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Fiscal seigniorage and price level determination in a currency union

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Abstract

I study price level determination in a currency union when some member countries' government securities earn a convenience yield. These "convenience assets" generate fiscal seigniorage revenues that, given appropriate fiscal and monetary policies, back the union's price level, much like primary surpluses and monetary seigniorage do. An exogenous drop in the private-sector demand for convenience assets reduces seigniorage revenues and raises the price level. It also results in a wealth transfer across countries owing to the heterogeneity in convenience yields.

Keywords: currency union, fiscal theory of the price level, convenience yield, cross-country heterogeneity (*JEL:* E31, E63, F45)

Non-technical summary

Investors value the government securities of countries like the United States and Germany not only for their pecuniary returns, but also for their liquidity and safety. As a result, investors accept lower returns on these securities compared to those on other assets without such properties, the return differential being referred to as the convenience yield. For governments, the ability to issue debt at a premium provides an additional source of revenue—fiscal seigniorage.

This paper explores the role of fiscal seigniorage revenues, arising from convenience yields, for price level determination in a currency union. My analysis is in the spirit of the fiscal theory of the price level. The fiscal theory emphasizes the role of fiscal conditions—notably the balance between the nominal value of government liabilities and future fiscal surpluses—in determining the price level. However, fiscal theory models typically abstract from convenience yields on government securities and the associated seigniorage revenues.

In a currency union like the euro area, with a common central bank and multiple national fiscal authorities, the convenience properties of government securities are likely to differ across member countries. This paper provides a tractable framework to study price level determination in a currency union with heterogeneous convenience yields.

The model economy consists of two countries forming a currency union. Each country is inhabited by a representative infinitely-lived household, a fiscal authority, and a national central bank. Together, the two national central banks form the union's common central bank. Households derive utility from consumption—each period they receive an endowment—and from holding convenience assets, notably central bank reserves and government bonds. The union is heterogeneous in that the ability to issue convenience assets is not shared uniformly by all fiscal authorities. Rather, only the government bonds of one member country are assumed to provide non-pecuniary benefits to the households in the union. These non-pecuniary benefits drive down the interest rate on the convenience bonds (and reserves) relative to the interest rate on the bonds that are valued solely for their pecuniary returns.

In equilibrium, the real value of total public sector liabilities in the union equals the present discounted value of the sum of primary surpluses, monetary seigniorage, and fiscal seigniorage. With an appropriate set of fiscal and monetary policies in place, the fiscal seigniorage revenues back the union price level, much like primary surpluses and monetary seigniorage revenues do, and

the price level is uniquely determined.

The presence of government bonds with a convenience yield also has implications for equilibrium allocations. In an otherwise symmetric currency union, the fact that member countries differ in terms of the convenience properties of their government bonds may lead to an asymmetric distribution of wealth across households. Both households find it optimal to hold convenience bonds. Thus, both households are giving up real resources to the issuing fiscal authority. These fiscal seigniorage revenues, all else equal, reduce the amount of real resources that said fiscal authority has to collect from other sources to back its debt. If the fiscal authority uses its seigniorage revenues to lower the tax burden for the domestic household, it effectively channels resources collected from the foreign household to the domestic household.

Exogenous changes in households' demand for convenience assets move the price level by increasing or reducing the amount of real resources that back it. They also lead to a redistribution of wealth across countries. An exogenous and symmetric drop in the demand for convenience assets reduces the convenience yield. The present value of total seigniorage revenues declines, putting upward pressure on the price level. The decline in seigniorage revenues is not symmetric across countries. Seigniorage revenues of the country whose government bonds earn a convenience yield drop by more than those of the country whose bonds are only valued for their pecuniary return. In equilibrium, the latter country makes a loan to the former country, and the loan is rolled over forever. If households' ex-ante wealth differs, then consumption levels change as well. The richer household raises consumption and the poorer household reduces consumption, permanently. Since the richer household holds more convenience assets, the drop in the convenience yield reduces the amount of real resources that she is giving up to the public sector by more than the amount of resources that the poorer household is giving up.

1 Introduction

Investors value the government securities of countries like the United States and Germany not only for their pecuniary returns, but also for their liquidity and safety (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Reis, 2022; Jiang et al., 2025). As a result, investors accept lower returns on these securities compared to those on other assets without such properties, the return differential being referred to as the convenience yield. For governments, the ability to issue debt at a premium provides an additional source of revenue—fiscal seigniorage.[1]

This paper explores the role of fiscal seigniorage revenues, arising from convenience yields, for price level determination in a currency union. My analysis is in the spirit of the fiscal theory of the price level. The fiscal theory emphasizes the role of fiscal conditions—notably the balance between the nominal value of government liabilities and future fiscal surpluses—in determining the price level. However, fiscal theory models typically abstract from convenience yields on government securities and the associated seigniorage revenues.

In a currency union like the euro area, with a common central bank and multiple national fiscal authorities, the convenience properties of government securities are likely to differ across member countries (e.g., Jiang et al., 2025). This paper provides a tractable framework to study price level determination in a currency union with heterogeneous convenience yields.

The model economy consists of two countries forming a currency union. Each country is inhabited by a representative infinitely-lived household, a fiscal authority, and a national central bank. Together, the two national central banks form the union's common central bank. Households derive utility from consumption—each period they receive an endowment—and from holding convenience assets, notably central bank reserves and government bonds. The union is heterogeneous in that the ability to issue convenience assets is not shared uniformly by all fiscal authorities. Rather, only the government bonds of one member country are assumed to provide non-pecuniary benefits to the households in the union. These non-pecuniary benefits drive down the interest rate on the convenience bonds (and reserves) relative to the interest rate on the bonds that are valued solely for their pecuniary returns.

In equilibrium, the real value of total public sector liabilities in the union equals the present

[1]Fiscal seigniorage revenue equals the interest rate premium times the real amount of government debt. Reis (2022) refers to fiscal seigniorage revenue as "debt revenue".

discounted value of the sum of primary surpluses, monetary seigniorage (arising from the convenience yield on central bank reserves), and fiscal seigniorage (arising from the convenience yield on government bonds). With an appropriate set of fiscal and monetary policies in place, the fiscal seigniorage revenues back the union price level, much like primary surpluses and monetary seigniorage revenues do, and the price level is uniquely determined. For fiscal policy, I assume that both national authorities run constant real primary surpluses—a common benchmark specification in fiscal theory models. Unlike in standard fiscal theory models, these surpluses may be strictly negative, i.e., fiscal authorities may run permanent primary deficits, provided that their present discounted value is smaller in magnitude than the present discounted value of seigniorage revenues.[2] For monetary policy, I assume that the common central bank adjusts the interest rate on reserves in response to changes in the supply of (and exogenous changes in the demand for) convenience assets so as to stabilize the interest rate on the non-convenience government bonds at the level that is consistent with inflation being at target.[3] If nominal interest rates are constrained by a lower bound, the public sector has to supply a sufficient amount of convenience assets for the constraint on the policy rate to be slack, and the central bank to be able to achieve its inflation target.

The presence of government bonds with a convenience yield also has implications for equilibrium allocations. In an otherwise symmetric currency union, the fact that member countries differ in terms of the convenience properties of their government bonds may lead to an asymmetric distribution of wealth across households. Both households find it optimal to hold convenience bonds. Thus, both households are giving up real resources to the issuing fiscal authority. These fiscal seigniorage revenues, all else equal, reduce the amount of real resources that said fiscal authority has to collect from other sources to back its debt. If the fiscal authority uses its seigniorage revenues to lower the tax burden for the domestic household, it effectively channels resources collected from the foreign household to the domestic household.

Exogenous changes in households' demand for convenience assets move the price level by increasing or reducing the amount of real resources that back it. They also lead to a redistribution

[2]The baseline model features log utility. In the case of power utility, the price level is uniquely determined if the present value of primary surpluses is positive, or, if the present value is strictly negative and households' demand for convenience assets is sufficiently inelastic.

[3]The interest rate on non-convenience government bonds and (future) inflation are linked by a Fisher equation. Monetary policy also decides about national central banks' government bond holdings, their remittances to national fiscal authorities, the issuance of reserves, and potential claims between the two national central banks.

of wealth across countries. An exogenous drop in the demand for convenience assets—a symmetric preference shock across households—reduces the convenience yield (both on central bank reserves and on convenience bonds). The present value of total seigniorage revenues declines, putting upward pressure on the price level. The decline in seigniorage revenues is not symmetric across countries. Seigniorage revenues of the country whose government bonds earn a convenience yield drop by more than those of the country whose bonds are only valued for their pecuniary return. In equilibrium, the latter country makes a loan to the former country, and the loan is rolled over forever.[4] If households' ex-ante wealth differs, for instance, because the household living in the country whose bonds earn a convenience yield pays less taxes than the other household (see previous paragraph), then consumption levels change as well, even so the preference shock is symmetric. The richer household raises consumption and the poorer household reduces consumption, permanently. Since the richer household holds more convenience assets, the drop in the convenience yield reduces the amount of real resources that she is giving up to the public sector by more than the amount of resources that the poorer household is giving up.

Changes in the present value of the sum of primary surpluses—fiscal shocks—also move the price level, and, when asymmetric, redistribute wealth across public sectors and households. The presence of convenience assets attenuates the wealth transfers across households. Intuitively, the household who benefits from the asymmetric fiscal shock raises not only consumption but also her convenience asset holdings, which means that the amount of resources that are extracted from her via seigniorage rises. The household who suffers from the asymmetric shock, instead, reduces her convenience asset holdings, which means that the amount of resources that are extracted from her via seigniorage decline.

This paper belongs to the literature on the fiscal theory of the price level. Early seminal contributions include Leeper (1991), Sims (1994), and Woodford (1994). Cochrane (2023) provides a comprehensive textbook treatment. Bassetto and Cui (2018), Berentsen and Waller (2018), Bonam (2020) and Kaplan et al. (2023) consider the fiscal theory in models where government debt provides non-pecuniary services.[5] Bassetto and Cui (2018) and Kaplan et al. (2023) show that their

[4] Changes in the demand for convenience government bonds *relative* to central bank reserves also result in a wealth transfer across countries. However, in the baseline model with log utility, the price level is not affected. In the model with power utility, the sign of the price level effect depends on the elasticity of households' demand for convenience assets.

[5] Bassetto and Cui (2018) and Berentsen and Waller (2018) revisit the fiscal theory in search-theoretic models

models are prone to equilibrium multiplicity when fiscal authorities run persistent fiscal deficits. Berentsen and Waller (2018) show how changes in the liquidity premium on government debt can move the price level. These papers do not consider a currency union. Bergin (2000), Sims (1997), Woodford (1998) section 5, and Maćkowiak and Schmidt (2025) study price level determination in a currency union.[6] They show that in a currency union, it is the balance between the nominal value of total public sector liabilities in the union and the sum of future surpluses across member countries that matters for the determination of the price level. These papers do not study convenience yields on government bonds and the backing that may be provided by the associated fiscal seigniorage revenues.

The remainder of the paper is organized as follows. Section 2 presents the model and defines equilibrium. Section 3 specifies fiscal and monetary policies, and studies price level determination and equilibrium allocations. Section 4 explores how changes in the convenience property of government securities, and changes in fiscal policy affect the price level and equilibrium allocations. Section 5 presents additional analysis. Section 6 concludes.

2 Model

I consider a currency union that consists of two countries indexed by $i = 1, 2$. Each country i is populated by an infinitely-lived representative household, a national fiscal authority and a national central bank (NCB). The two NCBs together comprise the currency union's central bank which issues the common fiat currency.

2.1 Households

Household i enters period $t = 0$ with some initial financial wealth. In each period $t \geq 0$, she receives an endowment of $Y_{it} \geq 0$ units of the common consumption good, pays lump-sum taxes S_{it}, consumes C_{it}, and saves. Financial markets are incomplete. The household has access to non-state-contingent, one-period, nominal bonds issued by the two national fiscal authorities and to

of monetary exchange. Kaplan et al. (2023) analyze equilibrium existence and uniqueness in a heterogeneous-agent model and, in Appendix B of their paper, in a representative-agent model with bonds-in-utility. Bonam (2020) studies equilibrium determinacy in a linearized New Keynesian model with bonds-in-utility.

[6] Jarociński and Maćkowiak (2018) and Bianchi et al. (2023) study macroeconomic stabilization policies in a currency union using fiscal theory models.

interest-paying central bank reserves (which can also be thought of as digital currency).[7] Reserves, and bonds issued by fiscal authority 1 provide non-pecuniary benefits to households, i.e. they are convenience assets. Bonds issued by fiscal authority 2 do not provide such benefits.

In period 0, household i maximizes expected lifetime utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U(C_{it}) + \theta V \left(\frac{X_{it}^H + \theta_B B_{i1t}^H}{P_t} \right) \right) \tag{1}$$

subject to a sequence of budget constraints

$$P_t C_{it} + P_t S_{it} + X_{it}^H + \sum_{j=1}^{2} B_{ijt}^H \leq P_t Y_{it} + R_{t-1} X_{it-1}^H + \sum_{j=1}^{2} R_{jt-1} B_{ijt-1}^H \tag{2}$$

and a no-Ponzi game condition. P_t is the price level. $X_{it}^H \geq 0$ denotes central bank reserves held by household i at the end of period t, which pay interest R_t at the beginning of period $t+1$. B_{ijt}^H are bonds of fiscal authority $j = 1, 2$ held by household i at the end of period t, which pay interest R_{jt} at the beginning of period $t+1$. $U(\cdot)$ and $V(\cdot)$ are differentiable, increasing and concave functions of their arguments. Parameter $\beta \in (0,1)$ is the subjective discount factor, $\theta > 0$ weighs the utility provided by the convenience assets, and $\theta_B \in (0,1]$ measures the convenience services provided by country 1 government bonds relative to central bank reserves. When $\theta_B = 1$, country 1 bonds and reserves are perfect substitutes.

