EUROPEAN CENTRAL BANK
EUROSYSTEM

WORKING PAPER SERIES
NO 1207 / JUNE 2010

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A REGIME SWITCHING APPROACH

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In 2010 all ECB publications feature a motif taken from the €500 banknote.

1 This paper has been earlier circulated under the title: "Money growth leads inflation regime changes". We thank Pooyan Amir Ahmadi, Jacopo Capra and Joanna Slawatyniec for providing valuable research assistance and seminar participants at the Central Bank of Ireland, ECB and Bank of Norway for comments on earlier drafts of the paper. We are particularly indebted to Luca Benati, Daniela Bragoli, Antonello D. Agostino, Michael Dueker, Bjorn Fischer, Domenico Giannone, Stefaan Idee, Marek Jarocinski, Michele Lenza, Huw Pill, Patrick Lunneman, Barbara Roffia, Frank Smets, Oreste Tristani and an anonymous referee for the ECB Working Paper series for very useful suggestions.

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ISSN 1725-2806 (online)

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Abstract

We develop a time-varying transition probabilities Markov Switching model in which inflation is characterised by two regimes (high and low inflation). Using Bayesian techniques, we apply the model to the euro area, Germany, the US, the UK and Canada for data from the 1960s up to the present. Our estimates suggest that a smoothed measure of broad money growth, corrected for real-time estimates of trend velocity and potential output growth, has important leading indicator properties for switches between inflation regimes. Thus money growth provides an important early warning indicator for risks to price stability.

Keywords: money growth, inflation regimes, early warning, time varying transition probabilities, Markov Switching model, Bayesian inference.

JEL codes: C11, C53, E31

Non-technical Summary

The long-run co-movement between money growth and inflation is a well-established empirical regularity. It has been found to apply across a range of countries and across different time periods. However, in recent years – characterised by low and relatively stable inflation – empirical evidence suggests that the value of monetary (and other) indicators for forecasting inflation is rather limited. Some observers have argued that such results suggest that the role of monetary indicators in monetary policy should accordingly be downgraded.

This paper questions this line of argument. It is true that, in a regime of price stability, the link between money growth and inflation may be weak, for example, being obscured by velocity shocks. Moreover, if the central bank responds effectively to counteract risks to price stability signalled by money (or other indicators), it is likely that, empirically, it will be difficult to establish leading indicator properties in the actual data. Indeed, one could even envisage a hypothetical situation in which the central bank was so successful in using leading information to calibrate its policy, that price stability would be maintained at all times. In such a case, it would be impossible to detect, using standard techniques, empirical leading indicator properties for inflation from the indicators used by the central bank. In order, therefore, to detect such indicator properties, the sample period of the analysis must include episodes in which inflation deviated from price stability and this deviation was signalled by the monetary indicator. This is a clear case where indeed one can learn from ones mistakes.

This line of reasoning suggests an alternative approach. Specifically, we model inflation as a regime-switching process, in which inflation is characterised by two regimes – low and high inflation. In our setup, the probability of moving from low to high inflation is allowed to depend on the rate of money growth. Money growth can thus act as a 'warning signal' of the risk of the departure of inflation from the price stability regime. Arguably for a central bank committed to price stability, such a signal may be a more valuable piece of information than a forecast for inflation at a specific horizon within the given regime.

We apply the model to data from Canada, the euro area, Germany, the US and the UK. We use using quarterly data from the early 1960s to 2009, which is a sufficiently long span to incorporate a number episodes of switches in inflation regime. We estimate the model parameters using Bayesian techniques. The results obtained support the view that money growth provides timely warning signals of transitions between inflation regimes and thus can be a useful indicator of risks to price stability. A number of robustness checks confirm this overall conclusion.

1 Introduction

The long-run co-movement between money growth and inflation is a well-established empirical regularity. It has been found to apply across a range of countries, including the euro area, and across different time periods. This link has been documented by a range of studies including Lucas (1980), Benati (2009) and Sargent and Surico (2008). The evidence on the co-movement between money growth and inflation has typically been examined by looking at the relationship between smoothed measures based either on moving averages or frequency domain techniques. While these techniques are valuable in identifying the relevant empirical relationships, they suffer from an important limitation, namely they may not be reliable at the end of the sample, precisely the point in time which is of most relevance to the policy-maker. In the light of this, a number of more standard forecasting techniques have also been used based either on extended Phillips curve models (see, for example, Gerlach (2004)) or linear single equation forecasting models (Nicoletti Altimari (2001) and Fischer, Lenza, Pill, and Reichlin (2008)). These studies suggest that money growth has in the past contained useful information about future inflation which enables these forecasts to outperform naive predictors. However, in recent years - characterized by relatively low and stable inflation - the relationship between money growth and inflation has weakened and the predictive power relative to naive benchmarks of money (and a range of other variables) for future inflation has weakened. In this regard see Stock and Watson (2006) for the US and Lenza (2006) for the euro area.

Do the weakened leading indicator properties of money growth for future inflation imply that money can now be safely disregarded by central banks? We argue that this would be an inappropriate conclusion. In a regime of low inflation, the correlation between money growth and inflation may indeed be found to be weak and money growth may be found to have limited value in terms of forecasting inflation – as long as the economy remains within the low inflation regime. This is not surprising, since, as pointed out by Estrella and Mishkin (1997), velocity shocks will tend to blur the signals coming from money in low inflation environments. However, based on the experience of a number of countries, we show that money growth has provided important and timely warning signals about the risk of the economy departing from a regime of price stability. Arguably, this risk is of greater concern to a stability-oriented central bank than variations in the inflation rate within a regime of low inflation. This way of thinking leads to a modelling of the inflation process which is different from the standard linear approach employed in much of the literature. Instead of modelling variations in inflation around a constant mean (or, alternatively, treating inflation as a unit root process), inflation is characterized by a regime-switching model in which the economy can potentially switch between regimes of low and high inflation. The approach aims to develop a model which exploits the well-established long-run co-movement between money and inflation but which can be used to provide policymakers with a money-based inflation warning indicator of shifts in inflation regime which is usable in real time.

We build on the extensive literature in which inflation is modelled as a Markov-Switching process (see, for example, Evans and Wachtel (1993) for the US, Ayuso, Kaminsky and López-Salido (2003) for Spain and Ricketts and Rose (2007) for G7 countries). We follow this literature in assuming that the inflation rate is governed by a regime-switching process, in which inflation shifts from regimes of low to high inflation and vice versa. In our case, we model inflation using a Bayesian Markov Switching framework. In contrast to the standard MS model, however, we follow Abiad (2003), Filardo (1994) and Kim and Nelson (1999) in allowing the transition probabilities to depend on other observable variables - in our case a smoothed measures of money growth (which can be calculated in real time). In this setting, we argue that inflation in a number of countries can be well represented as a regime-shifting process, characterized by two regimes - which one may loosely describe as price stability and high inflation. Within a regime of low inflation, money growth is not necessarily useful for predicting inflation in future periods. However, in our setup, money growth is allowed to play an important role in signaling the probability that the economy will move from a low inflation to a high inflation regime - thus providing a warning indicator of the risks to price stability. In this regard, we show that, based on experience in euro area and a number of OECD countries, money growth has provided important and timely warnings of shifts from low to high inflation regimes.

The resulting indicator is based on the past relation between money growth and inflation. Such a relationship is only likely to be found in data for samples in which the central bank has not responded adequately to inflationary risks and has not been fully successful in maintaining price stability. If, on the other hand, as pointed out by Woodford (1994) among others, the central bank responds in a timely manner to inflationary risks highlighted by money or other indicators and successfully maintains price stability, then standard empirical tests are likely to show weakened (or even negligible) leading indictor properties for future inflation. In such cases, an indicator based on historical data is best seen as a "warning signal" of inflationary risks rather than as a forecast of likely future inflation developments. Our paper thus has many parallels with the literature of early warning systems for financial crises (see Kaminsky, Lizondo and Reinhart (1998), for a review and Alessi and Detken (2009) for a recent application).

The remainder of this paper is structured as follows. In Section (2) we outline the econometric model employed. Section (3) presents our estimation methodology while Section (4) presents the dataset. Section (5) presents the results and in Section (6) we check the robustness of the results; Section (7) concludes. In Appendix (A) we provide the details of the Bayesian posterior simulation and in Appendix (B) we describe data sources.

2 The econometric model

Here we model inflation (y_t) as a stationary process that, conditional on a unobservable variable s_t , has a AR(1) dynamics

$$y_t = c_{s_t} + \phi_{s_t} y_{t-1} + \sigma_{s_t} e_t \tag{1}$$

$$e_t \backsim NID(0,1) \tag{2}$$

where s_t is a Markov Switching discrete process describing the regime of inflation. There are two possible regimes, $s_t = 1$ (low inflation) and $s_t = 2$ (high inflation) transition probabilities (henceforth TP) across regimes possibly depends on a conditioning variable, the early indicator variable z_{t-1}:

$$p(s_t = j|\underline{\mathbf{s}}_{t-1}, \underline{\mathbf{y}}_{t-1}, \theta) = \tag{3}$$

$$= p(s_t = j|s_{t-1} = i, z_{t-1}, \theta) = p_{ij,t} \tag{4}$$

It is useful to remember that the unconditional first and second moments of the AR process, in the case of fixed TPs, would be easy to describe (see Timmermann (2000)). Things are much more difficult when we have time-varying TPs (via dependence on some indicator variable z_t): in this case it is much harder to find exact analytical forms for moments.

