hat{l}_t$.

A.13. Extensions to the Flexible Economy

The flexible price and wage equations are obtained by assuming that the natural price and wage markup processes are zero and by setting $\xi_w = \xi_p = 0$ and $\imath_m = \imath_p = 0$, as in Section A.3. Inflation is equal to the steady-state inflation rate and real wages are, as mentioned before, set such that they are equal to the marginal rate of substitution between labor and consumption as well as to the marginal product of labor, with the effect that unemployment is zero.

[36] Analogous to the case for the price markup shock, this also separates the wage markup shock parameters better from the parameters entering w_4; GSW and Smets et al. (2014) use the shock ϵ_t^w as the wage markup shock.

The changes to the equations describing the evolution of the flexible price and wage economy are given by

$$
\begin{aligned}
\hat{\lambda}_t^f &= E_t\hat{\lambda}_{t+1}^f + \hat{r}_t^f - c_3^{-1}\varepsilon_t^b \\
\hat{\lambda}_t^f &= -\frac{\sigma_c}{1-(\lambda/\gamma)}\left[\hat{c}_t^f - \frac{\lambda}{\gamma}\hat{c}_{t-1}^f\right] + \frac{(\sigma_c-1)(w^h l/c)}{1-(\lambda/\gamma)}\hat{l}_t^f, \\
\hat{w}_t^f &= \sigma_l\hat{l}_t^f + \hat{x}_t^f - \hat{\lambda}_t^f + \varepsilon_t^s, \\
\hat{x}_t^f &= \hat{\kappa}_t^f - \frac{1}{1-(\lambda/\gamma)}\left[\hat{c}_t^f - \frac{\lambda}{\gamma}\hat{c}_{t-1}^f\right], \\
\hat{\kappa}_t^f &= (1-\upsilon)\hat{\kappa}_{t-1}^f + \frac{\upsilon}{1-(\lambda/\gamma)}\left[\hat{c}_t^f - \frac{\lambda}{\gamma}\hat{c}_{t-1}^f\right].
\end{aligned}
\tag{A.58}
$$

The first two equations for $\hat{\lambda}_t^f$ above replace the $\hat{c}_t^f$ equation in (A.17).

A.14. Augmented Measurement Equations

The steady-state values of the capital-output ratio, etc., are determined as in Section A.5 for the Smets and Wouters model. The model is consistent with a balanced steady-state growth path, driven by deterministic labor augmenting trend growth, and the vector of observed variables for the euro area is augmented with an equation for unemployment, denoted by u_t. Specifically,

$$u_t = \bar{u} + \hat{u}_t. \tag{A.59}$$

The steady-state parameter $\bar{u}$ is given by

$$\bar{u} = 100\left(\frac{\phi_w - 1}{\sigma_l}\right), \tag{A.60}$$

where $(\phi_w - 1)$ is the steady-state labor market markup and σ_l is the elasticity of labor supply with respect to the real wage. Apart from the parameter σ_c, four additional structural parameters are calibrated. These are $g_y = 0.18$, $\delta = 0.025$, and $\varepsilon_p = 10$ as in the Smets and Wouters model. Unlike Galí et al. (2012) and Smets et al. (2014) we estimate the persistence parameter of the labor supply shock, ρ_s, and calibrate $\phi_w = 1.5$. The latter parameter can also be estimated and yields posterior mean and mode estimates very close to 1.5 when using the same prior as in Galí et al. (2012) and Smets et al. (2014). We have opted to calibrate it in this study in order to treat it in the same way across the three models.[37]

[37] It may also be noted that the definition of $\bar{u}$ for the mean unemployment rate invites for a potentially high correlation between posterior draws of ϕ_w and σ_l, an issue we have observed for the euro area full sample data discussed in Sections 4. These two parameters are also closely connected in the definition of w_4 below equation (A.50) where the curvature of the Kimball labor market aggregator, $\varepsilon_w = \phi_w/(\phi_w - 1)$, is multiplied by σ_l.

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Peter McAdam
European Central Bank, Frankfurt am Main, Germany; email: peter.mcadam@ecb.europa.eu

Anders Warne
European Central Bank, Frankfurt am Main, Germany; email: anders.warne@ecb.europa.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

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