The first-order conditions for an interior solution to household i's optimization problem are

$$0 = U'(C_{it}) - \beta R_{2t} \mathrm{E}_t U'(C_{i,t+1}) \frac{P_t}{P_{t+1}} \tag{3}$$

$$\theta V' \left(\frac{X_{it}^H + \theta_B B_{i1t}^H}{P_t} \right) = U'(C_{it}) - \beta R_t \mathrm{E}_t U'(C_{it+1}) \frac{P_t}{P_{t+1}} \tag{4}$$

$$\theta \theta_B V' \left(\frac{X_{it}^H + \theta_B B_{i1t}^H}{P_t} \right) = U'(C_{it}) - \beta R_{1t} \mathrm{E}_t U'(C_{it+1}) \frac{P_t}{P_{t+1}} \tag{5}$$

and the transversality condition

$$\lim_{T \to \infty} \mathrm{E}_t Q_{itT} \frac{R_T X_{iT}^H + \sum_{j=1}^{2} R_{jT} B_{ijT}^H}{P_{T+1}} = 0, \tag{6}$$

[7] Explicitly accounting for central bank reserves in the model has the advantage that there is an obvious choice for the central bank's policy rate, namely the interest rate on reserves. See Section 2.2.

for all $t \geq 0$, where $Q_{itT} = \beta^{T-t} \frac{U'(C_{iT})}{U'(C_{it})}$ is the real stochastic discount factor in period t for period $T \geq t$, and $U'(\cdot)$ and $V'(\cdot)$ are the first derivatives of functions U and V.

By combining equations (3) and (4), we can relate the spread between the interest rate on country 2 bonds and the interest rate on reserves to the ratio of household i's marginal utility from holding reserves and her marginal utility of consumption

$$\frac{\theta V' \left(\frac{X_{it}^H + \theta_B B_{i1t}^H}{P_t} \right)}{U'(C_{it})} = \frac{R_{2t} - R_t}{R_{2t}}. \tag{7}$$

Household i's demand for convenience assets increases with her consumption and decreases with the interest rate spread. Likewise, by combining equations (3) and (5), we can relate the spread between the interest rate on country 2 bonds and the interest rate on country 1 bonds to the ratio of household i's marginal utility from holding country 1 bonds and her marginal utility of consumption

$$\frac{\theta \theta_B V' \left(\frac{X_{it}^H + \theta_B B_{i1t}^H}{P_t} \right)}{U'(C_{it})} = \frac{R_{2t} - R_{1t}}{R_{2t}}. \tag{8}$$

Combining the two optimality conditions, we obtain an equilibrium relationship between the two interest rate spreads

$$R_{2t} - R_{1t} = \theta_B (R_{2t} - R_t). \tag{9}$$

When reserves and country 1 government bonds are perfect substitutes, $R_{1t} = R_t$.

Finally, solving forward flow budget constraint (2) starting from period 0, and making use of conditions (3), (6) and (9), we obtain household i's intertemporal budget constraint (see Appendix A for more details)

$$\frac{\sum_j R_{j,-1} B_{ij,-1}^H + R_{-1} X_{i,-1}^H}{P_0} = \sum_{t=0}^{\infty} \mathrm{E}_0 Q_{i0t} \left(C_{it} - Y_{it} + S_{it} + \frac{R_{2t} - R_t}{R_{2t}} \frac{X_{it}^H + \theta_B B_{i1t}^H}{P_t} \right). \tag{10}$$

The term $(R_{2t} - R_t)/R_{2t}(X_{it}^H + \theta_B B_{i1t}^H)/P_t$ on the right hand side of equation (10) represents the real resources that the public sector extracts from household i on top of those extracted from her via direct taxation.

2.2 Public sector

Fiscal authorities. National fiscal authority i enters period 0 with nominal debt $R_{i,-1}B_{i,-1} > 0$. In each period $t \geq 0$, the authority collects lump-sum taxes from the domestic household, receives remittances Z_{it} from NCB i (or makes a transfer to NCB i if $Z_{it} < 0$), pays off its debt incurred from the previous period, and issues new bonds B_{it}. Its flow budget constraint in period t is

$$R_{it-1}B_{it-1} \leq P_t S_{it} + P_t Z_{it} + B_{it}. \tag{11}$$

Central banks. NCB i enters period 0 with nominal liabilities $X_{i,-1} > 0$ and nominal assets $B^{CB}_{i,-1} > 0$ in the form of government bonds. In each period $t \geq 0$, NCB i issues (or soaks up) reserves, buys government bonds, makes remittances to its national fiscal authority (or receives a transfer), and lends to or borrows from NCB $j \neq i$. I assume that NCB i only buys country i government bonds. The flow budget constraint of NCB i in period t then is

$$B^{CB}_{it} + P_t Z_{it} + L_{ijt} - X_{it} \leq R_{it-1}B^{CB}_{it-1} + R_{t-1}\left(L_{ijt-1} - X_{it-1}\right), \tag{12}$$

where B^{CB}_{it} denotes NCB i's purchases of country i government bonds, and L_{ijt} denotes a period t claim of NCB i on NCB j that pays interest R_t at the beginning of period $t+1$.[8]

Together, the two NCBs comprise the common central bank of the union. The common central bank sets the interest rate on reserves, henceforth also referred to as the policy rate. Summing (12) over i, and using $L_{12t} = -L_{21t}$, we obtain the flow budget constraint of the common central bank

$$\sum_i B^{CB}_{it} + P_t \sum_i Z_{it} - X_t \leq \sum_i R_{it-1}B^{CB}_{it-1} - R_{t-1}X_{t-1}, \tag{13}$$

where $X_t = \sum_i X_{it}$.

Valuation of public sector liabilities. Let us first consider the consolidated public sector of country i. Summing the flow budget constraints of fiscal authority i and NCB i in period 0, and

[8] This assumption is consistent with current practice in the euro area. The target balances of euro area member states' NCBs are remunerated at the deposit facility rate, which is also the interest rate paid on the reserves that banks hold in the ECB's deposit facility. The deposit facility rate is the ECB's main monetary policy instrument.

solving forward, we obtain

$$\frac{R_{i,-1}(B_{i,-1}-B_{i,-1}^{CB})+R_{-1}\left(X_{i,-1}-L_{ij,-1}\right)}{P_0}=\sum_{t=0}^{\infty}\mathrm{E}_0Q_{h0t}\Bigg(S_{it}+\frac{R_{2t}-R_{it}}{R_{2t}}\frac{B_{it}-B_{it}^{CB}}{P_t}+\frac{R_{2t}-R_t}{R_{2t}}\frac{X_{it}-L_{ijt}}{P_t}\Bigg)+\lim_{T\to\infty}\mathrm{E}_0Q_{h0T}\Bigg(\frac{R_{iT}\left(B_{iT}-B_{iT}^{CB}\right)}{P_{T+1}}+\frac{R_T\left(X_{iT}-L_{ijT}\right)}{P_{T+1}}\Bigg),\tag{14}$$

which has to hold in equilibrium for $h=1,2$. The last term on the right-hand side of the equation may equal zero—in which case the intertemporal budget constraint of the public sector in country i holds—or not, depending on fiscal and monetary policies.[9]

Next, consider the public sector of the union as a whole. Summing the flow budget constraint of the common central bank (13) and the flow budget constraints of the two national fiscal authorities (11), and solving forward, we obtain (see Appendix A for more details)

$$\frac{\sum_i R_{i,-1}(B_{i,-1}-B_{i,-1}^{CB})+R_{-1}\sum_i X_{i,-1}}{P_0}=\sum_{t=0}^{\infty}\mathrm{E}_0Q_{h0t}\Bigg(\sum_i S_{it}+\frac{R_{2t}-R_{1t}}{R_{2t}}\frac{B_{1t}-B_{1t}^{CB}}{P_t}+\frac{R_{2t}-R_t}{R_{2t}}\frac{\sum_i X_{it}}{P_t}\Bigg)\tag{15}$$

for $h=1,2$. Equation (15) is the valuation equation for total public sector liabilities in the currency union (or intertemporal budget constraint of the public sector in the union). It states that the real value of total public sector liabilities depends on the present discounted value of three terms: (i) the sum of real primary surpluses, (ii) fiscal authority 1's seigniorage revenues, and (iii) the sum of the two NCBs' seigniorage revenues.

2.3 Equilibrium

We are now ready to define equilibrium.

Assumptions. For the remainder, I make the following assumptions. First, I consider a deterministic economy so as to focus on perfect foresight equilibria. This implies $Q_{it,t+k}=\beta^k$ for

[9] That is, following the literature, I refer to the intertemporal budget constraint of public sector i as equation (14) with the last term on the right-hand side being equal to zero.

$i = 1, 2$, $t \geq 0$ and $k \geq 0$. Second, I assume that endowments are constant over time, $Y_{it} = Y_i$. Third, to obtain closed-form expressions, I assume log utility, $U(\cdot) = \log(\cdot)$ and $V(\cdot) = \log(\cdot)$.[10]

Equilibrium. A perfect foresight equilibrium is a sequence of allocations $\{C_{it}, B^H_{ijt}, X^H_{it}\}$, prices $\{P_t, R_{it}\}$ and policies $\{S_{it}, Z_{it}, R_t, B_{it}, B^{CB}_{it}, X_{it}, L_{ijt}\}$, $i, j = 1, 2$, $t \geq 0$ satisfying

$$B_{it} = B^H_{1it} + B^H_{2it} + B^{CB}_{it}, \quad X^H_{it} = X_{it}, \quad L_{12t} = -L_{21t}, \quad i = 1, 2 \tag{16}$$

$$Y_1 + Y_2 = C_{1t} + C_{2t} \tag{17}$$

$$R_{it-1}\frac{B_{it-1}}{P_t} = S_{it} + Z_{it} + \frac{B_{it}}{P_t}, \quad i = 1, 2 \tag{18}$$

$$\frac{B^{CB}_{it} + L_{ijt} - X_{it}}{P_t} + Z_{it} = \frac{R_{it-1}B^{CB}_{it-1} + R_{t-1}\left(L_{ijt-1} - X_{it-1}\right)}{P_t}, \quad i = 1, 2, \quad j \neq i \tag{19}$$

$$1 = \beta R_{2t}\frac{P_t}{P_{t+1}}, \quad C_{it+1} = C_{it}, \quad i = 1, 2 \tag{20}$$

$$\theta C_{it} = \frac{R_{2t} - R_t}{R_{2t}}\left(\frac{X^H_{it} + \theta_B B^H_{i1t}}{P_t}\right), \quad i = 1, 2 \tag{21}$$

$$R_{2t} - R_{1t} = \theta_B\left(R_{2t} - R_t\right), \tag{22}$$

$$(1 + \theta)C_{i0} = Y_i + (1 - \beta)\left(\frac{\sum_j R_{j,-1}B^H_{ij,-1} + R_{-1}X^H_{i,-1}}{P_0} - \sum_{t=0}^{\infty}\beta^t S_{it}\right), \quad i = 1, 2 \tag{23}$$

$$\frac{\sum_i R_{i,-1}(B_{i,-1} - B^{CB}_{i,-1}) + R_{-1}\sum_i X_{i,-1}}{P_0} = \sum_{t=0}^{\infty}\beta^t\left(\sum_i S_{it}\right) + \frac{\theta Y}{1 - \beta}, \tag{24}$$

given initial conditions and endowments $Y_i > 0$, where $Y = \sum_i Y_i$. The first row reports the five financial market clearing conditions, and the second row reports the goods market clearing condition. The third row reports the flow budget constraints of the two fiscal authorities, and the fourth row the flow budget constraints of the two NCBs. Rows five to seven report household optimality conditions. The first equation in row five is the Fisher equation, which follows from households' consumption Euler equations, goods market clearing, and the fact that endowments are constant. The eighth row reports the period 0 consumption functions of the two households (having used households' intertemporal budget constraints and optimality conditions). For sake of completeness, the ninth row reports the valuation equation for total public sector liabilities. In case

[10] Section 5.2 considers the more general case of power utility.

of log utility and constant endowments, the present discounted value of total seigniorage revenues in the union is a constant, and equal to $\theta Y/(1-\beta)$.

There are eight degrees of freedom to specify fiscal and monetary policies. I assume that the national fiscal authorities set $\{S_{it}\}$, $i=1,2$, and the system of central banks sets $\{R_t\}$ and five of the seven sequences $\{Z_{it}, B_{it}^{CB}, X_{it}, L_{12t}\}$, $i=1,2$, where central bank bond holdings and reserves must satisfy $B_{it}^{CB}, X_{it} \geq 0$, for $i=1,2$ and $t \geq 0$.

3 Monetary-fiscal policy arrangements and price level determination

This section specifies fiscal and monetary policies, and studies price level determination. I will consider a policy configuration that gives rise to a unique equilibrium.

Fiscal policy. Each national fiscal authority $i=1,2$ maintains a constant real primary balance, $S_{it} = S_i > -\theta Y_i$ for all $t \geq 0$.

Monetary policy. Let $\Pi_t = P_t/P_{t-1}$ denote the gross inflation rate between periods $t-1$ and t, and Π^* the common central bank's inflation target (for periods $t>0$). Furthermore, let R_2^* be the interest rate on country 2 government bonds that is consistent with the inflation target. Then, from equation (20), $R_2^* = \Pi^*/\beta$.