2.1 Time varying transition probabilities

We consider a situation in which an early warning (henceforth EW) $(r \times 1)$ indicator vector $\mathbf{z}_{t-1}$ affects TPs. A convenient parameterisation for this mechanism is the probit parameterisation:

$$p(s_t = 1|\underline{\mathbf{s}}_{t-1} = i, \underline{\mathbf{y}}_{t-1}, \mathbf{z}_{t-1}, \theta) = p_{1j,t} \tag{5}$$

$$= \Phi\left(\gamma_i^{\prime} \mathbf{z}_{t-1} \right), \tag{6}$$

$$\Phi(\omega) = \int_{-\infty}^{\omega} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\omega^2\right\} d\omega$$

In this way the parameter γ_{ri} $(r = 1, 2, ..k_z)$, i.e. the r^{th} element of vector γ_i, measures the sensitivity of probability $p_{1j,t}$ with respect to z_{rt-1}, i.e. the r-th element of the indicator variables vector $\mathbf{z}_{t-1}$. In the case in which there are two elements in the vector $\mathbf{z}_t$, an intercept term and a true EW variable z_{2t}, then the mechanism (5) requires four parameters which we organise in the (2×2) Γ matrix. The parameters γ_{11} and γ_{12} can be thought as state dependent intercepts and γ_{21} and γ_{22} as state dependent slope coefficients. We call this specification EW-MS (early warning-Markov Switching) model. It is worth noting that the standard Markov Switching model which is used in the literature (which implies time invariant transition probabilities) is a special case of our model and holds when $\gamma_{21} = \gamma_{22} = 0$.

When using this specification in a contexts where a small number of transitions is observed in the sample period, it might be profitable, in order to increase the efficiency of the estimates, that the slope coefficients are common across states:

$$\gamma_{21} = \gamma_{22} = \gamma_2 \tag{7}$$
$$p(s_t = 1|s_{t-1} = i, I_{t-1}) = \Phi(\gamma_{1i} + \gamma_2 z_{t-1}), i = 1, 2. \tag{8}$$

In spite of this restriction, the specification is flexible enough to generate sensible transition probabilities.

In order to better understand the mechanism, let us consider an example taken from the posterior mean estimate of the EW model for the US (see Table 7). Assume that the parameter values are:

$$\gamma_{11} = .99$$
$$\gamma_{21} = \gamma_{22} = -.22$$
$$\gamma_{12} = -.49$$

we can compute the transition probability matrix at different values for the z_{2t} indicator which is an adjusted and standardised measure of lagged money growth $\Delta\widetilde{m}_{t-k}$. We take into consideration three points: $z_{2t} = 0$, the sample mean of the standardised indicator, $z_{2t} = -2.08$, the minimum sample value for the EW attained in 1995Q2 and $z_{2t} = 2.15$, the sample maximum value of the indicator which occurred in 1985Q2. We also compute the vector of ergodic probabilities, $\pi(z_{2t})$, which would describe the ergodic probabilities of each state in the case that the indicator variable stayed indefinitely at level z_{2t}:

$$\mathbf{P}(z_{2t} = 0) = \begin{bmatrix} .81 & .19 \\ .33 & .67 \end{bmatrix}, \pi(z_{2t} = 0) = \begin{bmatrix} .64 \\ .36 \end{bmatrix}$$
$$\mathbf{P}(z_{2t} = -2.08) = \begin{bmatrix} .92 & .08 \\ .54 & .46 \end{bmatrix}, \pi(z_{2t} = -2.08) = \begin{bmatrix} .88 \\ .12 \end{bmatrix}$$
$$\mathbf{P}(z_{2t} = 2.15) = \begin{bmatrix} .62 & .38 \\ .16 & .84 \end{bmatrix}, \pi(z_{2t} = 2.15) = \begin{bmatrix} .30 \\ .70 \end{bmatrix}.$$

It is possible to see that when $z_{2t} = 0$, the probability of remaining in a low inflation regime is .81, corresponding to an expected duration of the low inflation regime of 5 quarters and an ergodic probability of .64. When the money growth index is at its minimum (-2.08) the probability of staying in low regime is .92, corresponding to an expected duration of the low inflation state of 13 quarters, and the ergodic probability of the low inflation state is .86. In situations of very pronounced money growth, for example when the EW indicator is equal to 2.15, then the conditional probability of staying in a low inflation state is equal to .62, the expected duration of that regime is 3 periods and the ergodic probability is .30.

Summing up, the EW affects transition probabilities via the slope parameter. Higher values of money growth reduce probabilities of remaining in (or moving to) the low inflation regime. The theoretical relationship is illustrated in Figure (1).

2.2 Marginal effects and elasticities

As in probit models, it is possible to obtain measures of the marginal effects of indicators on probabilities as follows:

$$\frac{\partial p_{i1,t}}{\partial z_{rt-1}} = \gamma_r \times \phi(\gamma_i' \mathbf{z}_{t-1}), \tag{9}$$
$$\phi(\omega) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\omega^2}{2}\right\}$$

Note that these measures are non linear in the indicators values and they are larger the closer to zero is $\gamma_i' \mathbf{z}_{t-1}$. In the same way, the elasticities of transition probabilities conditioned on the current state of indicator variables are:

$$\frac{\partial p_{i1,t+1}}{\partial z_{rt}} \times \frac{z_{rt}}{p_{ij,t+1}} = \gamma_{ri} \times \frac{\phi(w_t)}{\Phi(w_t)} \times z_{rt} \tag{10}$$

and the elasticity of the unconditional probability of being at t+1 in state 1 (low inflation) is

$$\frac{\partial p_{1,t+1}}{\partial z_{rt}} = \frac{\sum_{i=1}^{2} \frac{\partial p_{i1,t+1}}{\partial z_{rt}} \times \pi_{it|t}}{\sum_{i=1}^{2} p_{i1,t+1} \times \pi_{it|t}} = \frac{\sum_{i=1}^{2} \gamma_{ri} \times \phi(\gamma_i' \mathbf{z}_{t-1}) \times \pi_{it|t}}{\sum_{i=1}^{2} \Phi(\gamma_i' \mathbf{z}_{t-1}) \times \pi_{it|t}} \times z_{rt} \tag{11}$$

where $\pi_{it|t}$ are the filtered probabilities a time t of each state. Note also that all the above quantities are highly nonlinear functions of the parameters of the model.

3 Inference

In this paper, we adopt a Bayesian approach to estimation. Simulation-based Bayesian methods are well suited to estimate Markov Switching models. See for example Kim and Nelson (1999), Amisano and Giacomini (2007) and Geweke and Amisano (2010)). The Bayesian approach is particularly well suited to deal with the EW model. Like all Markov Switching models, the EW model has a very irregular likelihood surface which is not easily amenable to numerical maximisation. The Bayesian approach allows to easily obtain a joint posterior distribution for the parameters and the latent variables. This is done by using a Gibbs sampling based posterior simulation.

Calling $\underline{\mathbf{y}}_T = \{y_\tau, \tau = 1, 2, ...T\}$ and $\underline{\mathbf{z}}_T = \{\mathbf{z}_\tau, \tau = 1, 2, ...T\}$ the data on the endogenous variable and the indicators, θ the vector of the free parameters and $\underline{\mathbf{s}}_T = \{s_t, t = 1, 2, ...T\}$ the sample values of the discrete MS process describing the regime, the object of the Bayesian posterior simulation is to obtain a large sample from the joint posterior distribution of $\underline{\mathbf{s}}_T$ and θ:

$$p(\underline{\mathbf{s}}_T, \theta | \underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T) \propto p(\underline{\mathbf{y}}_T | \underline{\mathbf{z}}_T, \underline{\mathbf{s}}_T, \theta) \times p(\underline{\mathbf{s}}_T | \underline{\mathbf{z}}_T, \theta) \times p(\theta)$$

Sampling from the joint posterior distribution can be achieved by implementing a Gibbs sampling-data augmentation algorithm that iterates on two steps:

1. sampling from the conditional posterior distribution of the parameters

$$p(\theta|\underline{\mathbf{s}}_T, \underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T)$$

2. sampling from the conditional posterior distribution of the latent states

$$p(\underline{\mathbf{s}}_T|\underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T, \theta)$$

How to implement these steps is described in detail in Appendix (A).

It is interesting to note that, with a sample of latent variables and parameters from their joint posterior distribution, it is possible to obtain the posterior distribution of nonlinear functions of the state and the parameters, such as marginal effects and elasticities of transition probabilities and the predictive distribution for the endogenous variable.

3.1 Forecasting

To obtain the predictive distribution of the dependent variable we need to compute:

$$\begin{aligned} p(y_{t+1}|\underline{\mathbf{y}}_t^*, \theta) = \sum_{j=1}^{m}\sum_{i=1}^{m} & \underbrace{p(y_{t+1}|\underline{\mathbf{y}}_t^*, s_{t+1}=j, s_t=i, \theta)}_{\text{state conditional predictive density}} \times \\ & \times \underbrace{p(s_{t+1}=j|s_t=i, \underline{\mathbf{y}}_t^*, \theta)}_{\text{time varying TPs}} \times \underbrace{p(s_t=i|\underline{\mathbf{y}}_t^*, \theta)}_{\text{filtered probabilities}} \end{aligned} \tag{12}$$

where $\underline{\mathbf{y}}_t^* = \{y_\tau, z_\tau, \tau = 1, 2...., t\}$ is the data information up to period t. All the factors in the above expression can be easily obtained since

- the state conditional predictive density $p(y_{t+1}|\underline{\mathbf{y}}_t^*, s_{t+1}=j, s_t=i, \theta)$ is Gaussian;
- the time-varying TPs $p(s_{t+1}=j|s_t=i, \mathbf{z}_t, \theta)$ are generated by the probit specification (5);
- the filtered probabilities $p(s_t = i|\underline{\mathbf{y}}_t^*, \theta)$ are obtained via application of the filtering recursion documented in Appendix (A).