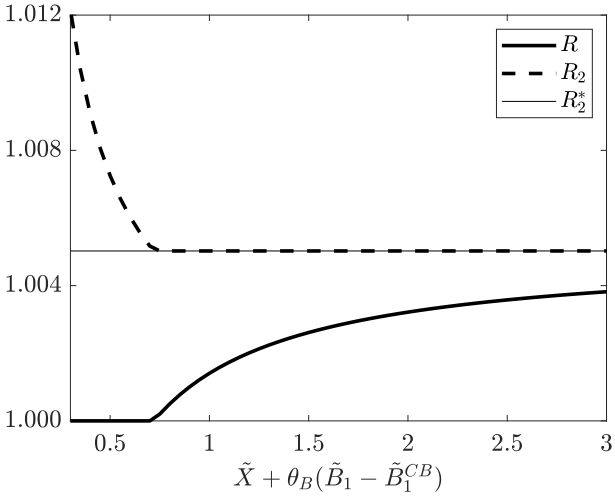
While monetary policy does not directly control the interest rates on government bonds, the central bank can use its control over the interest rate on reserves R_t to steer the interest rate on country 2 bonds towards R_2^*. Summing equation (21) over i, and making use of financial market clearing (16) and resource constraint (17), we obtain the union-wide demand function for convenience assets

$$\theta Y = \frac{R_{2t} - R_t}{R_{2t}} \left(\tilde{X}_t + \theta_B \left(\tilde{B}_{1t} - \tilde{B}_{1t}^{CB} \right) \right), \tag{25}$$

where for any variable A_t, $\tilde{A}_t = A_t/P_t$. When the supply of convenience assets is low, the marginal utility of convenience assets is high. As a result, households accept a high convenience yield. The opposite holds true when the supply of convenience assets is high.

Replacing R_{2t} with R_2^* in demand function (25), and solving for R_t, we obtain the common

Figure 1: **Interest rates and the supply of convenience assets**



central bank's interest rate rule

$$R_t = \max\left(1, R_2^*\left(1 - \frac{\theta Y}{\tilde{X}_t + \theta_B\left(\tilde{B}_{1t} - \tilde{B}_{1t}^{CB}\right)}\right)\right). \tag{26}$$

To stabilize inflation at target, the central bank adjusts its policy rate in response to fluctuations in the amount of convenience assets held by households so that the interest rate on country 2 bonds equals the interest rate target R_2^*.[11] The policy rule includes a zero lower bound constraint, which, in principle, could be formally justified by explicitly introducing cash in the model. The lower bound applies to all nominal interest rates, and thus also constrains the nominal interest rates on government bonds, $R_{it} \geq 1$ for $i = 1, 2$ and $t \geq 0$.

Figure 1 plots the interest rate on reserves (solid line) and the interest rate on country 2 government bonds (dashed line) as a function of the real value of reserves and country 1 bonds supplied to households. The difference between the solid line and the dashed line represents the convenience yield. The convenience yield is declining in the supply of convenience assets to households. There exists a threshold value for convenience assets, such that if the supply of convenience assets is above the threshold, the lower bound constraint on the policy rate is slack. In this case, the policy rate

[11] If the common central bank, instead, pegged the interest rate on reserves, a common assumption in standard fiscal theory models, then variations in the supply of convenience assets would lead to changes in the interest rate on country 2 bonds. Variations in the interest rate on country 2 bonds, in turn, would lead to fluctuations in the inflation rate.

adjusts to attain the level of the convenience yield consistent with convenience asset supply, while the interest rate on country 2 bonds stays at the target level R_2^* (thin horizontal line). Below the threshold value for convenience assets, the lower bound constraint on the policy rate is binding. In this case, the interest rate on country 2 bonds is above the target level R_2^*, and inflation exceeds the central bank's target Π^*.

Turning to the other policy instruments, I assume that, in addition to the interest rate on reserves, the system of central banks chooses the NCBs' government bond holdings, the remittances to the national fiscal authorities, and the distribution (but not the overall amount) of central bank reserves.[12] Suppose then that NCB $i = 1, 2$ holds a constant (non-negative) share of country i bonds

$$\tilde{B}_{it}^{CB} = \max\left(\alpha \tilde{B}_{it}, 0\right), \tag{27}$$

where $\alpha \in (0, 1)$.

Furthermore, NCB i pays a constant remittance to fiscal authority i, $Z_{it} = Z_i$, $t \geq 0$. As an example of such a constant remittance policy, suppose that NCB i sets Z_i such that *given* P_0, fiscal authority i's intertemporal budget constraint is satisfied

$$Z_i = (1 - \beta)\left(\frac{R_{i,-1}B_{i,-1}}{P_0} - \sum_{t=0}^{\infty} \beta^t \left(S_{it} + \frac{R_{2t} - R_{it}}{R_{2t}} \tilde{B}_{it}\right)\right). \tag{28}$$

The total amount of central bank reserves at the end of each period $t \geq 0$, $\tilde{X}_t$, is then determined by the common central bank's flow budget constraint (13). I assume that reserves are distributed proportionally to member countries' GDP, $\tilde{X}_{it} = \frac{Y_i}{Y}\tilde{X}_t$, $t \geq 0$. Finally, the net claims between the two NCBs at the end of period $t \geq 0$ are determined by flow budget constraint (18).

Equilibrium existence and uniqueness. Given the assumptions about monetary and fiscal policies, we can establish analytically that there exists a unique perfect foresight equilibrium.

Proposition 1 *Suppose that the two fiscal authorities maintain time-invariant real primary surpluses, $S_{it} = S_i$ for all $t \geq 0$ where $\sum_i S_i > -\theta Y$, and that the system of central banks follows interest rate rule (26) and asset purchasing rule (27) for $i = 1, 2$. Suppose, furthermore, that NCBs make time-invariant remittances to the fiscal authorities, and maintain a constant share of total*

[12] Due to the substitutability of reserves and country 1 bonds, equation (21) only determines household i's total demand for convenience assets.

central bank reserves. Initial conditions satisfy $B_{i,-1}, X_{i,-1} > 0$, $i = 1, 2$. *There exists a unique perfect foresight equilibrium.*

Proof: See Appendix B.

Note that if some government securities earn a convenience yield, equilibrium existence does not require the sum of the primary surpluses to be strictly positive. Fiscal authorities can run permanent primary deficits provided that they are smaller in magnitude than seigniorage revenues.[13]

There is a tight link between the sign of the sum of primary surpluses and the sign of the effective real interest rate on total public sector liabilities $\tilde{A} = \sum_i (\tilde{B}_i - \tilde{B}_i^{CB} + \tilde{X}_i)$.

Corollary 1 *The equilibrium net real interest rate on total public sector liabilities between periods* t *and* $t+1$ *is given by* $r_t^* = 1/\beta - 1 - \theta Y/(\beta \tilde{A}_t)$. *If* $\sum_i S_i \geq 0$, *then* $r_t^* \geq 0$, *else,* $r_t^* < 0$, *for all* $t \geq 0$.

Proof: See Appendix B.

Hence, if the sum of primary surpluses is negative, the effective real interest rate on total public sector liabilities will be negative, too. Since the growth rate of the economy equals zero, this pertains to the case where the real interest rate is smaller than the growth rate.[14]

For the remainder of the paper, I consider a parameterized version of the model. Where applicable, accompanying propositions state results in more general terms.

Parameterization. Table 1 lists the parameter values. One period corresponds to one quarter. The value for the subjective discount factor is consistent with an annualized real interest rate of 2 percent. Reserves and country 1 government bonds are perfect substitutes ($\theta_B = 1$). I assume that the two member countries enter period 0 with the same amount of government debt, $R_{i,-1} B_{i,-1} = 2$ for $i = 1, 2$. Both households receive the same amount of the endowment good, $Y_1 = Y_2 = 0.5$. Initially, household i holds 80% of country i government bonds, and NCB i holds the remaining 20%. Initial central bank government bond holdings are matched by an equivalent amount of reserves. Initial cross-border asset positions of households and NCBs are zero. If the price level

[13] In the absence of government liabilities with non-pecuniary benefits, equilibrium existence would require $\sum_i S_i > 0$.

[14] This case has recently received renewed attention in the literature. See, e.g., Blanchard (2021) and Bassetto and Cui (2018).

equals unity—which turns out to be the case in the baseline equilibrium presented below—initial total public sector liabilities amount to 100% of annualized GDP of the union. The preference parameter θ is calibrated to obtain a steady state annualized convenience yield on country 1 bonds of 0.6 percentage points. For comparison, Jiang et al. (2025) estimate an average convenience yield of 0.64 percentage points for German government bonds over the period 2008-2024.

Table 1: **Baseline parameterization**

Parameter	Value	Economic interpretation
β	0.995	Subjective discount factor
θ	0.0036	Preference parameter wealth in the utility
θ_B	1	Substitutability between reserves and country 1 bonds
Y_i	0.5	Endowment of country i, $i = 1, 2$
$R_{i,-1}B_{i,-1}$	2	Country i initial government debt, $i = 1, 2$
$R_{-1}X_{i,-1}$	0.4	NCB i initial reserves, $i = 1, 2$
$B^{CB}_{i,-1}/B_{i,-1}$	0.2	Initial share of government bonds held by NCB i, $i = 1, 2$
$B^{H}_{ii,-1}/B_{i,-1}$	0.8	Initial share of country i bonds held by household i, $i = 1, 2$
$R_{-1}L_{12,-1}$	0	Initial net claims of NCB 1 on NCB 2
Π^*	1	Gross inflation target
R^*_2	1.005	Interest rate target
α	0.2	Share of government bonds held by NCB i, $i = 1, 2$
S_1	$0.0140Y_1$	Primary surplus country 1
S_2	$0.0188Y_2$	Primary surplus country 2

The common central bank aims at price stability, $\Pi^* = 1$, implying a target of 2 percent for the annualized interest rate on country 2 government bonds ($R^*_2 = 1.005$). I set $\alpha = 0.2$, consistent with NCBs' initial government bond holdings. Finally, I choose the primary surpluses of the two fiscal authorities such that initial national government debt of country $i = 1, 2$ is fully backed by the seigniorage revenues of the public sector in country i and the surplus, implying $S_1/Y_1 = 0.0140$ (a real primary surplus of 1.40 percent of GDP in country 1) and $S_2/Y_2 = 0.0188$ (a real primary surplus of 1.88 percent of GDP in country 2).

Baseline equilibrium. Table 2 presents the baseline equilibrium. The period 0 price level equals unity, and the inflation rate coincides with the common central bank's target. The interest rates on reserves and country 1 bonds both equal 1.4 percent in annualized terms, consistent with a convenience yield of 60 basis points. Remittances, which are chosen according to rule (28), coincide with NCBs' net seigniorage revenues, whose closed-form expressions are provided in Table 2. NCB 1's net seigniorage revenues are zero (hence, $Z_1 = 0$), reflecting the fact that the interest rate on country 1 bonds and the interest rate on reserves are identical. NCB 2's seigniorage revenue equals

Table 2: **Baseline equilibrium**

Parameter	Value	Economic interpretation
P_0	1	Price level in period $t = 0$
Π_t	1	Gross inflation rate, $t \geq 1$
R_t	1.0035	Interest rate on reserves, $t \geq 0$
R_{1t}	1.0035	Interest rate on country 1 bonds, $t \geq 0$
R_{2t}	1.005	Interest rate on country 2 bonds, $t \geq 0$
Z_1	0	Remittances from NCB 1 to fiscal authority 1
Z_2	$\frac{\bar{Y}_2\alpha}{\alpha+(1-\alpha)\bar{Y}_1}\theta Y$	Remittances from NCB 2 to fiscal authority 2
$\frac{R_{2t}-R_{1t}}{R_{2t}}\tilde{B}_{1t}$	$\frac{\bar{Y}_1}{\alpha+(1-\alpha)\bar{Y}_1}\theta Y$	Fiscal authority 1's seigniorage revenue, $t \geq 0$
C_{1t}	0.5012	Household 1 consumption, $t \geq 0$
C_{2t}	0.4988	Household 2 consumption, $t \geq 0$
$\tilde{B}^H_{11t}$	0.8001	Country 1 bonds held by household 1, $t \geq 0$
$\tilde{B}^H_{21t}$	0.7943	Country 1 bonds held by household 2, $t \geq 0$
NFA_{1t}	-0.0012	Net foreign asset position of country 1, $t \geq 0$

Note: $\bar{Y}_i = Y_i/Y$, $i = 1, 2$.

0.12 percent of national GDP. Fiscal authority 1's seigniorage revenue equals 0.6 percent of national GDP. Thus, for each country, the real primary surplus, monetary seigniorage, and fiscal seigniorage sum to 2 percent of GDP.

Household 1's consumption exceeds her endowment, $C_{1t} > Y_1$, and household 2's consumption falls short of her endowment, $C_{2t} < Y_2$, for all $t \geq 0$. What explains the cross-country heterogeneity in consumption? The two households enter period 0 with the same amount of financial wealth ($\sum_j R_{j,-1}B^H_{1j,-1} + R_{-1}X^H_{1,-1} = \sum_j R_{j,-1}B^H_{2j,-1} + R_{-1}X^H_{2,-1}$), and obtain the same amount of the endowment good ($Y_1 = Y_2$). Yet, the two households differ in terms of the amount of real resources that the public sector extracts from them. Household 1's tax obligations are lower than household 2's tax obligations ($S_1 < S_2$) since the convenience property of country 1 government bonds allows fiscal authority 1 to issue debt at a lower interest rate than fiscal authority 2. Fiscal authority 1 obtains the associated fiscal seigniorage revenues from both households. While household 2 does not hold any country 1 bonds when entering period 0, in period 0, she reallocates part of her financial wealth towards country 1 bonds ($\tilde{B}^H_{21t} > 0$ for all $t \geq 0$). Overall, household 1 is richer than household 2, because for household 1, the present value of reduced tax payments exceeds the present value of real resources extracted from her via fiscal seigniorage, whereas household 2 does not benefit from lower taxation while giving up real resources to fiscal authority 1 via fiscal seigniorage. The following proposition summarizes the result.

Proposition 2 *Consider a currency union with symmetric initial conditions. The two national public sectors enter period* 0 *with the same amount of liabilities, and the two households enter period* 0 *with the same amount of financial assets. Both households receive an equal amount of the endowment good. Fiscal and monetary policies follow the configuration of Section 3. Consider an equilibrium where i. the intertemporal government budget constraints of the national public sectors are satisfied (i.e., equation (14) holds for* $i = 1, 2$ *with the last term on the right-hand side being equal to zero), ii. net claims between NCBs equal zero (*$L_{12t} = 0, \forall t$*), and iii. the present discounted value of fiscal authority 1's seigniorage revenue is strictly positive,* $\sum_{t=0}^{\infty} \beta^t (R_{2t} - R_{1t}) R_{2t}^{-1} \tilde{B}_{1t} > 0$. *Then,* $S_1 < S_2$, *and* $C_1 > C_2$.

Proof: See Appendix B.

Finally, the (end-of-period) net foreign asset position of country 1 is negative, $NFA_{1t} = \tilde{B}^H_{12t} - \tilde{B}^H_{21t} + \tilde{L}_{12t} < 0$, and the net foreign asset position of country 2 is positive, $NFA_{2t} = -NFA_{1t}$. Net foreign asset positions are constant over time, reflecting the fact that $R_{2t}\tilde{B}^H_{12t} = R_{1t}\tilde{B}^H_{21t}$ (household 1 earns a higher return on her foreign assets than household 2), and $\tilde{L}_{12t} = 0$ for all $t \geq 0$.

4 Scenarios

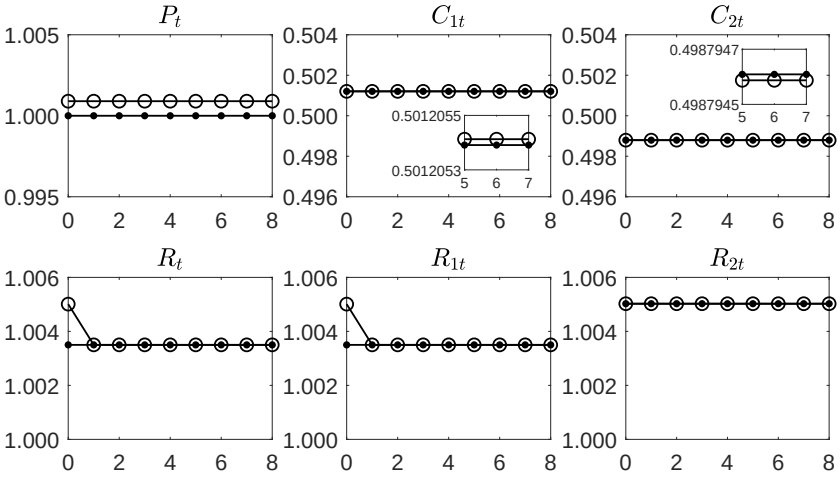
This section explores how exogenous shifts in households' demand for convenience assets, and changes in fiscal policies affect the price level and allocations.

4.1 Decrease in the demand for convenience assets

Suppose that households' preferences for convenience assets are potentially time-varying, i.e. we replace θ with θ_t. Specifically, suppose that in period 0, households' desire for the convenience assets supplied by the public sector decreases temporarily, $\theta_0 < \theta$, and $\theta_t = \theta$ for $t \geq 1$.

Figures 2 and 3 show the equilibrium paths, assuming that $\theta_0 = \theta/100$ (lines with circles), together with the baseline equilibrium (lines with points). The price level rises in period 0 and stays permanently higher. As a result of the change in household preferences, the convenience yields on country 1 bonds and reserves decline on impact, and so does the present discounted value of total monetary and fiscal seigniorage revenues in the union–there is less backing than in the

Figure 2: **Decrease in convenience asset demand**



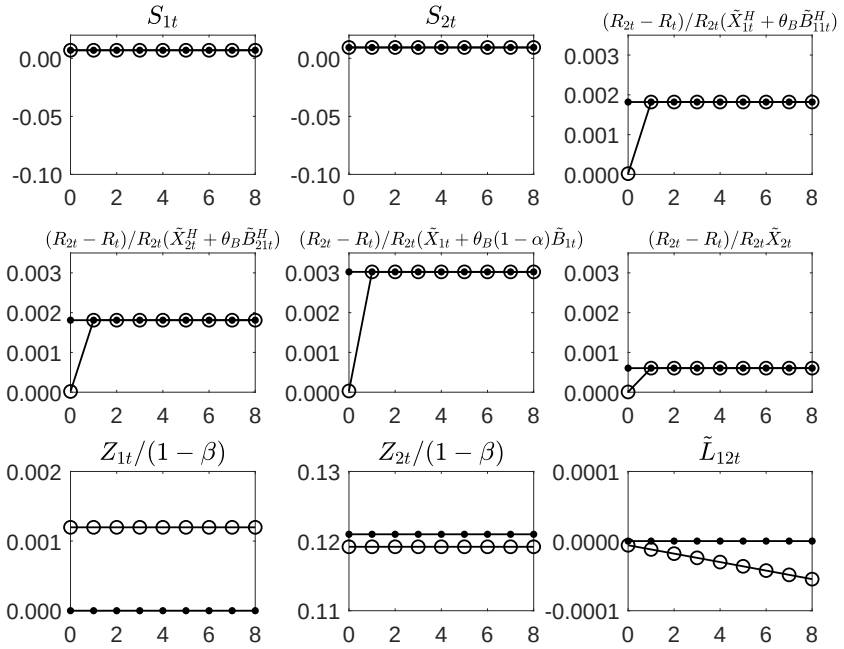
Notes: Lines with points: baseline. Lines with circles: decrease in households' desire for convenience assets.

baseline (see equation (24)). The increase in the price level equilibrates the real value of total public sector liabilities with the reduced amount of real resources backing them.

The temporary drop in convenience services leads to a permanent change in consumption levels. Household 1 consumes more than in the baseline and household 2 consumes less than in the baseline (see Figure 2). Both households experience a symmetric decrease in their financial wealth from the increase in the price level. At the same time, both households transfer less real resources to the public sector (see Figure 3). Household 1 holds more convenience assets than household 2, hence, the present value of real resources extracted from household 1 via seigniorage declines by more than the present value of real resources extracted from household 2 via seigniorage (though the difference is too small to be visible in Figure 3). Overall, household 1 becomes richer because the decrease in the present value of real resources extracted from her is larger than the decrease in the real value of her asset holdings. Household 2, instead, becomes poorer because the decrease in the present value of real resources extracted from her is smaller than the decrease in the real value of her asset holdings. Quantitatively, the changes in consumption are very small because household 1 holds only a slightly larger amount of country 1 bonds than household 2, and both hold the same amount of reserves.

The following proposition summarizes the effects of a change in θ_0 on the price level and house-

Figure 3: **Decrease in convenience asset demand**



Notes: Lines with points: baseline. Lines with circles: decrease in households' desire for convenience assets.

hold consumption. Define household i's relative financial wealth at the beginning of period 0 as

$$\mathcal{W}_i = \frac{\sum_j R_{j,-1} B^H_{ij,-1} + R_{-1} X^H_{i,-1}}{\sum_j R_{j,-1} \sum_h B^H_{hj,-1} + R_{-1} \sum_h X^H_{h,-1}}. \tag{29}$$

Proposition 3 *Suppose the non-pecuniary benefits of the convenience assets decrease in period* 0, $\theta_0 < \theta$. *Then, the price level increases. Suppose, furthermore, that households have the same initial financial wealth,* $\mathcal{W}_1 = \mathcal{W}_2$, *receive the same endowment,* $Y_1 = Y_2$, *and household 1 pays less taxes than household 2* $S_1 < S_2$. *Then, consumption of household* 1 *increases and consumption of household* 2 *decreases permanently.*

Proof: See Appendix B.

Finally, how are the consolidated public sectors of the two countries affected by the shift in household preferences? The increase in the price level lowers the real value of public sector liabilities in period 0 symmetrically across the two countries. At the same time, the decrease in the interest rate spreads (i.e., the decline in the convenience yields) reduces the seigniorage revenues of the public

sectors in the two countries (see Figure 3). The decrease in seigniorage revenues is asymmetric. Since public sector 1 (consisting of fiscal authority 1 and NCB 1) has issued more convenience assets than public sector 2 (consisting of fiscal authority 2 and NCB 2), the seigniorage revenues of public sector 1 fall by more than the seigniorage revenues of public sector 2. While the increase in the period 0 price level ensures that the valuation equation for total public sector liabilities in the union holds, the rise in the price level is too small for public sector 1 and too large for public sector 2. Following remittance rule (28), NCB 1 increases its remittances to fiscal authority 1 (from 0 to strictly positive). NCB 2 reduces its remittances to fiscal authority 2, and makes a loan to NCB 1. The claim of NCB 2 on NCB 1 keeps growing forever, and the growth rate converges to the interest rate on reserves. Since the rate on reserves is strictly smaller than the subjective discount rate, in the limit as time goes to infinity the present discounted value of NCB 2's claim on NCB 1 converges to zero. Hence, the intertemporal budget constraints of the two public sectors hold, i.e., the last term on the right-hand side of equation (14) equals zero for $i = 1, 2$.

4.2 Decline in the substitutability of bonds and reserves

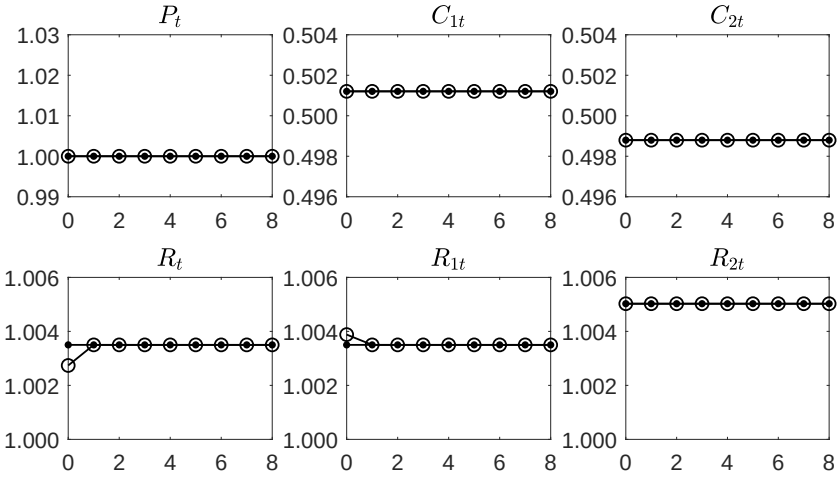
Let us now consider the case where households' preferences for country 1 government bonds relative to reserves are potentially time-varying. We replace θ_B with θ_{Bt}. Suppose that in period 0, country 1 government bonds become less convenient, $\theta_{B0} < \theta_B$, and $\theta_{Bt} = \theta_B$ for $t \geq 1$.

Figures 4 and 5 show the equilibrium paths, assuming that $\theta_{B0} = \theta_B/2$ (lines with circles). The present value of the sum of primary surpluses and total seigniorage revenues is unaffected by the change in the convenience services of country 1 bonds, see equation (24). Consequently, the period 0 price level coincides with the one in the baseline equilibrium.

Consumption paths also coincide with those in the baseline. The change in θ_{B0} does neither affect the real value of households' asset holdings nor the amount of real resources extracted from them via taxation and seigniorage, see equation (23). Specifically, the two households adjust their holdings of country 1 bonds and reserves such that the present value of real resources extracted from each of them via seigniorage remains unchanged in spite of a change in interest rate spreads in period 0.

Spreads move in opposite directions. The spread between the interest rates on country 2 bonds and country 1 bonds declines, and the spread between the interest rates on country 2 bonds and

Figure 4: **Decline in the substitutability of bonds and reserves**



Notes: Lines with points: baseline. Lines with circles: decline in relative convenience of country 1 government bonds.

reserves increases in period 0 (see Figure 4). While total seigniorage revenues in the union are unaffected, the movements in spreads have a bearing on the allocation of seigniorage revenues across countries. Seigniorage revenues of public sector 1 decline (because the increase in NCB 1's seigniorage revenue is more than offset by the decline in fiscal authority 1's seigniorage revenue), and seigniorage revenues of public sector 2 increase (see Figure 5). Following remittance rule (28), NCB 1 raises its transfers to fiscal authority 1 (from zero to strictly positive). The transfers from NCB 2 to fiscal authority 2, instead, are unchanged. NCB 2 lends its additional seigniorage revenue to NCB 1, and the claim of NCB 2 on NCB 1 is rolled over forever.