This means that, conditional on a given value for the parameters, the predictive density is a discrete mixture of Gaussian distributions. In order to obtain the predictive density $p(y_{t+1}|\underline{\mathbf{y}}_t^*)$ it is necessary to use MCMC integration as follows:

$$p(y_{t+1}|\underline{\mathbf{y}}_t^*) = \frac{1}{M}\sum_{m=1}^{M} p(y_{t+1}|\underline{\mathbf{y}}_t^*, \theta^{(m)}) \tag{13}$$

where $\theta^{(m)}, m = 1, ..., M$, is a sample from the posterior distribution of the parameters. In the same way, we can obtain multistep ahead predictive densities and the posterior probabilities of any other more structured event, e.g. the expected duration of low inflation regime, conditional on all information available at time t.

4 The dataset

We run our EW model for different countries separately. In particular, we chose to analyse Canada, the Euro area (EA) Germany, the UK and the US.

This group of countries has been chosen for two reasons. First, data on the relevant variables are readily available for sufficiently long spans. Second, focusing on a set of countries instead of a single country should throw light on the robustness of the results. In particular, the group of countries chose have had diverse inflation experiences.

The data set for each of the country considered contains inflation, money and output. In particular output (and its deflator) is needed for the velocity adjustment described in Section 4.1. In Table (1) we give summary information on the series being used, while in Appendix (B) we give more details on the sources and the transformations applied to the data. In particular we should emphasise that sample for Germany ends with the inception of the EMU (1998Q4).

For the euro area, any empirical analysis has to address the issue as to how to construct historical backdata. A number of approaches are available for this purpose (see Beyer, Doornik and Hendry (2000) for a review of alternative methods) and typically involve some weighted average of the series of the historical data of the participating countries. However, since our primary interest is in developing a warning indicator for use as in input into the ECB's monetary policy, we chose a method suggested by Bruggemann, Luetkepohl and Marcellino (2008) which is found to lead to more accurate inflation forecasts over the period since the start of the monetary union. Specifically, this method involves splicing German backdata onto the official euro area data published by the ECB. The rationale for this approach is that the monetary policy regime of the ECB, in particular its commitment to maintaining price stability, more closely resembles the historical policy of Bundesbank over a longer period rather than a "weighted average" of the past monetary regimes of all the participating countries. Thus our data for the euro area comprises official area-wide data from 1992 onwards linked to German data for the earlier period.

For the monetary variables to be included as a potential indicator in our early warning model, there are in principle a number of possible aggregates. However, for each country we chose the broad money aggregate which is the main aggregate used by the respective central banks (M3 in Germany and the Euro Area, M2 in the US, M4 in the UK and M3 in Canada).

4.1 Accounting for trends in real growth and velocity

The standard quantity equation expressed in logs and in first differences is:

$$\Delta m_t + \Delta v_t = \Delta p_t + \Delta y_t \tag{14}$$

where m is the respective monetary aggregate, v is velocity, p the price level and y output. This can be rearranged as:

$$\Delta p_t = \Delta m_t + \Delta v_t - \Delta y_t \tag{15}$$

Headline money growth (Δm_t) per se may not be a good indicator of inflationary pressure if velocity or potential output are subject to changes in trend. In order to deal with this problem, it is useful to adjust raw money growth for such changes. We follow the practice of the ECB (in calculating its reference value) or the former practice of the Bundesbank (in calculating its monetary target), in correcting raw money growth for velocity and output trends. The rationale for this procedure is discussed more fully in Orphanides and Porter (2001). Specifically, we define a measure of adjusted money growth which is given by:

$$\Delta m_t^* = \Delta m_t - \Delta \widetilde{y}_t + \Delta \widetilde{v}_t \tag{16}$$

where $\Delta \widetilde{v}_t$ and $\Delta \widetilde{y}_t$ are time-varying growth rates of, respectively, output and velocity.

In the simplest case, where the trend in velocity and output were constant, this would simply involve adjusting money growth using the average growth rates of velocity and output. However, it is well known that the trend of money velocity may change over time (see Bordo and Jonung (2004)) and, similarly, changes in potential output growth are well documented (for the case of the US, see, for example, Edge, Laubach and Williams (2007)) Therefore we need to calculate these trends using a method which i) allows for changes in trends and ii) can be computed in real time. In this paper, $\Delta \widetilde{v}_t$ and $\Delta \widetilde{y}_t$ are computed as the sample means of Δv_t and Δy_t using a rolling window of $w = 40$ observations. This is easy, as opposed to using more sophisticated models, for instance an unobserved component model. The disadvantage is that we lose the first 40 observations in order to compute the rolling window based adjustments. For this reason, we use data from 1950Q1 for all countries being considered. This allows us to perform the velocity trend adjustment and to be able to use an effective sample size that can start as early as in 1963Q1 accounting for lags.

In the upper left panel of Figures (3) to (7) we report the estimated trend growth of output and velocity and in the upper right panel of the same figures we report the unadjusted and adjusted money growth indicators, together with the actual inflation series. This is done country by country. The differences between the adjusted and unadjusted money indicator series are, by definition, due to the evolution of the trend growth rates of output and velocity.

In particular, for all countries we see that trend growth rate of velocity has dramatically decreased since the mid 1990s, with the notable exception of the US, where velocity is roughly constant up to the mid 1990s and then briefly increasing (upper left panel of Figure 7). We see also some important movements in the trend output growth rate. As an example (see the upper left panel of Figure 4), in the EA trend output growth decreases from a value around 6% in the 1960s to values around 2% from the early 1980s onwards.

The joint influence of the variations of trend growth rates for velocity and output generate relevant differences between the adjusted and unadjusted money growth series. Looking at the EA data (top right panel of Figure 4) for all the period from the 1995 onwards using the unadjusted money growth series would lead to understate the risks of transition to a high inflation regime. For the US (see top right panel of Figure 7), in the last 8 quarters the two indicators (adjusted and unadjusted) have roughly the same behaviour, but in the second half of the 1990s using the unadjusted money growth series would have understated the risk to price stability.

4.2 Lagging and smoothing

In order to model the leading properties of money growth on inflation, the adjusted money growth indicator must be appropriately lagged. In addition, since money growth may be affected by temporary shocks with no implications for future inflation, it is advisable to work with a smoothed money growth indicators.

Regarding the causality relationship between money and inflation, the smoothing of inflation and lagging of money, a very good clarifying reference is Reichlin and Lenza (2007), where it is shown that for the Euro area data, the optimal predictive content of money on inflation is given between 6 and 12 quarters ahead and for a three year average of inflation.

In this context, we are not much concerned on the smoothness of inflation, since we use a model which is capable of assigning observed inflation to one of two possible regimes, but we are indeed concerned about using an appropriately lagged and smoothed measure of money growth.

In synthesis, the monetary growth indicator is subject to the following transformations:

1. we apply the velocity and output growth rate adjustments:

$$\Delta m_t^* = \Delta m_t - \Delta \widetilde{y}_t + \Delta \widetilde{v}_t \tag{17}$$

2. we use a $MA(q)$ on Δm_t^*:

$$\Delta m_t^{**} = \frac{\sum_{i=0}^{q} \Delta m_{t-i}^*}{q+1} \tag{18}$$

3. we lag it p periods and in the EW model we use

$$\mathbf{z}_t = \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} 1 \\ \Delta m_{t-k}^{**} \end{bmatrix} \tag{19}$$

In the applications contained in this paper we decided to use $q = 5$ and $p = 9$. The particular choice of the lag order ($p = 9$) is motivated by the fact that we believe that the monetary signal takes some time to unfold. In order to give (partial) support to this belief, in Figure (2) we provide the cross correlogram between inflation and lagged adjusted money growth for all the countries being considered. This figure shows that the dynamic correlations tend to be moderately increasing until lag 10-11 and then mildly decreasing. We use p^{th} lag on the money indicator variable for all countries where p is suggested by the shape of the cross-correlogram between money growth and inflation. Robustness with respect to other choices has been examined and the experience so far seems to point out that results do not qualitatively depend on the choice of the lag order.

An alternative way of proceeding, in order to get rid of the dependence of results on particular choices of q and k is to run the model for each possible (and sensible) choice of these indexes and then average results by using the posterior probabilities of each models. We have done that and documented the results in Section 6.1.

5 Results

As already stated, we conduct our analysis on quarterly data for Canada, Euro Area, Germany, UK and USA. The series and the sample sizes used are described in Table (1). Allowing for the lags needed to compute velocity and potential output trends, the effective samples start in 1963Q4 and end in 2009Q4[1]. The upper left panels of Figures (3) to (7) describe the sample behaviour of the inflation series for these countries.

5.1 Priors

For all five datasets analysed we used exactly the same prior. The prior specification is synthesised in Table (2) Note that the prior is specified in a way to give parameters conditional conjugate posteriors, i.e. conditional posterior distributions having the same analytical form as the corresponding prior. This is done in the interest of keeping computations as simple as possible and in any case priors can be specified in a very flexible way also in the conditional conjugate framework.

Going more in the details of the prior, we specify a truncated Gaussian distribution for the AR coefficient ϕ, centered on .5 and with support $(-1, 1)$ to rule out ex ante non-stationary behaviour for the inflation series. The prior standard deviation (.2) seems to be appropriate to reflect an ex ante plausible range for the values of ϕ.

The prior on the state specific intercepts is Gaussian with mean equal to zero and standard deviations equal to 1.5. Note that the priors on c_1 and c_2 are not independent since on them we impose the labelling constraint that $c_1 < c_2$. This is to allow interpreting the first state as low inflation regime. This of course will generate a prior mean and standard deviations which are different from

[1] In the case of Germany the sample ends in 1998, reflecting the entry of the country into the monetary union.

the moments of the marginal distributions of c_1 and c_2 which would hold if the labelling constraint was removed.

On the precision of the shocks h_1 and h_2,we impose the constraint that the two precisions be equal across states, i.e. $h_1 = h_2$, and we use a Gamma prior with parameters 1 and 5, which covers with high probability mass a wide range of possible values: the prior 95% confidence bound is $(.82, 12.86)$ or, in terms of the standard error of shocks would be $(.28, 1.10)$.

A little more explanation is required for the prior on the parameters determining transition probabilities. First of all, as mentioned earlier, the slope coefficients are constrained to be equal and not to depend on the starting state, i.e. $\gamma_{21} = \gamma_{22}$ and on γ_{12} we impose a prior centered on zero and with sharp shrinking (prior s.d. equal to .1). This is meant to replicate an ex ante standpoint in which the researcher is very skeptical of the leading properties of money growth but does not rule it out at all.

The intercepts of the transition probabilities probit equation (5) are allowed to differ and they are both given Gaussian priors centered on 1.5 and -1.0, for γ_{11} and γ_{12} respectively, with very tight standard deviations: 0.05 for both. This tight prior is used to reflect a prior belief that when money growth assumes values close to the sample mean ($z_{2t} = 0$), or when the slope coefficient is zero), then the 95% prior confidence set for persistence probabilities of each states are respectively $[.91, .93]$ and in $[.84, .86]$ This is coherent with a median value of the associated ergodic probabilities of .7 and .3 for low and high inflation states respectively. We believe that these priors are centered then on sensible values and we cannot eschew being quite informative on these parameters since in each dataset the number of transition across regimes is very limited and the posterior distribution of these slope parameters is not dominated by data evidence, as would happen in a situation in which the sample contained very many transitions.

5.2 Estimation results

We implement MCMC posterior simulation of the parameters. We use the Gibbs sampling scheme described in Appendix (B) and compute 1,300,000 iterations starting from the prior distribution of the parameters. We discard the first 300,000 iterations and compute results using the remaining 1,000,000 iterations. In order to reduce the serial correlation of the draws, we keep every 50^{th} draw to compute features of the posterior distributions. Given that, like in the probit model, the transition probabilities are invariant to linear transformations of the regressors in the probability equation, we standardize the money growth indicator prior to running the posterior simulation. Let us call the standardized adjusted money growth indicator z_t. This implies that the intercept parameters give us directly the transition probabilities evaluated at the sample mean of the adjusted money growth indicator ($z_{2t} = 0$):

$$p^0_{1i} = [p_{1i,t}]_{z_t=0} = \Phi(\gamma_{1i}), i = 1, 2 \tag{20}$$

$$\Phi(\nu) = \int_{-\infty}^{\nu} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\omega^2}{2}\right) d\omega \tag{21}$$

The results are contained in Tables (3) to (7) for Canada, EA, Germany, UK and US. For each of the parameters being estimated the tables report mean, standard deviation and 2.5% and 97.5% quantiles of the prior and posterior distributions.

In the middle panel of Tables (3) to (7) we report the state specific means of inflation, obtained as

$$\mu_i = \frac{c_i}{1-\phi}, i = 1, 2 \tag{22}$$

which is computed at the posterior mean value of the parameters. These values would be the mean of the AR process in the case the system remained indefinitely in each of the state and can be conceptually thought of as the mean of the distribution around which the series tends to oscillate within each state.

In the bottom panel of Tables (3) to (7) we report the transition probabilities computed at the sample mean for the money growth indicator and the corresponding ergodic probabilities. These are the state ergodic probabilities which would attain should the adjusted money growth indicator remain for good at its sample mean value

$$\mathbf{P}^0 = \begin{bmatrix} \Phi(\gamma_{11}) & 1-\Phi(\gamma_{11}) \\ \Phi(\gamma_{12}) & 1-\Phi(\gamma_{12}) \end{bmatrix} \tag{23}$$

$$p^0_1 = \frac{1-p^0_{22}}{2-p^0_{11}-p^0_{22}}, p^0_2 = 1 - p^0_1 \tag{24}$$

These probabilities are all computed at the posterior sample mean for the parameters. As an example (see Table 4), these computations tell us that in the EA the low inflation state mean is 1.43 % and the high inflation mean is 7.65%. The diagonal elements of the transition probabilities are .82 and .68 which correspond to ergodic probabilities equal to .64 and .36. All this implies that the system would be 2/3 of the time in the low inflation regime.

In addition report also elasticities of TPs computed at the end of the sample as in equation (10). These are computed in a fully Bayesian way, i.e. by taking the appropriate averages over the posterior simulation sample:

$$\frac{\partial p_{12}(z_{2T})}{\partial p z_{2T}} = \frac{1}{M}\sum_{m=1}^{M}\left(\gamma^{(m)}_{21} \times \frac{\phi\left(\gamma^{(m)}_{11} + \gamma^{(m)}_{21} \times z_T\right)}{\Phi\left(\gamma^{(m)}_{11} + \gamma^{(m)}_{21} \times z_T\right)}\right) \tag{25}$$

$$\frac{\partial p_{21}(z_{2T})}{\partial p z_{2T}} = -\frac{1}{M}\sum_{m=1}^{M}\left(\gamma^{(m)}_{21} \times \frac{\phi\left(-\gamma^{(m)}_{12} - \gamma^{(m)}_{22} \times z_T\right)}{\Phi\left(-\gamma^{(m)}_{12} - \gamma^{(m)}_{22} \times z_T\right)}\right) \tag{26}$$

The estimated elasticities of TPs are a way to convey the size and the relevance of the effects of changes in the EW variable on the probabilities of switching regimes. As an example (see bottom panel of Table (4)), the posterior means for the elasticities of $p_{12,T}$ and $p_{21,T}$ are respectively .21 and -.36 for the EA. This tells us that if the adjusted money growth indicator were to increase by 10% this would impart a 2.1% increase in the probability of leaving the low inflation state. Another interesting aspect is the fact that, although the posterior 95% confidence set for both elasticities does include zero, the corresponding 90% confidence sets do not include zero, and this can be interpreted as informally supporting the conjecture that money growth is empirically relevant to predict regime changes.

Some features of the results are common across the countries being analysed. Firstly, the posterior mean of the slope coefficient is always negative and relevant. Taking again as an example the Euro Area results, we see that the slope coefficient has posterior mean equal to -.22 and the posterior 95% confidence set is $[-.58, .07]$. The posterior probability of the slope parameter being less than zero is 92%.

For all the other countries slope coefficients elasticities of transition probabilities are all sizeable and relevant and their posterior 95% confidence sets never contain zero.

Another interesting feature is that the parameters which describe the distribution of inflation conditional on each state (i.e. c_1,c_2, ϕ and h) are very precisely estimated with posterior standard deviations which are much smaller than their prior counterparts. This is a signal that these parameters are well identified in the model. As an example in the US case, the posterior standard deviations of c_1,c_2, ϕ and h are .03, .13,.38, .49, while the corresponding prior standard deviations are .19, 1.23,1.23 and 3.14.

The parameters determining transition probabilities, i.e. $\gamma_{11}, \gamma_{21}$and γ_{12}, are clearly less well identified. It is clear that their posterior standard deviations are in the same order of magnitude as their corresponding prior values: for the US (see Table 7) we have .06, .12 and .08 as posterior standard deviations and .05, .10 and .05 as corresponding prior standard deviations. But it is also clear that the data steer away the posterior means of these parameters from their prior values: taking again the US example the posterior means for those parameters are .88, -.26 and -.44, while the corresponding prior means are 1.5, 0 and -1.00.

We believe that most of the problems in estimating these parameters are due to the low number of observed transitions in the sample and that a multicountry approach with partial pooling of the coefficients might be useful to solve this problem. This issue is discussed further in Section 6.2.

An interesting feature relates to the estimated means of inflation in the respective regimes. In all countries, the posterior mean of mean inflation is below 2%, consistent with current definitions of price stability and/or inflation targets. Estimates for mean inflation in the high inflation regimes are higher for the Anglo-saxon countries compared with the euro area, consistent with historical experience during the Great Inflation era.

Regarding state allocation, the posterior simulation of the model allows us to obtain the posterior smoothed probabilities of each regime at each point in time conditioned on the whole sample, namely

$$p(s_t = i|\underline{\mathbf{y}}_T, \underline{z}_T) \tag{27}$$

These probabilities can be obtained as a by-product of the posterior simulation in a number of different ways, but the most straightforward way is to compute them on the basis of the simulation smoother which is used to sample from the joint posterior distribution of $\underline{\mathbf{s}_T}$ conditional on parameters

$$p(s_t = i|\underline{\mathbf{z}_T}, \underline{\mathbf{y}_T}) = \frac{1}{M}\sum_{m=1}^{M} I(s_t^{(m)} = 1) \tag{28}$$

where $s_t^{(m)}$, $m = 1, 2, ..., M$ are draws from the posterior distribution of the latent variables. The details of the procedure are contained in the appendix.

In principle, it is also possible to obtain the filtered probabilities

$$p(s_t = 1|\underline{\mathbf{y}}_t, \underline{z}_t) \tag{29}$$

but these are harder to obtain since they require sequential estimation of the parameters of the model.

In the bottom left panels of Figures (3) to (7), we report the smoothed probabilities of low inflation at each date in the sample.

It is interesting to note that in most cases the smoothed probabilities are quite polarised, i.e. close to zero or one. For Canada (see Figure 3) we see that there the model signals (and anticipates) clearly the transition to high inflation in the early 1970s. Then, there is a clear transition to low inflation after 1983, a return to the high inflation regime from 1988 to 1992 and the prevalence of the low inflation regime afterwards, with the exception of 2007-2008. We note also that the model generates a predicted regime change from low to high inflation at the end of 2002 as a result of sharp rise of the adjusted money growth indicator.