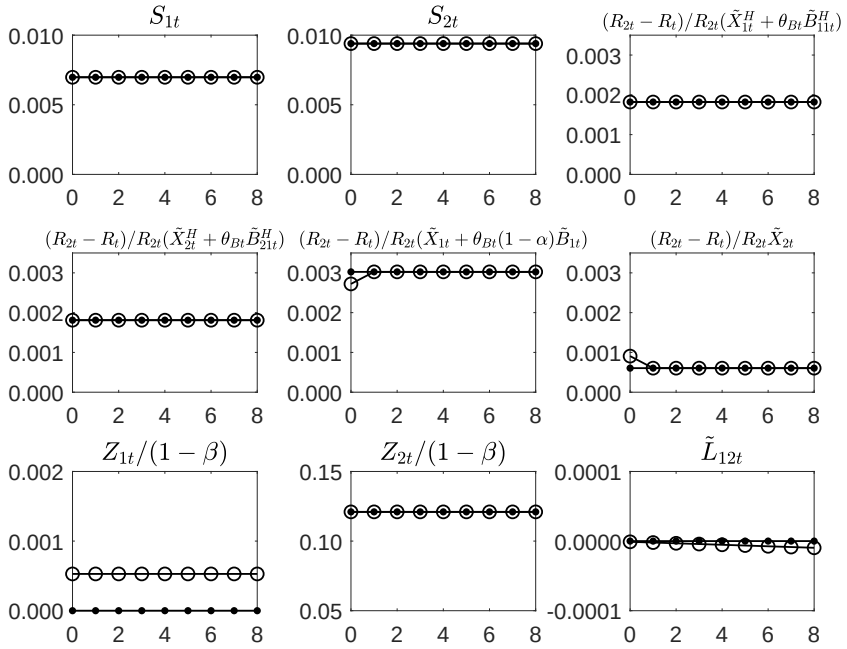
4.3 Asymmetric fiscal expansion

We now turn to the macroeconomic effects of exogenous changes in fiscal policies. Suppose that in period 0, fiscal authority 1 runs a temporary primary deficit of 5 percent of annual GDP, $S_{10} = -0.2Y_1$, while the primary surplus in country 2 remains the same as in the baseline.[15]

Figures 6 and 7 show the equilibrium paths for the scenario (lines with circles) together with the baseline equilibrium (lines with points). The price level increases in period 0. As a result of the fiscal intervention, the present discounted value of the sum of primary surpluses is lower than in the baseline–there is less backing. The increase in the price level equilibrates the real value of

[15] Maćkowiak and Schmidt (2025) consider this scenario in a monetary union model without convenience yields. In what follows, we will focus on the role of convenience yields for the propagation of the fiscal shock.

Figure 5: **Decline in the substitutability of bonds and reserves**



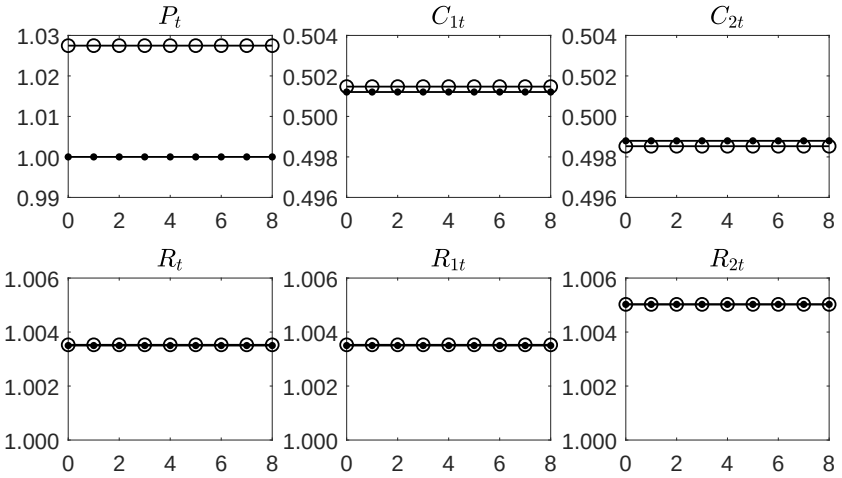
Notes: Lines with points: baseline. Lines with circles: decline in relative convenience of country 1 government bonds.

total public sector liabilities with the reduced amount of real resources backing them. After the inflation spike in period 0, inflation returns to target in period 1. Hence, the union price level remains permanently higher.

The asymmetric fiscal expansion also affects consumption. Household 1 raises consumption, whereas household 2 reduces consumption, permanently. Both households suffer from the symmetric decline in the real value of their asset holdings. Overall, however, household 1 becomes richer, because the present value of her tax obligations declines by more than the real value of her assets. Household 2, instead, becomes poorer, because the real value of her assets falls while the present value of her tax obligations remains unchanged.

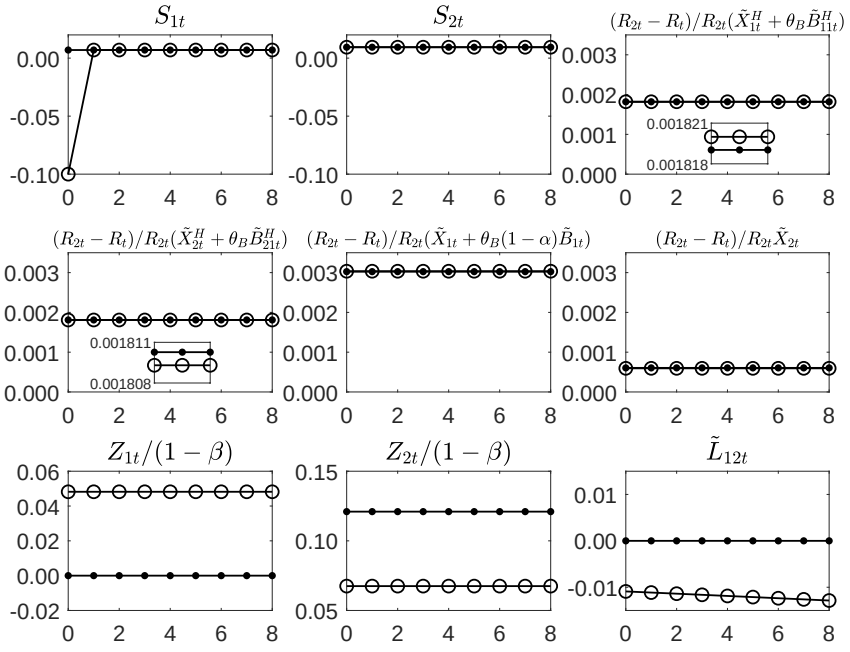
The presence of convenience assets attenuates the consumption responses to the asymmetric fiscal expansion. Since household 1 raises consumption, she also increases her demand for convenience assets for a given interest rate spread, see equation (21). Household 2 reduces consumption, and, thus wants to hold less convenience assets for a given interest rate spread. In equilibrium, the interest rate spread declines (slightly), and both households absorb part of the increased supply

Figure 6: **Asymmetric fiscal expansion**



Notes: Lines with points: baseline. Lines with circles: asymmetric fiscal expansion in country 1.

Figure 7: **Asymmetric fiscal expansion**



Notes: Lines with points: baseline. Lines with circles: asymmetric fiscal expansion in country 1.

of country 1 bonds (not shown).[16] However, household 1 increases her country 1 bond holdings by more than household 2. As a consequence, the present value of real resources extracted from household 1 via seigniorage increases (see Figure 7), dampening the rise in household 1's consumption, and the present value of real resources extracted from household 2 via seigniorage declines, dampening the reduction in household 2's consumption.[17]

The following proposition summarizes the effects of an asymmetric fiscal expansion on the price level and consumption.

Proposition 4 *Suppose fiscal authority i lowers its primary surplus in period 0, $S_{i0} < S_i$, and fiscal authority $j \neq i$ keeps its surplus unchanged, $S_{j0} = S_j$. The period 0 price level increases, consumption of household i rises, and consumption of household j decreases. The presence of convenience assets ($\theta > 0$) mitigates the changes in consumption.*

Proof: See Appendix B.

How does the asymmetric fiscal expansion affect the public sectors of the two countries? On the one hand, the increase in the price level reduces the real value of public sector liabilities symmetrically. On the other hand, the present value of public sector 1's tax revenues declines whereas the tax revenues of public sector 2 remain unchanged (see Figure 7). While the increase in the period 0 price level ensures that the valuation equation for total public sector liabilities in the union holds, the increase in the price level is too small for public sector 1 and too large for public sector 2. NCB 2 reduces its remittances to fiscal authority 2, and makes a loan to NCB 1. NCB 1 increases its remittances to fiscal authority 1. The claim of NCB 2 on NCB 1 keeps growing forever. In the long run, it grows at the interest rate on reserves. Since the rate on reserves is strictly smaller than the subjective discount rate, in the limit as time goes to infinity the present discounted value of NCB 2's claim on NCB 1 converges to zero, like in the previous two scenarios.

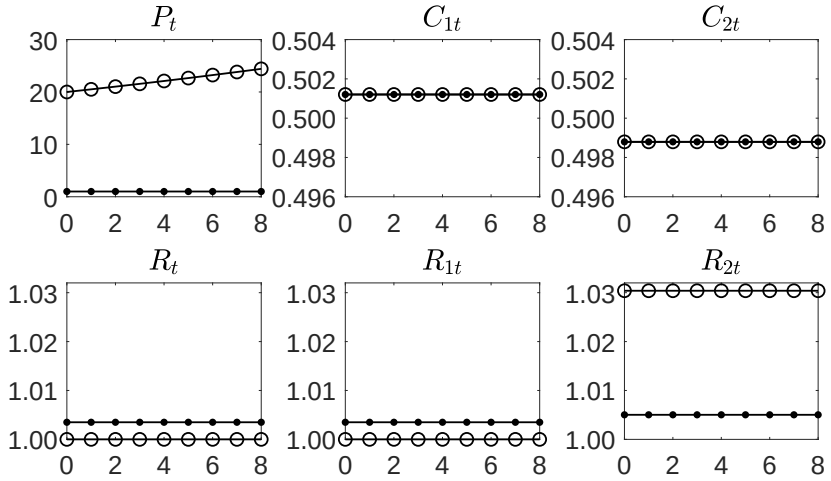
4.4 Permanent fiscal deficits

Under the baseline parameterization, both national fiscal authorities run strictly positive primary surpluses. However, equilibrium existence does not require the present discounted value of the sum

[16] The change in the interest rate spread is too small to be visible in Figure 6.

[17] The total amount of real resources extracted from households via monetary and fiscal seigniorage is constant. See equation (24).

Figure 8: **Permanent primary deficits**



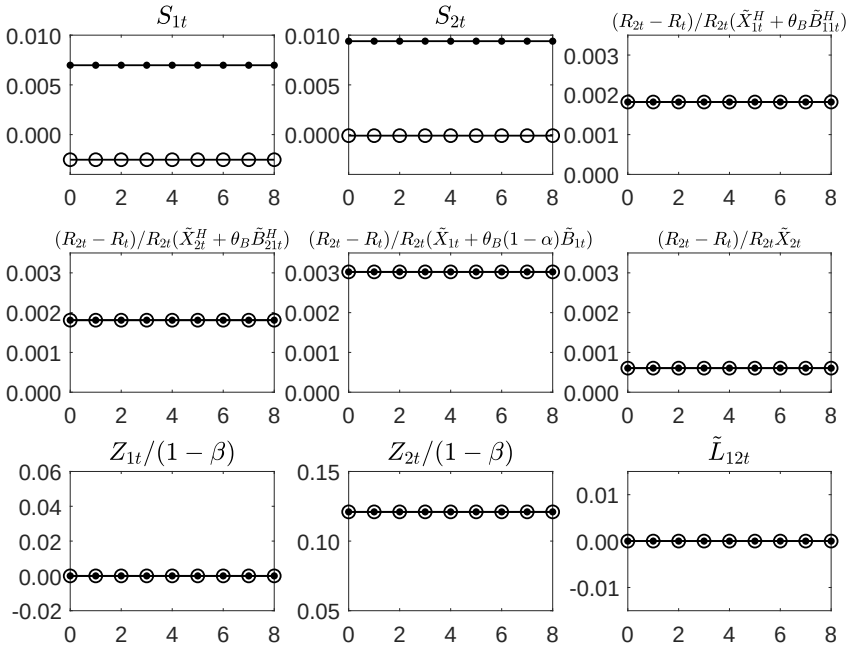
Notes: Lines with points: no shocks. Lines with circles: permanent primary deficits in both countries.

of member countries' primary surpluses to be positive, see Proposition 1. We now consider the case where both countries switch to permanent fiscal deficits. Specifically, suppose that $S_{it} = S_i' < 0$ for all $t \geq 0$ and $i = 1, 2$, where $S_i' = S_i - 0.0095$. Note that $\sum_i S_i' = -0.0026 > -\theta Y = -0.0036$. Hence, a unique equilibrium exists.

Figures 8 and 9 show the equilibrium paths for the fiscal deficit scenario (lines with circles) and for the baseline (lines with points). In period 0, the price level increases by the factor 20. The shift from permanent primary surpluses to permanent deficits leads to a substantial decline in fiscal-monetary backing (which is now solely based on seigniorage revenues). The jump in the price level equilibrates the real value of total public sector liabilities with the new much smaller amount of real resources backing them.

From period 1 onward, the price level grows at 2.5 percent, i.e. actual inflation permanently exceeds the central bank's target. The price level jump in period 0 drastically reduces the real value of convenience assets available to households. In order for households' demand for convenience assets to fall accordingly, the convenience yields on reserves and country 1 bonds must increase substantially. The central bank would like to engineer the increase in the interest rate spreads solely by means of a reduction in the policy rate and the interest rate on country 1 bonds so as to keep the interest rate on country 2 bonds at the level consistent with its price stability objective. However, the lower bound on nominal interest rates prevents a sufficiently large cut in

Figure 9: **Permanent primary deficits**



Notes: Lines with points: no shocks. Lines with circles: permanent primary deficits in both countries.

the policy rate. In the new equilibrium, the policy rate and the interest rate on country 1 bonds are permanently stuck at the lower bound, and the interest rate on country 2 bonds is 2.5 percentage points higher than in the baseline—leading to the permanent inflation target overshoot.