Looking at the German case (Figure 5), we observe the transition to high inflation in the early 1970s. Subsequently, we see that there is a regime switch from low to high inflation in 1978 which is reversed at the beginning of 1981. These transitions are clearly anticipated by the dynamics of the adjusted money growth indicator (top-right panel of the same figure). In the same way the adjusted money growth indicator leads the transition from low to high inflation that occurs at the beginning of 1988, but it does not seem to have a great role in the subsequent bout of inflation taking place in 1991.

The UK results (Figure 6) is somehow dominated by the huge spikes of inflation in the mid and late 1970s. The model correctly signals an increase in high inflation risk in the late 1960s. The estimated model then assigns nearly all the period from 1972 to 1990 to the high inflation regime.

Much clearer results emerge from looking at the Euro Area in Figure (4). For the pre-EMU period, the state allocations are roughly in line with those emerging from the German dataset with the EMU starting in a regime of low inflation and the risk to price stability remaining very low until 2007. Then we see a sharp drop in the probability of low inflation during 2007. This was correctly signalled by the monetary growth acceleration taking place in 2005 and reversing only in the course 2008. This leads to the recent change of regime from high to low inflation which occurred at the end of 2008.

As for the US case (Figure 7) we see that the run up to inflation in the early 1970s and inflationary spell of the late seventies are clearly picked up by the adjusted money growth indicator. Also the acceleration of inflation taking place from 1987 until 1991 is led by money growth acceleration. Finally, the sharp acceleration of inflation in 2007 and its reversal in 2008 give rise to two rapid regime changes (from low to high and then from high to low) which are clearly led by money growth movements.

6 Checking the model

6.1 Robustness

As already pointed out in Section (5), the results of the analysis are conditioned on the particular choice of the lag order $p = 9$ and the MA order $q = 5$. In order to shed some light on the robustness of the results with respect to alternative choices of k and q we adopt the principle of Bayesian Model Averaging and we compute the results for a grid of values for p from 7 to 11 and a grid of value for q from 3 to 7. We call each of the model being estimated $M_{p,q}$ and we have therefore a model space which consists of 25 different models. Then for all models we compute the model marginal likelihoods:

$$p(\underline{\mathbf{y}}_T|\underline{\mathbf{z}}_T, M_{p,q}) \tag{30}$$

which are then used as weights to attach to each of the model being considered:

$$p(M_{p_0,q_0}|\underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T) = \frac{p(\underline{\mathbf{y}}_T|\underline{\mathbf{z}}_T, M_{p_0,q_0}) \times p(M_{p_0,q_0})}{\sum_{p=7}^{11}\sum_{q=3}^{7} p(\underline{\mathbf{y}}_T|\underline{\mathbf{z}}_T, M_{p,q}) \times p(M_{p,q})} \tag{31}$$

We then use these weights to weight the output of the MCMC outputs from each model and we can compute marginal the posterior distribution of parameters and other features of interest of the model in a very intuitive way as:

$$p(\theta|\underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T) = \sum_{p=7}^{11}\sum_{q=3}^{7} p(\theta|\underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T, M_{p,q}) p(M_{p_0,q_0}|\underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T) \tag{32}$$

Then we have results which are not obtained by conditioning on the choices of p and q but they marginalise with respect to a range of plausible values for these choices. The results largely confirm those reported in Section (5).

We report the results of this exercise for the euro area. Table (8) reports posterior probabilities of all models. We can see that the results reported in Section 5 are not generated by the model with the highest posterior probability: the choice of $p = 9$ and $q = 5$ yields a posterior model probability of .05, while choosing $p = 10$ and $q = 4$ or $q = 5$ generates models with posterior probabilities equal to .06. We would like to point out here that we chose to report the results of the model with $p = 9$ because we thought that the choice of this lag was appropriate in our initial analysis and we were not trying to maximise model fit but merely looking for predictive signal of money growth on inflation regime changes.

In any case, looking at the marginal posterior distribution of the parameters contained in Table (9), the model averaging results confirm that the results reported with $p = 9$ and $q = 5$ reported in Table (4). In particular, the coefficient γ_{12} is negative and relevant, with posterior mean equal -.19 and posterior standard deviation equal to .17, while in Table 4) we have posterior mean of -.22 and posterior standard deviation of .17 Marginalising with respect to different lag orders and MA order adjustment for money growth produces a somehow less strong predictive power of money with respect to inflation (as expected) but the signal is still relevant.

6.2 A panel probit approach

An important problem of our univariate approach is that each of the samples being considered includes very few transitions across different regimes and this renders problematic the estimation of the parameters determining transition probabilities. A sensible solution would be to extend the EW-MS model to a panel framework and, although we have not already a multivariate version of the model, we are currently investigating some alternative specifications.

A preliminary step in this direction is to use a two step dynamic panel probit model. We consider jointly Canada, Germany, UK and the US and the sample period is the intersection of the respective sizes, i.e. 1968Q1 to 1998Q4. Then we apply the following two steps:

1. for each of the countries we use a MS model with fixed transition probabilities and we use these estimated models to classify the observations on equation as belonging to high or low inflation. For each country we create a dichotomous variable d_{it} equal to 1 if in period t and country i inflation was in the low regime.

2. Then we take these dichotomous variables for each country and we estimate the following panel

probit model

$$p(d_{it} = 1|\underline{\mathbf{d}}_{t-1}, \underline{\mathbf{z}}_{t-1}) = \Phi(c_i + \gamma_0 + \gamma_1 d_{it-1} + \gamma_2 z_{2t-1})$$
$$c_i = \alpha_i + \beta_1 d_{it-1} + \beta_2 z_{2t-1}, \alpha_i \backsim N(0, h_\alpha^{-1})$$

The coefficients c_i are unit specific intercept terms which are potentially correlated with the other covariates. Note that we insert the lagged dependent variable d_{it-1} to allow for dependence of the current state on the previous one, in order to mimic the EW-MS specification adopted country by country.

The model is estimated using a Bayesian approach and the details can be found in Amisano and Giorgetti (2010). Note that in this way we pool over slope coefficients and we allow for cross-country differentials in the form of the intercept shifts. The prior reflects little prior knowledge and it is described in Table (10).

The results of this approach are collected in Table (11). First of all we see that country specific shifts do not seem to be important, given that the zero value is within their 95% posterior confidence bounds. Another interesting feature is that the lagged state is strongly relevant and positive, with [2.03, 2.98] as 95% posterior interval, and this is a clear sign of the positive correlation in the discrete inflation regime process. The coefficient on lagged adjusted money growth has $[-.51, -.06]$ as 95% interval with posterior mean equal to -.28.

In order to gauge the importance of adjusted money growth, we also computed average partial effects (APE), namely the derivatives of estimated probabilities with respect to each single regressor: Given that the money indicator is already expressed in logs, this APE can be readily transformed into an elasticity just providing a starting point. Let us suppose that we are in a situation in which the model predicts a probability of low inflation of 85% and in the past period the regime was high. This is a reasonable starting point. Then the posterior mean of the APE for the adjust money indicator (-.03) turns into an elasticity of -0.035. This is a small and significant value, even if the elasticity of estimated probability with respect to the lagged state is much higher in absolute value (-.91).

In synthesis, the results of this panel analysis show us that there our results are robust to panel extension. In a sense, it would be interesting to see whether a truly multivariate extension of the EW-MS could be used to eschew the small sample problems encountered in the single country analysis. This will be pursued in future research.

6.3 Forecasting performance evaluation

It is important to emphasise that the EW model we propose in this paper should not be interpreted as a forecasting model: the aim of the model is to provide signals of risk to price stability based on the evolution of monetary aggregates. Therefore, we do not want to engage in forecasting competitions on the level of inflation with competing models. We are nevertheless interested in assessing the role of the monetary growth indicator in predicting regime changes and for this reason we compare our EW-MS model to the simple, fixed transition probabilities, Markov Switching model.

In order to provide some measure of how the EW-MS model performs in forecasting, we compute the predictive density of each observation conditioned only on the past value of the observable variables:

$$\log p(y_t|\underline{\mathbf{y}}_{t-1}, \underline{\mathbf{z}}_{t-1}, \mathcal{M}_h) = \log \frac{1}{M}\sum_{m=1}^{M} p(y_t|\underline{\mathbf{y}}_{t-1}, \underline{\mathbf{z}}_{t-1}, \theta^{(m)}, \underline{\mathbf{s}}_{t-1}^{(m)}, \mathcal{M}_h) \tag{33}$$

In the expression above $\mathcal{M}_h$ denotes the model being estimated and $\theta^{(m)}, \underline{\mathbf{s}}_{t-1}^{(m)}$ are drawn from the joint posterior distribution of parameters and discrete states conditioned on observable data up to time $t-1$. We compute these quantities bot for $\mathcal{M}_{EW-MS}$, our EW-MS model, and $\mathcal{M}_{MS}$ the simple MS model which attains as a particular case of the EW-MS by restricting the slope coefficients in the probit specification (5) to be equal to zero.

Note that if we sum expression (33) over T from t=1 up to T, we get the complete predictive distribution over all the available observations

$$\log p(\underline{\mathbf{y}}_T|\underline{\mathbf{z}}_T, \mathcal{M}_h) = \sum_{t=1}^{T} \log p(y_t|\underline{\mathbf{y}}_{t-1}, \underline{\mathbf{z}}_t, \mathcal{M}_h) \tag{34}$$

while if we sum over t from τ to T gives us a measure of the predictive performance from τ to T

$$\log p(\underline{\mathbf{y}}^{\tau}|\underline{\mathbf{y}}_{\tau-1}, \underline{\mathbf{z}}_T, \mathcal{M}_h) = \sum_{t=\tau}^{T} \log p(y_t|\underline{\mathbf{y}}_{t-1}, \underline{\mathbf{z}}_t, \mathcal{M}_h) \tag{35}$$

where $\underline{\mathbf{y}}^{\tau} = \{y_t : t = \tau, \tau+1, ..., T\}$. This is called the predictive density conditional on sample information up to time $\tau - 1$. This quantity is particularly useful to monitor forecasting performance over the last $T - \tau + 1$ observations.

We have computed these quantities both for model EW-MS and model MS and in Figure (8) we report

$$log\left[\frac{p(\underline{\mathbf{y}}^{\tau}|\underline{\mathbf{y}}_{\tau-1}, \underline{\mathbf{z}}_T, \mathcal{M}_{EW-MS})}{p(\underline{\mathbf{y}}^{\tau}|\underline{\mathbf{y}}_{\tau-1}, \underline{\mathbf{z}}_T, \mathcal{M}_{MS})}\right] \tag{36}$$

i.e. the log predictive odds ratio between model EW-MS and model MS conditional on information up to time $\tau - 1$. When $\tau = 1$, i.e. in the case of no conditioning on any subsample of observations, in this way we compute the full posterior odds ratio.

We have computed (36) for the Euro Area data (top panel of Figure 8) and for the US data (bottom panel of Figure 8)

The two panels of the figure can be easily interpreted: when the log predictive odds ratio is positive, this measure favours the EW-MS model over the simple MS model. On the X axis we read the time $\tau - 1$ up to which conditioning is done. Conditioning on the first observations is done for two reasons:

1. this is the easiest way to conduct model comparison without being too much influenced by the prior specification: the first $\tau - 1$ observations are used to train the prior and to put the two models being compared on equal grounds.

2. In this way it is possible to monitor how different subsamples contribute to the relative forecasting performance.

For the EA, the whole sample (the posterior odds ratio) favours the EW-MS model but it is very evident that excluding the observations from 1982 the forecast performance of the EW model deteriorates markedly with respect to the simple MS model. For the US, model the $\mathcal{M}_{EW-MS}$ is nearly equivalent to the $\mathcal{M}_{MS}$ and becomes clearly inferior excluding observations until 1986. The weaker performance of the EW-MS model from the mid-1980s is not surprising given that this period was generally characterised as a low inflation regime. This is in line with the literature documenting a decline in the relative predictability of inflation mentioned in the introduction.

7 Conclusion

Building on the existing literature which establishes a long-run link between inflation and monetary growth, this paper has developed a money-based early warning indicator for shifts in inflation regimes. The model is based on money growth with a correction for velocity and output trends which can be computed in real time. We modelled inflation as a process characterised by two regimes - low and high inflation - in which the probability of shifting from one regime to the other depends on a measure of lagged money growth which can be computed in real time. We applied the model to data from Canada, the euro area, Germany, the US and the UK using quarterly data from the early 1960s to the present. We estimated its parameters using Bayesian techniques. The results obtained support the view that money growth provides timely warning signals of transitions between inflation regimes. A number of robustness checks confirm this overall conclusion.

At the same time, our results show that the signals coming from money growth are noisy and, in particular, given the limited number of transitions observed in the sample, it is difficult to obtain precise estimates of some of the parameters. Further research along these lines could therefore benefit from examining measures of money growth which could reduce the noise contained in the data, e.g. by adjusting for portfolio shifts. In addition, a fully fledged Bayesian model averaging of models with different degrees of smoothing and lags would be useful. These caveats notwithstanding, we believe that the results are sufficiently robust to support the claim that money growth is a leading indicator of shifts in inflation regime. Thus it would be unwise of central banks to neglect the information contained in money because of recent evidence on the limited forecasting performance of money for inflation in the recent low-inflation period.

A Bayesian analysis of the EW-MS model

In this appendix we provide some details of the posterior simulation of the EW-MS model. We use a Gibbs sampling - data augmentation algorithm to obtain MCMC draws from the joint posterior distribution of latent variables and parameters, conditional on the data, namely

$$p(\underline{\mathbf{s}}_T, \theta | \underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T) \tag{37}$$

This can be done in two main steps:

1. simulate from the posterior distribution of discrete states conditional on parameters $p(\underline{\mathbf{s}}_T|\theta,\underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T$
2. simulate from the posterior distribution of parameters conditional on discrete states $p(\theta|\underline{\mathbf{s}}_T, \underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T$

This is a Gibbs sampling-data augmentation MCMC scheme (see Kim and Nelson (1999)) which converges to the target joint posterior distribution (37). In the next two subsection we give details on how to implement these two steps.

A.1 Simulation of the discrete state variables

Following Kim and Nelson (1999), the algorithm consists of two steps:

- runing the filter to obtain the relevant filtered and predicted probabilities:

$$\pi_{i,t|t} = p(s_t = i | \underline{\mathbf{y}}_t, \underline{\mathbf{z}}_t, \theta), \tag{38}$$

$$\pi_{j,t+1|t} = p(s_{t+1} = j | \underline{\mathbf{y}}_t, \underline{\mathbf{z}}_t, \theta); \tag{39}$$

- running a posterior simulation smoother to obtain a draw from the joint posterior distribution of state variables conditional on data and parameters $p(\underline{\mathbf{s}}_T|\theta,\underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T)$.

A.1.1 Filtering

The filtering is standard and need only be extended to account for time variability of transition probabilities which depend on EW variable(s) in $\mathbf{z}_t$:

$$\text{(Prediction)} : \pi_{j,t+1|t} = \sum_{i=1}^{m} p_{ij,t+1} \pi_{i,t|t} \tag{40}$$

$$\text{(Update)} : \pi_{j,t+1|t+1} = \frac{\pi_{j,t+1|t} \times p(\mathbf{y}_{t+1} | s_{t+1} = j, \underline{\mathbf{y}}_t, \theta)}{\sum_{h=1}^{m} \pi_{h,t+1|t} \times p(\mathbf{y}_{t+1} | s_{t+1} = h, \underline{\mathbf{y}}_t, \theta)} \tag{41}$$

Initialisation of the filter A special problem arising in this EW model is how to initialise the filter. To this end, we consider a situation in which the indicator variable assumes its unconditional mean:

$$\mathbf{z}_t^{(0)} = \begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} \tag{42}$$

i.e. we set the indicator variables to their mean values. We then compute:

$$\mathbf{P}^{(0)} = \mathbf{P}(\mathbf{z}_t^{(0)} = \mathbf{z}_t^{(0)}) = \begin{bmatrix} \Phi(\gamma_{11}) & 1-\Phi(\gamma_{11}) \\ \Phi(\gamma_{12}) & 1-\Phi(\gamma_{12}) \end{bmatrix} \tag{43}$$

$$= \begin{bmatrix} p_{11}^{(0)} & 1-p_{11}^{(0)} \\ 1-p_{22}^{(0)} & p_{22}^{(0)} \end{bmatrix} \tag{44}$$

and we use the ergodic probabilities corresponding to this transition probability matrix

$$\pi^{(0)} = \begin{bmatrix} \frac{1-p_{22}^{(0)}}{2-p_{11}^{(0)}-p_{22}^{(0)}} \\ \frac{1-p_{11}^{(0)}}{2-p_{11}^{(0)}-p_{22}^{(0)}} \end{bmatrix} \Leftrightarrow \pi^{(0)\prime}\mathbf{P}^{(0)} = \pi^{(0)\prime} \tag{45}$$

A.1.2 State simulation

In order to draw from the distribution $p(\underline{\mathbf{s}}_T|\theta,\underline{\mathbf{y}}_T,\underline{\mathbf{z}}_T)$, we can factorise it as:

$$p(\underline{\mathbf{s}}_T|\theta,\underline{\mathbf{y}}_T,\underline{\mathbf{z}}_T) = p(s_T|\theta,\underline{\mathbf{y}}_T,\underline{\mathbf{z}}_T) \times \prod_{t=1}^{T-1} p(s_t|\underline{\mathbf{s}}^{t+1}\underline{\mathbf{y}}_T,\underline{\mathbf{z}}_T,\theta) \tag{46}$$

$$\underline{\mathbf{s}}^{t+1} = \{s_\tau, \tau = t+1, t+2, ..., T\}.$$

Also in the EW-MS model, like in the fixed transition probabilities case, following Chib (1996) we can exploit the conditional Markov property and write

$$\begin{aligned} p(s_t = i|\underline{\mathbf{s}}^{t+1}\underline{\mathbf{y}}_T,\underline{\mathbf{z}}_T,\theta) &= p(s_t = i|s_{t+1} = j,\underline{\mathbf{y}}_t,\underline{\mathbf{z}}_t,\theta) \\ &= \frac{p(s_t = i|\underline{\mathbf{y}}_t,\underline{\mathbf{z}}_t,\theta) \times p(s_{t+1} = j|s_t = i, z_t,\theta)}{\sum_{h=1}^{m} p(s_t = h|\underline{\mathbf{y}}_t,\underline{\mathbf{z}}_t,\theta) \times p(s_{t+1} = j|s_t = h, z_t,\theta)} = \\ &= \frac{\pi_{it|t} \times p_{ij,t+1}}{\sum_{h=1}^{m} \pi_{ht|t} \times p_{hj,t+1}} = \frac{\pi_{it|t} \times p_{ij,t+1}}{\pi_{jt+1|t}}. \end{aligned} \tag{47}$$

This means that, like in the fixed transition probabilities case, we can draw $\underline{\mathbf{s}}_T$ by drawing s_T from its filtered distribution and iterate backwards using the probabilities (47) which depend only on filtered and projected probabilities computed using the filter.