Since the permanent fiscal expansion is symmetric across the two countries—both national fiscal authorities reduce their primary surplus by 1.9 percent of GDP—there is no wealth transfer across countries. The increase in interest rate spreads and the decline in the real value of reserves and country 1 bonds exactly offset each other at the country level so that the present value of fiscal-monetary seigniorage revenues of both governments remains unchanged relative to the baseline.

5 Additional analysis

The first subsection discusses the role of central bank government bond purchases and NCBs' remittance policies for price level determination and equilibrium allocations. The second subsection relaxes the assumption of log utility, and considers price level determination in the case of power

utility.

5.1 Central bank bond purchases and remittances

Bond purchases. So far, we have assumed that the NCBs hold a constant share of national government bonds (subject to a non-negativity constraint), see equation (27). We can consider alternative rules for NCBs' government bond purchases. For instance, we could contemplate a rule whereby the NCBs fully absorb any "excess" supply of government bonds so as to keep the real value of government bonds held by households constant, $\tilde{B}_{it}^{CB} = \max(0, \tilde{B}_{it} - (1-\alpha)\tilde{B}_{i,-1})$. The responses of the price level and households' consumption to a preference or fiscal shock would be the same as under the baseline rule (27). See equations (23) and (24).

Alternatively, we could contemplate a rule whereby NCBs hold a constant real amount of government bonds, so that households fully absorb any "excess" bond supply, $\tilde{B}_{it}^{CB} = \tilde{B}_{i}^{CB} \geq 0$. Again, the effects of a preference or fiscal shock on the price level and consumption would be the same as under the baseline rule.

Remittances. The NCBs' remittance policies influence how wealth is transferred between countries. Under remittance rule (28), any wealth transfer between the two countries operates via the system of central banks. In case of other remittance policies, a wealth transfer may operate via households' lending to and borrowing from fiscal authorities.[18] The specifics of the remittance policies, however, do not affect the period 0 price level and households' consumption.

5.2 Preferences

We relax the assumption that households' preferences are represented by log utility. Suppose that utility functions $U(\cdot)$ and $V(\cdot)$ have the following functional forms

$$U\left(C_{it}\right) = \frac{C_{it}^{1-\sigma} - 1}{1-\sigma}, \quad V\left(\tilde{X}_{it}^{H} + \theta_B \tilde{B}_{i1t}^{H}\right) = \frac{\left(\tilde{X}_{it}^{H} + \theta_B \tilde{B}_{i1t}^{H}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{30}$$

for $i = 1, 2$, where $\sigma \neq 1$. Equilibrium conditions (21), (23), and (24) then become

$$\theta^{\frac{1}{\sigma}} C_{it} = \left(\frac{R_{2t} - R_t}{R_{2t}}\right)^{\frac{1}{\sigma}} \left(\frac{X_{it}^{H} + \theta_B B_{i1t}^{H}}{P_t}\right), \quad i = 1, 2 \tag{31}$$

[18] See also Maćkowiak and Schmidt (2025).

$$\left(\frac{1}{1-\beta}+\sum_{t=0}^{\infty}\beta^t\left(\frac{R_{2t}-R_t}{R_{2t}}\right)^{\frac{\sigma-1}{\sigma}}\theta^{\frac{1}{\sigma}}\right)C_{i0}=\frac{Y_i}{1-\beta}+\frac{\sum_j R_{j,-1}B^H_{ij,-1}+R_{-1}X^H_{i,-1}}{P_0}$$
$$-\sum_{t=0}^{\infty}\beta^t S_{it},\quad i=1,2 \tag{32}$$

$$\frac{\sum_i R_{i,-1}(B_{i,-1}-B^{CB}_{i,-1})+R_{-1}\sum_i X_{i,-1}}{P_0}=\sum_{t=0}^{\infty}\beta^t\left(\sum_i S_{it}+\left(\frac{R_{2t}-R_t}{R_{2t}}\right)^{\frac{\sigma-1}{\sigma}}\theta^{\frac{1}{\sigma}}Y\right). \tag{33}$$

The remaining equilibrium conditions are identical to those in the log-utility model. With power utility, the present value of total monetary and fiscal seigniorage revenues in the union is no longer a constant, but rather depends on the path of the interest rate spread, see equation (33). When $\sigma > 1$, an increase in the interest rate spread raises seigniorage revenues for the public sector in the union, and vice versa when $\sigma < 1$. That is because the smaller σ, the more elastic is households' demand for convenience assets, see equation (31).

The monetary and fiscal policy arrangements follow those in Section 3, with interest rate rule (26) being replaced by

$$R_t=\max\left(1,R_2^*\left(1-\frac{\theta Y^{\sigma}}{\left(\tilde{X}_t+\theta_B\left(\tilde{B}_{1t}-\tilde{B}^{CB}_{1t}\right)\right)^{\sigma}}\right)\right). \tag{34}$$

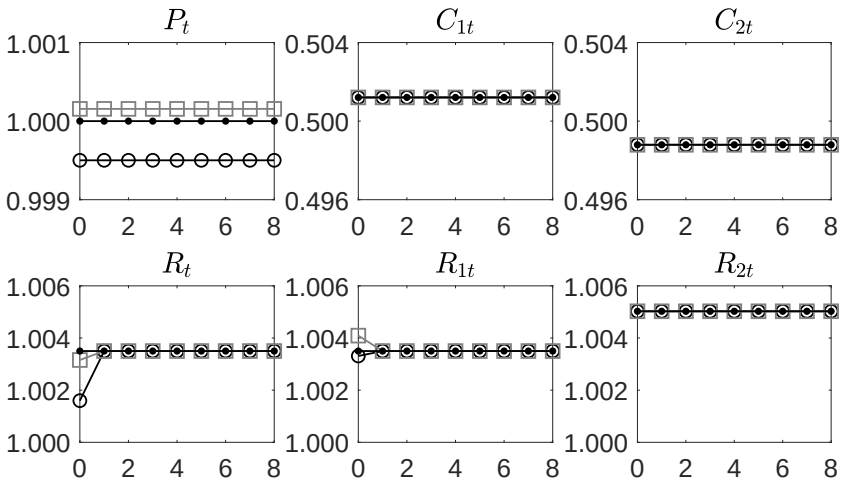
Establishing conditions for equilibrium existence and uniqueness is more involved than in the log utility case. In Appendix C, I provide analytical results for the case where NCBs hold all government bonds and households, consequently, only hold reserves. Then, a unique perfect foresight equilibrium exists when (i) $\sum_i S_i \geq 0$, or (ii) $\sum_i S_i < 0$ and $\sigma > 1$.[19]

Let us reconsider the scenario of a temporary decline in the convenience services provided by country 1 government bonds, $\theta_{B0} = \theta_B/2$ (see section 4.2). Figures 10 and 11 show equilibrium paths for the cases $\sigma = 2$ (lines with circles), and $\sigma = 0.5$ (lines with squares). In each case, I recalibrate θ such that the equilibrium in the absence of shocks coincides with the baseline equilibrium of Section 3 (lines with dots).

When $\sigma > 1$, the price level declines in period 0. This is in contrast to the model with log utility, where the price level remained unchanged (see Figure 4). Since country 1 bonds are only imperfect substitutes for central bank reserves in period 0, a spread arises between the interest rate

[19] When $\sum_i S_i < 0$ and $\sigma < 1$, then, generically, if an equilibrium exists, the equilibrium will not be unique.

Figure 10: **Decline in the substitutability of bonds and reserves - power utility**



Notes: Lines with points: no shocks. Lines with circles (squares): decline in relative convenience of country 1 government bonds when $\sigma = 2$ ($\sigma = 0.5$).

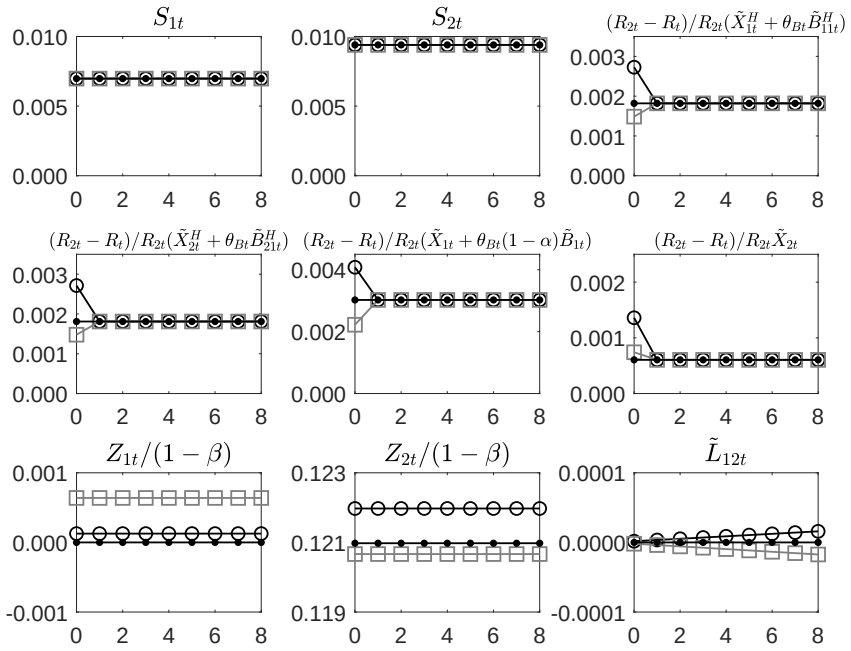
on country 1 bonds and the policy rate. Interest rate rule (34) implies that the common central bank lowers the interest rate on reserves in response to the change in households' preferences. As a consequence of the policy rate cut, the spread between the interest rate on country 2 bonds and the interest rate on reserves increases. Since $\sigma > 1$, the increase in the interest rate spread raises the present discounted value of total seigniorage revenues in the union—there is more backing (see Figure 11). The decline in the price level equilibrates the real value of total public sector liabilities with the increased amount of real resources backing them.

In contrast to the model with log utility, the temporary decline in the convenience services provided by country 1 bonds also leads to a (small) wealth transfer between households. On the one hand, the fall in the price level raises households' financial wealth symmetrically. On the other hand, the increase in the interest rate spread leads to an increase in the present value of real resources extracted from households via seigniorage. For household 1, who holds more convenience assets than household 2, the increase in the present value of real resources extracted from her via seigniorage exceeds the rise in the real value of her financial wealth. For household 2, instead, the increase in the present value of real resources that are extracted from her via seigniorage is smaller than the increase in the real value of her financial wealth. Hence, the shift in preferences makes household 1 poorer and household 2 richer, lowering consumption of household 1 and raising

consumption of household 2, see equation (32). Quantitatively, the wealth transfer from household 1 to household 2, and the associated changes in consumption are very small in the present example, and indeed not visible in Figure 10.

Finally, turning to the public sectors of the two countries, on the one hand, the fall in the price level raises the real value of their liabilities symmetrically. On the other hand, the change in interest rate spreads raises public sector seigniorage revenues asymmetrically. When $\sigma > 1$, both the policy rate and the interest rate on country 1 bonds decline. Hence, both, monetary and fiscal seigniorage revenues increase (see Figure 11). Overall, then, seigniorage revenues of public sector 1 increase by more than seigniorage revenues of public sector 2. NCB 1 increases its remittances to fiscal authority 1 and makes a loan to NCB 2. NCB 2 increases its remittances to fiscal authority 2. The claim of NCB 1 on NCB 2 is rolled over forever.

Figure 11: **Decline in the substitutability of bonds and reserves - power utility**



Notes: Lines with points: no shocks. Lines with circles (squares): decline in relative convenience of country 1 government bonds when $\sigma = 2$ ($\sigma = 0.5$).

Next, consider the case where $\sigma < 1$. The price level now increases in period 0. When $\sigma < 1$, the increase in the spread between the interest rates on country 2 bonds and on reserves lowers the present discounted value of total seigniorage revenues in the union—there is less backing. The

increase in the price level equilibrates the real value of total public sector liabilities with the reduced amount of real resources backing them.

As before, there is a wealth transfer between households, but with $\sigma < 1$ the transfer is from household 2 to household 1. On the one hand, the increase in the price level lowers households' financial wealth symmetrically. On the other hand, the increase in the interest rate spread leads to a decline in the present value of real resources extracted from households (see Figure 11). For household 1, who holds more convenience assets than household 2, the decline in the present value of real resources extracted from her via seigniorage exceeds the fall in the real value of her financial wealth. For household 2, instead, the decline in the present value of real resources that are extracted from her via seigniorage is smaller than the decline in the real value of her financial wealth. Hence, the shift in preferences makes household 1 richer and household 2 poorer. As in the case of $\sigma > 1$, quantitatively, the wealth transfer and the associated changes in consumption turn out to be very small in the present example.

Finally, turning to the national public sectors, when $\sigma < 1$, the real value of their liabilities falls symmetrically in response to the shift in household preferences. Seigniorage revenues, instead, fall for public sector 1, and increase for public sector 2 (see Figure 11). Since the interest rate on country 1 bonds rises when $\sigma < 1$—reducing the spread between the interest rates on country 2 and country 1 bonds—fiscal seigniorage revenues of public sector 1 decline, and the decline in fiscal seigniorage revenues is larger in magnitude than the increase in public sector 1's monetary seigniorage revenues. NCB 2 lowers its remittances to fiscal authority 2 and makes a loan to NCB 1. NCB 1 increases its remittances to fiscal authority 1. The claim of NCB 2 on NCB 1 is rolled over forever.

6 Conclusion

I extended the fiscal theory of the price level to explore the role of convenience yields on government securities in determining the price level of a currency union. Government securities that earn a convenience yield generate additional resources for the public sector—fiscal seigniorage—that can back the union's price level, alongside primary surpluses and traditional monetary seigniorage. The present value of member countries' primary surpluses may be negative, i.e. backing may be pro-

vided solely by fiscal and monetary seigniorage revenues, as long as the present value of seigniorage revenues is larger in magnitude than the present value of fiscal deficits. Cross-country heterogeneity in the convenience property of government securities may give rise to an asymmetric wealth distribution that benefits the residents of the country issuing the convenience bonds. Changes in households' demand for convenience assets move the price level and redistribute wealth across countries. Fiscal shocks also move the price level, and, when asymmetric, redistribute wealth across countries, as in standard fiscal theory models. Compared to these standard models, the presence of convenience assets mitigates the redistributive effects of asymmetric fiscal shocks for households.

Appendix

A Intertemporal budget constraints

I derive the intertemporal budget constraints (IBC) of household i—equation (10)—and of the public sector as a whole—equation (15).

A.1 IBC of household i

We can write the flow budget constraint of household i in period $t = 0$ as

$$\begin{aligned}\frac{\sum_{j=1}^{2} R_{j,-1} B_{ij,-1}^{H} + R_{-1} X_{i,-1}^{H}}{P_0} =& C_{i0} - Y_{i0} + S_{i0} + \frac{(R_{20} - R_{10})}{R_{20}} \frac{B_{i10}^{H}}{P_0} + \frac{(R_{20} - R_0)}{R_{20}} \frac{X_{i0}^{H}}{P_0} \\ &+ \frac{\sum_{j=1}^{2} R_{j0} B_{ij0}^{H} + R_0 X_{i0}^{H}}{P_1} \frac{P_1}{R_{20} P_0} \end{aligned} \tag{A.1}$$

Solving equation (A.1) forward, taking expectations conditional on period 0, and making use of household optimality conditions (3) and (9), we obtain

$$\begin{aligned}\frac{\sum_{j=1}^{2} R_{j,-1} B_{ij,-1}^{H} + R_{-1} X_{i,-1}^{H}}{P_0} =& \sum_{t=0}^{T} \mathrm{E}_0 Q_{i0t} \left(C_{it} - Y_{it} + S_{it} + \frac{(R_{2t} - R_t)}{R_{2t}} \frac{(X_{i0}^{H} + \theta_B B_{i1t}^{H})}{P_t} \right) \\ &+ \mathrm{E}_0 Q_{i0T} \left(\frac{\sum_j R_{jT} B_{ijT}^{H} + R_T X_{iT}^{H}}{P_{T+1}} \right) \end{aligned} \tag{A.2}$$

Taking the limit of both sides of equation (A.2) as T goes to infinity, and imposing transversality condition (6), we obtain

$$\frac{\sum_j R_{j,-1} B_{ij,-1}^{H} + R_{-1} X_{i,-1}^{H}}{P_0} = \sum_{t=0}^{\infty} \mathrm{E}_0 Q_{i0t} \left(C_{it} - Y_{it} + S_{it} + \frac{(R_{2t} - R_t)}{R_{2t}} \frac{X_{it}^{H} + \theta_B B_{i1t}^{H}}{P_t} \right),$$

which is household i's intertemporal budget constraint.

A.2 IBC of public sector in the union

Summing the flow budget constraint of the common central bank (13) and the flow budget constraints of the two national fiscal authorities (11) in period $t = 0$, we obtain

$$\frac{\sum_{i=1}^{2} R_{i,-1}\left(B_{i,-1} - B_{i,-1}^{CB}\right) + R_{-1}X_{-1}}{P_0} = \sum_{i=1}^{2} S_{i0} + \frac{(R_{20} - R_{10})}{R_{20}} \frac{B_{10} - B_{10}^{CB}}{P_0} + \frac{(R_{20} - R_0)}{R_{20}} \frac{X_0}{P_0} + \frac{\sum_i R_{i0}(B_{i0} - B_{i0}^{CB}) + R_0 X_0}{P_1} \frac{P_1}{R_{20} P_0} \tag{A.3}$$

Solving equation (A.3) forward, taking expectations conditional on period 0, and making use of optimality conditions (3) and (9), we obtain

$$\frac{\sum_{i=1}^{2} R_{i,-1}\left(B_{i,-1} - B_{i,-1}^{CB}\right) + R_{-1}X_{-1}}{P_0} = \sum_{t=0}^{T} \mathrm{E}_0 Q_{h0t} \left(\sum_i S_{it} + \frac{(R_{2t} - R_t)}{R_{2t}} \frac{X_t + \theta_B (B_{1t} - B_{1t}^{CB})}{P_t} \right) + \mathrm{E}_0 Q_{h0T} \frac{\sum_i R_{iT}(B_{iT} - B_{iT}^{CB}) + R_T X_T}{P_{T+1}}, \tag{A.4}$$

which holds for $h = 1, 2$.

Using asset market clearing, we can rewrite the last term on the right-hand side of equation (A.4) as

$$\begin{aligned} \mathrm{E}_0 Q_{h0T} \frac{\sum_i R_{iT}(B_{iT} - B_{iT}^{CB}) + R_T X_T}{P_{T+1}} &= \sum_{k=1}^{2} \mathrm{E}_0 Q_{h0T} \frac{\sum_i R_{iT} B_{kiT}^H + R_T X_{kT}^H}{P_{T+1}} \\ &= \sum_{k=1}^{2} \mathrm{E}_0 Q_{k0T} \frac{\sum_i R_{iT} B_{kiT}^H + R_T X_{kT}^H}{P_{T+1}}, \end{aligned} \tag{A.5}$$

where the second equality in (A.5) follows from no arbitrage. Households' transversality condition (6) implies that the limit of the term in the second row on the right-hand side of (A.5) as T goes to infinity equals zero.

Hence, taking the limit of both sides of (A.4) as time goes to infinity, we obtain the valuation equation for total public sector liabilities

$$\frac{\sum_{i=1}^{2} R_{i,-1}\left(B_{i,-1} - B_{i,-1}^{CB}\right) + R_{-1}X_{-1}}{P_0} = \sum_{t=0}^{\infty} \mathrm{E}_0 Q_{h0t} \left(\sum_i S_{it} + \frac{(R_{2t} - R_t)}{R_{2t}} \frac{X_t + \theta_B (B_{1t} - B_{1t}^{CB})}{P_t} \right),$$

for $h = 1, 2$.

B Proofs

In the following, I prove Propositions 1-4, and Corollary 1.

B.1 Proof of Proposition 1

Given initial conditions $B_{i,-1}, B^{CB}_{i,-1}, X_{i,-1} > 0$ and fiscal policies $S_{it} = S_i \geq -\theta Y_i$ for $t \geq 0$ and $i = 1, 2$, the valuation equation for total public sector liabilities (24) determines P_0

$$P_0 = \frac{1-\beta}{\sum_i S_i + \theta Y} \left(\sum_i R_{i,-1}(B_{i,-1} - B^{CB}_{i,-1}) + R_{-1} \sum_i X_{i,-1} \right)$$

where the floor on the surpluses ensures that P_0 is strictly positive. For $i = 1$, consumption equation (23) then determines C_{10}, and the second equation in (20) determines C_{1t} for $t > 0$. The resource constraint (17) then determines C_{2t} for $t \geq 0$. Given time-invariant remittances and the price level, the fiscal authorities' flow budget constraints (18) determine B_{it}, $t \geq 0$, $i = 1, 2$. The flow budget constraint of the common central bank, i.e. the sum of the two NCBs' flow budget constraints (19), determines X_t, $t \geq 0$, and the distribution of reserves follows from $X_{it} = \frac{Y_i}{Y} X_t$, $i = 1, 2$. X^H_{it} then follows from the market clearing condition. The policy rate R_t, $t \geq 0$, is determined by interest rate rule (26). R_{2t}, $t \geq 0$, is then determined by households' optimality condition for reserves, equation (21), summed over i, and R_{1t}, $t \geq 0$, is determined by equation (22). The price level P_t in periods $t > 0$ follows from Fisher equation (20). For $i = 1$, equation (21) determines B^H_{11t}, and B^H_{21t} then follows from financial market clearing (the first equation in (16)) for all $t \geq 0$. The flow budget constraint of household 1 determines B^H_{12t}, and B^H_{22t}, $t \geq 0$, follows from financial market clearing. Finally, L_{12t} then follows from NCB 1's flow budget constraint (18), and L_{21t} follows from (16).

B.2 Proof of Corollary 1

Under constant primary surpluses, we can write the flow budget constraint of the public sector in the union for period $t+1$ as

$$\begin{aligned}\tilde{A}_{t+1} &= \frac{1}{\beta}\left(\tilde{A}_t - \theta Y\right) - \sum_i S_i \\ &= (1 + r_t^*)\,\tilde{A}_t - \sum_i S_i,\end{aligned}$$

where $r_t^* = 1/\beta - 1 - \theta Y/(\beta \tilde{A}_t)$. In steady state, the flow budget constraint becomes

$$\frac{\sum_i S_i}{\tilde{A}} = r^*,$$

where $\tilde{A} = (\beta \sum_i S_i + \theta Y)/(1-\beta) > \theta Y > 0$. Hence, $r^* > 0$, if $\sum_i S_i > 0$; $r^* = 0$, if $\sum_i S_i = 0$; and $r^* < 0$, if $\sum_i S_i < 0$.

Finally, note that $\tilde{A}_t = \tilde{A}$, and, hence, $r_t^* = r^*$ for all $t \geq 0$. We have

$$\frac{d\tilde{A}_{t+1}}{d\tilde{A}_t} = \frac{1}{\beta} > 1.$$

Hence, if $\tilde{A}_t > \tilde{A}$, then $\lim_{T\to\infty} \beta^T \tilde{A}_T > 0$, violating households transversality conditions. If $\tilde{A}_t < \tilde{A}$, then $\lim_{T\to\infty} \beta^T \tilde{A}_T < 0$, violating the no-Ponzi scheme conditions.

B.3 Proof of Proposition 2

The intertemporal budget constraints of public sectors 1 and 2 with $\tilde{L}_{12t} = 0$, and $\tilde{X}_{it} = 1/2\tilde{X}_t$ for all $t \geq 0$, and $i = 1, 2$ and are

$$\begin{aligned}\frac{(1-\alpha)R_{1,-1}B_{1,-1} + 1/2R_{-1}X_{-1}}{P_0} &= \frac{S_1}{1-\beta} + \sum_{t=0}^{\infty}\beta^t\left(\frac{R_{2t}-R_t}{R_{2t}}\left(\frac{1}{2}\tilde{X}_t + \theta_B(1-\alpha)\tilde{B}_{1t}\right)\right) \\ \frac{(1-\alpha)R_{2,-1}B_{2,-1} + 1/2R_{-1}X_{-1}}{P_0} &= \frac{S_2}{1-\beta} + \sum_{t=0}^{\infty}\beta^t\left(\frac{R_{2t}-R_t}{R_{2t}}\frac{1}{2}\tilde{X}_t\right)\end{aligned}$$

When the two public sectors enter period 0 with the same amount of liabilities, we have

$$\frac{S_1}{1-\beta}+\sum_{t=0}^{\infty}\beta^t\frac{R_{2t}-R_t}{R_{2t}}\left(\theta_B(1-\alpha)\tilde{B}_{1t}\right)=\frac{S_2}{1-\beta}$$

From $\sum_{t=0}^{\infty}\beta^t\frac{R_{2t}-R_t}{R_{2t}}\theta_B\tilde{B}_{1t}>0$ and $\alpha\in(0,1)$ follows $S_1<S_2$.

If households enter period 0 with the same amount of financial wealth and receive an equal share of the endowment, their consumption functions (23) imply

$$(1+\theta)(C_1-C_2)=S_2-S_1$$

Hence, it follows from $S_1<S_2$ that $C_1>C_2$.

B.4 Proof of Proposition 3

When $\theta_0\neq\theta$, and $S_{it}=S_i$ for $t\geq 0$, we can write equations (23) and (24) as

$$\left(1+\theta_0+\frac{\beta}{1-\beta}(1+\theta)\right)C_i=\frac{\sum_j R_{j,-1}B^H_{ij,-1}+R_{-1}X^H_{i,-1}}{P_0}+\frac{Y_i-S_i}{1-\beta}$$
$$\frac{\sum_j R_{j,-1}(B_{j,-1}-B^{CB}_{j,-1})+R_{-1}\sum_j X_{j,-1}}{P_0}=\frac{\sum_j S_j}{1-\beta}+\left(\theta_0+\frac{\beta}{1-\beta}\theta\right)Y$$

It follows from the second equation that $\partial P_0/\partial\theta_0<0$. Combining the two equations to substitute out P_0, we obtain

$$\left(1+\theta_0+\frac{\beta}{1-\beta}(1+\theta)\right)C_i=\mathcal{W}_i\left(\frac{\sum_j S_j}{1-\beta}+\left(\theta_0+\frac{\beta}{1-\beta}\theta\right)Y\right)+\frac{Y_i-S_i}{1-\beta}$$

where $\mathcal{W}_i=\frac{\sum_j R_{j,-1}B^H_{ij,-1}+R_{-1}X^H_{i,-1}}{\sum_j R_{j,-1}\sum_k B^H_{kj,-1}+R_{-1}\sum_k X^H_{k,-1}}$, and $\sum_j\mathcal{W}_j=1$.