A.2 Simulation of the parameters

Let us collect the parameters of the model in the following subsets:

1. the parameters describing the conditional mean of the dependent variable:

$$\delta = \begin{bmatrix} c_1 \\ c_2 \\ \phi_1 \end{bmatrix}$$

2. the parameters describing the conditional variance of the dependent variable, i.e. the precision h or the vector $\mathbf{h}$ with the state specific precisions;

3. the parameters determining the TPs, i.e. the element of the $(k \times 2\ )$ matrix $\mathbf{\Gamma}$.

We then use a 3 block Gibbs sampling algorithm in order to draw from each of these three blocks of parameters. It is evident that, conditional on state variables, the parameters in δ and $\mathbf{h}$ are independent of Γ and can be sampled by using the standard Normal-Gamma conjugate results, provided that δ is endowed with a Gaussian prior and $\mathbf{h}$ with a Gamma prior.

As for Γ, conditional on the discrete state variables, its posterior distribution is independent of δ and $\mathbf{h}$ and it coincides with the posterior distribution of the parameters in a probit model which has been described by Albert and Chib (2007). We follow their approach and simulate the latent vector of sign variables:

$$\begin{aligned} &\underline{\mathbf{s}}_T^* = \{s_t^*, t = 1, 2, ..., T\} \\ &s_t^* > 0 \Longleftrightarrow s_t = 1. \end{aligned} \tag{48}$$

The latent variables $\underline{\mathbf{s}}_T^*$ can be easily simulated, given that the structure of the model implies

$$\begin{aligned} p(s_t^* | \underline{\mathbf{s}}_{\tau \neq t}^*, \underline{\mathbf{s}}_T, \theta, \underline{\mathbf{y}}_T, \underline{\mathbf{z}}_T) &= p(v_t | s_t, s_{t-1} = i, \mathbf{\Gamma}, \mathbf{z}_t) \\ &= N(0,1) \left[(2 - s_t) I_{v_t}(v_t > -\gamma_i' \mathbf{z}_t) + (s_t - 1) I_{v_t}(v_t \leq -\gamma_i' \mathbf{z}_t) \right] \\ \underline{\mathbf{s}}_{\tau \neq t}^* &= \{s_\tau^*, \tau = 1, 2, ...t-1, t+1, ...T\}, v_t = s_t^* - \gamma_i' \mathbf{z}_t. \end{aligned} \tag{49}$$

Then we can conveniently draw the elements of $\underline{\mathbf{s}}_T^*$ from appropriately truncated standard Gaussian distributions. Conditional on $\underline{\mathbf{s}}_T^*$, we have a linear model with IID standardised Gaussian disturbances. Hence, the posterior distribution of the free elements of Γ is Gaussian, provided that they have been endowed with a Gaussian prior distribution.

A.3 Label switching

It is well known (see for instance Geweke and Amisano (2009)) that Markov Switching models are subject to a particular non-identification problem: the likelihood is invariant to permutations in the labelling of the discrete states. This is usually referred to as the label switching problem. This is not a problem for the Bayesian estimation of the model but it complicates the issue of interpreting the results when there is interest in assigning the discrete states a "structural" interpretation. In this study we interpret the two regimes as "high" and "low" inflation and then we need to impose constraints to achieve identification and, hence the possibility to interpret the latent states.

In order to achieve identification, in the case of a two regime model it is sufficient to impose one constraint. There are possible options in this regard:

1. we can order the elements on a single row of $\mathbf{\Gamma}$, i.e. order the sensitivities with respect to an indicator variable. If we choose to order the first row of $\mathbf{\Gamma}$ (containing the intercepts), then we are imposing a constraint on the persistences of the states associated to the steady state value of $\mathbf{z}_{t-1}$, i.e. $\mathbf{z}_{t-1}^{(SS)} = \left[1, \mathbf{0}'\right]'$ and this in turn imposes a constraint that one of the states has a higher ergodic probability than the other.

2. Another possibility is to impose inequality constraints on the standard errors (or the precision) associated to different regimes.

3. We can impose that the coefficient $\beta_j^{(1)}$, associated to regressor x_{jt} under regime 1 is higher (or lower) than $\beta_j^{(2)}$, the coefficient associated to the same regressor under regime 2. This is what we do in this paper, associating to regime 1 (low inflation) an intercept that is lower than the intercept in regime 2 (high inflation).

This is what we do in this paper: we impose the constraint:

$$c_1 < c_2 \tag{50}$$

and therefore we call $s_t = 1$ the low inflation state.

A.3.1 Label switching and ex-post relabelling

One possibility to tackle the label switching problem is to work with a prior which is completely symmetric across states, i.e. a prior which is invariant with respect to state labelling permutations. Then, the desired identification constraint is imposed ex post on the MCMC output to achieve interpretability of the states.

In the case of the EW-MS model, we should keep in mind that for any cumulative density function which is symmetric around zero we have:

$$1 - F(z) = F(-z) \tag{51}$$

Therefore, we can write the TP matrix in a EW-MS model as:

$$\begin{aligned}\mathbf{P}(\mathbf{z}_t, \mathbf{\Gamma}) &= \begin{bmatrix} \Phi(\gamma_1'\mathbf{z}_t) & 1-\Phi(\gamma_1'\mathbf{z}_t) \\ \Phi(\gamma_2'\mathbf{z}_t) & 1-\Phi(\gamma_2'\mathbf{z}_t) \end{bmatrix} \\ &= \begin{bmatrix} \Phi(\gamma_1'\mathbf{z}_t) & \Phi(-\gamma_1'\mathbf{z}_t) \\ \Phi(\gamma_2'\mathbf{z}_t) & \Phi(-\gamma_2'\mathbf{z}_t) \end{bmatrix} \\ \underset{(h\times 2)}{\mathbf{\Gamma}} &= [\gamma_1\gamma_2]\end{aligned}$$

Hence when label switching takes place we have:

$$\mathbf{P}^*(\mathbf{z}_t, \mathbf{\Gamma}) = \begin{bmatrix} \Phi(-\gamma_2'\mathbf{z}_t) & \Phi(\gamma_2'\mathbf{z}_t) \\ \Phi(-\gamma_1'\mathbf{z}_t) & \Phi(\gamma_1'\mathbf{z}_t) \end{bmatrix}$$

Invariance across label switching is safeguarded if the prior is such that:

$$p(\gamma_1) = p(-\gamma_2) \tag{52}$$

In the case in which $k = 2$ case (an intercept and a single z_{2t} EW variable), we have that:

$$\mathbf{P}^*(\mathbf{z}_t, \mathbf{\Gamma}) = \begin{bmatrix} \Phi(\gamma_{11} + \gamma_{21}z_t) & \Phi(-\gamma_{11} - \gamma_{21}z_t) \\ \Phi(\gamma_{12} + \gamma_{22}z_t) & \Phi(-\gamma_{12} - \gamma_{22}z_t) \end{bmatrix} \tag{53}$$

$$p(\gamma_{11}) = p(-\gamma_{12}) \tag{54}$$

$$p(\gamma_{21}) = p(\gamma_{22}) \tag{55}$$

B Data appendix

B.1 Canada

Data for Canada come from IMF-IFS statistics for the period 1968Q1 to 2009Q4. Data for real and nominal GDP for the period 1951 to 1967 come from Global Data Finder. For the 1951 to 1956 period we have only yearly data and we therefore computed quarterly data by using the BFL interpolation procedure described in Quilis (2009).

Money data come from earlier period come from Metcalf, Redish and Shearer (1998) and were kindly provided to us by Luca Benati.

B.2 Germany

Data for Germany come from the Bundesbank real time data base for the period 1962Q1 to 1998Q4. Earlier data were obtained from Bundesbank (1988), and quarterly data for the same period were obtained by using the BFL interpolation procedure.

B.3 Euro area

Data for the 1992q1 to 2009q4 period come from the ECB. We used German data for the 1950 to 1991Q4 period.

B.4 UK

For the period 1957q1 to 2009q4 data come from IMF-IFS. Earlier data, available only on a yearly basis come from Global Data Finder (GDP) and Mitchell (1998a) (money) and they were quarterly interpolated using the BFL interpolation procedure.

B.5 USA

All data come from FRED at the St. Louis Fed. Quarterly data on money are available since 1959Q4; therefore we used yearly data on M2 from Mitchell (1998b), subject to quarterly BFL interpolation.

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Table 1: Data used in the application

country	source	initial date	terminal date	money	inflation	lag order	MA order	eff. sample start
Canada	IFS, MRS (1998)	1950Q1	2009Q4	M3	CPI	9	5	1963Q4
Germany	Bundesbank	1950Q1	1998Q4	M3	CPI	9	5	1963Q4
Euro Area	ECB, Bundesbank	1950Q1	2009Q4	M3	HICP	9	5	1963Q4
UK	IFS, Mitchell (1998a)	1950Q1	2009Q4	M4	CP1	9	5	1963Q4
USA	FRED, Mitchell (1998b)	1950Q1	2009Q4	M2	CPI	9	5	1963Q4

MRS(1998) denotes Metcalf et al. (1998). Datasets and codes available at `http://www.eco.unibs.it/~amisano/Amisano_Fagan.zip`

Table 2: Prior specification

parameter	support	prior type	hp1	hp2	mean	std. dev.
ϕ	$[-1, +1]$	Truncated Gaussian	.5	.2	.5	.2
c_1	$(-\infty, +\infty)$	Gaussian	.0	1.5	.0	1.5
c_2	$(-\infty, +\infty)$	Gaussian	.0	1.5	.0	1.5
h_1	$[0, +\infty)$	Gamma	1	5	5.0	3.1
h_2	$[0, +\infty)$	Gamma	1	5	5.0	3.1
γ_{11}	$(-\infty, +\infty)$	Gaussian	1.5	.05	1.5	.05
γ_{21}	$(-\infty, +\infty)$	Gaussian	.0	.1	.0	.1
γ_{12}	$(-\infty, +\infty)$	Gaussian	-1.0	.05	-1.0	.05
γ_{22}	$(-\infty, +\infty)$	Gaussian	.0	.1	.0	.1

Note: hp1 and hp2 are the hyperparameters of the prior (mean and std dev if Gaussian, otherwise as appropriate).
Note: for identification we impose that c_1 be smaller than c_2.
Note: h_1 and h_2 are constrained to be equal.
Note: γ_{21} and γ_{22} are constrained to be equal.