Then,

$$\frac{\partial C_i}{\partial\theta_0}=\frac{(Y_h-S_h)\mathcal{W}_i-(Y_i-S_i)\mathcal{W}_h}{(1-\beta)\left(1+\theta_0+\beta/(1-\beta)(1+\theta)\right)^2},$$

where $h\neq i$. When $\mathcal{W}_i=\mathcal{W}_h$, and $Y_i=Y_h$

$$\frac{\partial C_i}{\partial\theta_0}=\frac{1}{2}\frac{S_i-S_h}{(1-\beta)\left(1+\theta_0+\beta/(1-\beta)(1+\theta)\right)^2}.$$

In the neighborhood of $\theta_0 = \theta$

$$\frac{\partial C_i}{\partial \theta_0 | \theta_0 = \theta} = \frac{1}{2}(1-\beta)\frac{S_i - S_h}{(1+\theta)^2}.$$

Note that $S_1 < S_2$. Hence

$$\frac{\partial C_1}{\partial \theta_0} < 0, \quad \frac{\partial C_2}{\partial \theta_0} > 0.$$

B.5 Proof of Proposition 4

When $S_{it} = S_i$ for $t > 0$, we can write equations (23) and (24) as

$$(1+\theta)C_i = Y_i + (1-\beta)\left(\frac{\sum_j R_{j,-1}B^H_{ij,-1} + R_{-1}X^H_{i,-1}}{P_0} - S_{i0}\right) - \beta S_i$$

$$P_0 = (1-\beta)\frac{\sum_j R_{j,-1}(B_{j,-1} - B^{CB}_{j,-1}) + R_{-1}\sum_j X_{j,-1}}{(1-\beta)\sum_j S_{j0} + \beta\sum_j S_j + \theta Y}$$

It follows from the second equation that $-\partial P_0/\partial S_{i0} > 0$. Combining the two equations to substitute out P_0, we obtain

$$\begin{aligned}(1+\theta)C_i &= Y_i - (1-\beta)S_{i0} - \beta S_i + \mathcal{W}_i\left((1-\beta)\sum_j S_{j0} + \beta\sum_j S_j + \theta Y\right)\\ &= Y_i + \mathcal{W}_i\theta Y + (1-\beta)(\mathcal{W}_i - 1)S_{i0} + (1-\beta)\mathcal{W}_i S_{h0} + \beta(\mathcal{W}_i - 1)S_i + \beta\mathcal{W}_i S_h\\ &= Y_i + \mathcal{W}_i\theta Y - (1-\beta)\mathcal{W}_h S_{i0} + (1-\beta)\mathcal{W}_i S_{h0} - \beta\mathcal{W}_h S_i + \beta\mathcal{W}_i S_h\end{aligned}$$

where $h \neq i$, $\mathcal{W}_i = \frac{\sum_j R_{j,-1}B^H_{ij,-1} + R_{-1}X^H_{i,-1}}{\sum_j R_{j,-1}\sum_k B^H_{kj,-1} + R_{-1}\sum_k X^H_{k,-1}}$, and $\sum_j \mathcal{W}_j = 1$. Hence,

$$\frac{\partial C_i}{\partial S_{i0}} = -\frac{1-\beta}{1+\theta}\mathcal{W}_h < 0 \tag{B.1}$$

$$\frac{\partial C_i}{\partial S_{h0}} = \frac{1-\beta}{1+\theta}\mathcal{W}_i > 0. \tag{B.2}$$

Finally

$$\frac{\partial^2 C_i}{\partial S_{i0}\partial\theta} > 0 \tag{B.3}$$

$$\frac{\partial^2 C_i}{\partial S_{h0}\partial\theta} < 0. \tag{B.4}$$

C Additional results

C.1 Symmetric fiscal expansion

Proposition 5 *Suppose, households have the same initial financial wealth, $\mathcal{W}_i = 1/2$, $i = 1, 2$. Then a symmetric reduction (increase) of the two fiscal authorities' primary surpluses in period 0, $S_{i0} - S_i = \Delta S < 0$ (> 0), $i = 1, 2$, raises (lowers) the period 0 price level, and leaves household consumption unaffected.*

Proof: The effect on the period 0 price level follows from equation (24). The absence of an effect on consumption when $\mathcal{W}_1 = \mathcal{W}_2$ follows from equations (B.1) and (B.2).

C.2 Power utility: equilibrium existence and uniqueness

The flow budget constraint of the public sector in the currency union can be written as

$$\sum_i \left(\tilde{B}_{it} - \tilde{B}^{CB}_{it} + \tilde{X}_{it} + S_{it}\right) = \frac{1}{\beta}\left(\sum_i \left(\tilde{B}_{it-1} - \tilde{B}^{CB}_{it-1} + \tilde{X}_{it-1}\right) - \frac{R_{2t-1} - R_{t-1}}{R_{2t-1}}\left(\tilde{X}_{t-1} + \theta_B(\tilde{B}_{1t-1} - \tilde{B}^{CB}_{1t-1})\right)\right) \tag{C.1}$$

Using equation (31) summed over i to substitute out the interest rate spread, we obtain

$$\sum_i \left(\tilde{B}_{it} - \tilde{B}^{CB}_{it} + \tilde{X}_{it} + S_{it}\right) = \frac{1}{\beta}\left(\sum_i \left(\tilde{B}_{it-1} - \tilde{B}^{CB}_{it-1} + \tilde{X}_{it-1}\right) - \theta Y^{\sigma}\left(\tilde{X}_{t-1} + \theta_B(\tilde{B}_{1t-1} - \tilde{B}^{CB}_{1t-1})\right)^{1-\sigma}\right) \tag{C.2}$$

In a deterministic steady state

$$\sum_i \left(\tilde{B}_i - \tilde{B}_i^{CB}\right) + \tilde{X} = \frac{\beta}{1-\beta}\sum_i S_i + \frac{\theta Y^\sigma}{1-\beta}\left(\tilde{X} + \theta_B(\tilde{B}_1 - \tilde{B}_1^{CB}\right)^{1-\sigma} \tag{C.3}$$

Let us assume, for tractability, that the NCBs hold all government bonds, $\tilde{B}_i^{CB} = \tilde{B}_i$, $i = 1, 2$. Then

$$\frac{\sum_i S_i}{\tilde{X}} = \frac{1}{\beta} - 1 - \frac{\theta Y^\sigma}{\beta}\frac{1}{\tilde{X}^\sigma} \tag{C.4}$$

Note that both sides of the above equation are continuous for $\tilde{X} > 0$. It is useful to distinguish two cases.

Case I: $\sum_i S_i \geq 0$**.** If $\sum_i S_i > 0$, the left-hand side of the above equation is strictly decreasing in $\tilde{X}$ with $\lim_{\tilde{X}\to 0^+} \sum_i S_i/\tilde{X} = \infty$, and $\lim_{\tilde{X}\to\infty} \sum_i S_i/\tilde{X} = 0$. If $\sum_i S_i = 0$, the left-hand side equals zero for all $\tilde{X} > 0$. The right-hand side is strictly increasing in $\tilde{X}$ with $\lim_{\tilde{X}\to 0^+} f(\tilde{X}) = -\infty$, and $\lim_{\tilde{X}\to\infty} f(\tilde{X}) = 1/\beta - 1$, where $f(\tilde{X}) = \frac{1}{\beta} - 1 - \frac{\theta Y^\sigma}{\beta}\frac{1}{\tilde{X}^\sigma}$. Hence, there exists a unique $\tilde{X} > 0$, denoted by $\tilde{X}^*$, for which the above equation holds.

The gross real interest rate on reserves in this steady state equals

$$r^* = \frac{1}{\beta} - \frac{\theta Y^\sigma}{\beta}(\tilde{X}^*)^{-\sigma} \geq 1.$$

Let us now return to the dynamic system. Define $\Delta\tilde{X}_t = \tilde{X}_t - \tilde{X}_{t-1}$. Then we can write the public sector flow budget constraint as

$$\Delta\tilde{X}_t = \left(\frac{1}{\beta} - 1 - \frac{\theta Y^\sigma}{\beta}\tilde{X}_{t-1}^{-\sigma}\right)\tilde{X}_{t-1} - \sum_i S_i. \tag{C.5}$$

It holds that $\Delta\tilde{X}_t > 0$ when $\tilde{X}_{t-1} \in (\tilde{X}^*, \infty)$, and $\Delta\tilde{X}_t < 0$ when $\tilde{X}_{t-1} \in (0, \tilde{X}^*)$. To see this, note that

$$\frac{d\Delta\tilde{X}_t}{d\tilde{X}_{t-1}}|_{\tilde{X}_{t-1}=\tilde{X}^*} = r^* - 1 + \sigma\frac{\theta Y^\sigma}{\beta}\left(\tilde{X}^*\right)^{-\sigma} > 0. \tag{C.6}$$

Existence of a unique steady state and continuous differentiability of $\Delta\tilde{X}_t$ for $\tilde{X}_{t-1} > 0$ then rules out that there exists a $\tilde{X}_{t-1} \in (\tilde{X}^*, \infty)$ for which $\Delta\tilde{X}_t < 0$, or a $\tilde{X}_{t-1} \in (0, \tilde{X}^*)$ for which $\Delta\tilde{X}_t > 0$.

We can then show that no explosive equilibria exist, using proof by contraction.

Suppose, there exists an equilibrium with $\tilde{X}_{t'} < \tilde{X}^*$ for some $t' > 0$. Then from the above discussion $\tilde{X}_{t''} < \tilde{X}_{t'}$ for all $t'' > t'$, and the constraint $\tilde{X}_t > 0 \ \forall t \geq 0$ would be violated in finite time. Thus, such an equilibrium cannot exist.

Now suppose that there exists an equilibrium with $\tilde{X}_{t'} > \tilde{X}^*$ for some $t' > 0$. Then $\tilde{X}_{t''} > \tilde{X}_{t'}$ for all $t'' > t'$, and $\lim_{T\to\infty} r^*_T(\tilde{X}_T) = 1/\beta$, i.e. in the limit as time goes to infinity the real value of total public sector liabilities would grow at the inverse of the subjective discount factor. But then $\lim_{T\to\infty} \beta^T \tilde{X}_T > 0$, violating households' transversality condition.

Case II: $\sum_i S_i < 0$. Let us re-write the steady state flow budget constraint of the public sector (C.4) as follows

$$\sum_i S_i = \left(\frac{1}{\beta} - 1\right)\tilde{X} - \frac{\theta Y^\sigma}{\beta}\tilde{X}^{1-\sigma} \tag{C.7}$$

The left-hand side of the above equation is strictly negative and constant. For the right-hand side, we have to distinguish between $\sigma > 1$ and $\sigma < 1$.

Case II.a: $\sigma > 1$. If $\sigma > 1$, the right-hand side of equation (C.7) is strictly increasing in $\tilde{X}$, for all $\tilde{X} > 0$, with $\lim_{\tilde{X}\to 0^+} g(\tilde{X}) = -\infty$, and $\lim_{\tilde{X}\to\infty} g(\tilde{X}) = \infty$, where $g(\tilde{X}) = \left(\frac{1}{\beta} - 1\right)\tilde{X} - \frac{\theta Y^\sigma}{\beta}\tilde{X}^{1-\sigma}$. Hence, there exists a unique $\tilde{X}$ for which the above equation holds. The steady state (net) real interest rate on reserves is negative, $r^* < 1$.

The remainder of the proof establishing equilibrium uniqueness then follows the proof in Case I. In particular, note that

$$\frac{d\Delta\tilde{X}_t}{d\tilde{X}_{t-1}}|_{\tilde{X}_{t-1}=\tilde{X}^*} = r^* - 1 + \sigma\frac{\theta Y^\sigma}{\beta}\left(\tilde{X}^*\right)^{-\sigma} = \frac{1}{\beta} - 1 + (\sigma - 1)\frac{\theta Y^\sigma}{\beta}\left(\tilde{X}^*\right)^{-\sigma} > 0. \tag{C.8}$$

Case II.b: $\sigma < 1$. If $\sigma < 1$, the right-hand side of equation (C.7) is not monotonic in $\tilde{X}$. Moreover, $\lim_{\tilde{X}\to 0^+} g(\tilde{X}) = 0$, and $\lim_{\tilde{X}\to\infty} g(\tilde{X}) = \infty$.

Let $\underline{\tilde{X}} = \arg\min_{\tilde{X}>0} g(\tilde{X})$. Define $\underline{S} = g(\underline{\tilde{X}}) = -\frac{\sigma}{1-\sigma}\frac{1-\beta}{\beta}\left(\frac{\theta(1-\sigma)}{1-\beta}\right)^{1/\sigma} Y < 0$, where $\underline{\tilde{X}} = \left(\frac{\theta(1-\sigma)}{1-\beta}\right)^{1/\sigma} Y$. Hence, no steady-state equilibrium exists when $\sum_i S_i < \underline{S}$. In the knife-edge case where $\sum_i S_i = \underline{S}$, one steady state exists. And when $\sum_i S_i > \underline{S}$, two steady states exist. Denote the real value of reserves in the two steady states by $\tilde{X}^*_I$ and $\tilde{X}^*_{II}$, respectively. Then, $\tilde{X}^*_I \in (0, \underline{\tilde{X}})$, and $\tilde{X}^*_{II} \in (\underline{\tilde{X}}, \infty)$. Hence, generically, when $\sum_i S_i < 0$ and $\sigma < 1$, there does not exist a unique

equilibrium.

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