Table 3: Posterior simulation, Canada

	prior mean	prior sd	prior low q	prior up q	post mean	post sd	post low q	post up q
ϕ	0.50	0.19	0.13	0.87	0.94	0.03	0.89	0.99
c_1	-0.86	1.23	-3.32	1.45	0.06	0.09	-0.12	0.23
c_2	0.83	1.23	-1.50	3.33	0.45	0.28	0.03	1.04
h_1	4.99	3.15	0.85	12.74	2.46	0.37	1.87	3.31
γ_{11}	1.50	0.05	1.40	1.60	0.67	0.07	0.54	0.80
γ_{21}	0.00	0.15	-0.29	0.29	-0.95	0.17	-1.27	-0.60
γ_{12}	-1.00	0.05	-1.10	-0.90	-0.49	0.06	-0.62	-0.37
γ_{22}	0.00	0.15	-0.29	0.29	-0.95	0.17	-1.27	-0.60
elasticity p_{12}	0.00	0.29	-0.58	0.56	1.12	0.18	0.76	1.46
elasticity p_{21}	0.00	0.23	-0.46	0.44	-1.15	0.21	-1.55	-0.72
inflation mean, low	1.04							
inflation mean, high	8.11							
transition probabilities at indicator mean					ergodic probabilities			
	low	high						
low	0.75	0.25				low	0.55	
high	0.31	0.69				high	0.45	

Results based on 130,000 MCMC replications. First 30,000 are rejected. Transition probabilities and corresponding ergodic probabilities are computed at sample mean value of money indicator.

Table 4: Posterior simulation, Euro Area

	prior mean	prior sd	prior low q	prior up q	post mean	post sd	post low q	post up q
ϕ	0.50	0.19	0.13	0.87	0.90	0.04	0.82	0.97
c_1	-0.84	1.24	-3.39	1.52	0.15	0.10	-0.05	0.34
c_2	0.87	1.23	-1.48	3.35	0.79	0.37	0.13	1.42
h_1	5.00	3.15	0.83	12.80	4.33	0.82	2.92	5.95
γ_{11}	1.50	0.05	1.40	1.60	0.93	0.10	0.72	1.08
γ_{21}	0.00	0.10	-0.20	0.19	-0.22	0.17	-0.58	0.07
γ_{12}	-1.00	0.05	-1.10	-0.90	-0.47	0.09	-0.65	-0.32
γ_{22}	0.00	0.10	-0.20	0.19	-0.22	0.17	-0.58	0.07
elasticity p_{12}	-0.01	0.19	-0.44	0.32	0.21	0.13	-0.12	0.37
elasticity p_{21}	-0.02	0.15	-0.36	0.24	-0.36	0.33	-1.20	0.07
inflation mean, low	1.43							
inflation mean, high	7.65							
transition probabilities at indicator mean						ergodic probabilities		
	low	high						
low	0.82	0.18				low	0.64	
high	0.32	0.68				high	0.36	

Results based on 130,000 MCMC replications. First 30,000 are rejected. Transition probabilities and corresponding ergodic probabilities are computed at sample mean value of money indicator.

Table 5: Posterior simulation, Germany

	prior mean	prior sd	prior low q	prior up q	post mean	post sd	post low q	post up q
ϕ	0.50	0.19	0.12	0.88	0.92	0.04	0.82	0.98
c_1	-0.84	1.24	-3.33	1.49	0.12	0.11	-0.09	0.35
c_2	0.86	1.24	-1.50	3.36	0.43	0.28	0.07	1.27
h_1	5.05	3.19	0.85	12.87	2.73	0.48	2.04	4.02
γ_{11}	1.50	0.05	1.40	1.60	0.80	0.08	0.67	0.98
γ_{21}	0.00	0.15	-0.29	0.29	-0.90	0.20	-1.24	-0.42
γ_{12}	-1.00	0.05	-1.10	-0.90	-0.58	0.06	-0.70	-0.46
γ_{22}	0.00	0.15	-0.29	0.29	-0.90	0.20	-1.24	-0.42
elasticity p_{12}	-0.01	0.29	-0.59	0.54	1.04	0.19	0.58	1.35
elasticity p_{21}	0.00	0.23	-0.46	0.43	-1.26	0.31	-1.80	-0.54
inflation mean, low	1.51							
inflation mean, high	5.22							
transition probabilities at indicator mean					ergodic probabilities			
	low	high						
low	0.79	0.21				low	0.57	
high	0.28	0.72				high	0.43	

Results based on 130,000 MCMC replications. First 30,000 are rejected. Transition probabilities and corresponding ergodic probabilities are computed at sample mean value of money indicator.

Table 6: Posterior simulation, UK

	prior mean	prior sd	prior low q	prior up q	post mean	post sd	post low q	post up q
ϕ	0.50	0.19	0.12	0.87	0.94	0.02	0.89	0.98
c_1	-0.84	1.23	-3.35	1.49	0.04	0.16	-0.27	0.35
c_2	0.85	1.25	-1.50	3.41	0.91	0.52	0.14	2.09
h_1	4.93	3.12	0.84	12.70	0.87	0.17	0.63	1.26
γ_{11}	1.50	0.05	1.40	1.60	0.70	0.09	0.54	0.88
γ_{21}	0.00	0.15	-0.30	0.29	-0.87	0.23	-1.28	-0.39
γ_{12}	-1.00	0.05	-1.10	-0.90	-0.48	0.06	-0.61	-0.36
γ_{22}	0.00	0.15	-0.30	0.29	-0.87	0.23	-1.28	-0.39
elasticity p_{12}	0.00	0.29	-0.57	0.57	1.05	0.24	0.52	1.48
elasticity p_{21}	0.00	0.23	-0.46	0.44	-1.04	0.29	-1.55	-0.43
inflation mean, low	0.63							
inflation mean, high	14.52							
transition probabilities at indicator. mean						ergodic probabilities		
	low	high						
low	0.81	0.19				low	0.57	
high	0.33	0.67				high	0.43	

Results based on 130,000 MCMC replications. First 30,000 are rejected. Transition probabilities and corresponding ergodic probabilities are computed at sample mean value of money indicator.

Table 7: Posterior simulation, US

	prior mean	prior sd	prior low q	prior up q	post mean	post sd	post low q	post up q
ϕ	0.50	0.19	0.13	0.87	0.92	0.03	0.87	0.97
c_1	-0.86	1.23	-3.33	1.47	0.10	0.13	-0.16	0.33
c_2	0.83	1.23	-1.51	3.32	1.16	0.38	0.46	1.85
h_1	5.00	3.14	0.82	12.79	2.98	0.49	2.06	3.96
γ_{11}	1.50	0.05	1.40	1.60	0.88	0.06	0.75	1.00
γ_{21}	0.00	0.10	-0.19	0.19	-0.26	0.12	-0.53	-0.04
γ_{12}	-1.00	0.05	-1.10	-0.90	-0.44	0.08	-0.60	-0.30
γ_{22}	0.00	0.10	-0.19	0.19	-0.26	0.12	-0.53	-0.04
elasticity p_{11}	0.01	0.19	-0.36	0.40	0.42	0.21	0.06	0.88
elasticity p_2	0.00	0.15	-0.28	0.32	-0.25	0.11	-0.49	-0.05
inflation mean, low	1.25							
inflation mean, high	14.25							
transition probabilities at indicator. mean					ergodic probabilities			
	low	high						
low	0.81	0.19				low	0.64	
high	0.33	0.67				high	0.36	

Results based on 130,000 MCMC replications. First 30,000 are rejected. Transition probabilities and corresponding ergodic probabilities are computed at sample mean value of money indicator.

Table 8: EA: posterior probabilities of models for different lags and MA order of money indicator

$q \rightarrow$ / $p \downarrow$	3	4	5	6	7
7	0.01	0.01	0.02	0.03	0.04
8	0.02	0.02	0.03	0.05	0.04
9	0.02	0.04	0.05	0.05	0.04
10	0.04	0.06	0.05	0.06	0.05
11	0.05	0.06	0.05	0.06	0.05

p is the lag order of the indicator variable and q is the MA order of its transformation in the EW model.

Table 9: EA: results of model averaging

	prior mean	prior sd	prior low q	prior up q	post mean	post sd	post low q	post up q
ϕ	0.50	0.19	0.13	0.87	0.89	0.04	0.83	0.96
c_1	-0.85	1.23	-3.37	1.50	0.15	0.10	-0.02	0.32
c_2	0.83	1.23	-1.50	3.36	0.84	0.36	0.21	1.37
h_1	4.94	3.12	0.82	12.75	4.43	0.80	3.15	5.76
γ_{11}	1.50	0.05	1.40	1.60	0.94	0.10	0.76	1.08
γ_{21}	0.00	0.10	-0.19	0.19	-0.19	0.17	-0.50	0.06
γ_{12}	-1.00	0.05	-1.10	-0.91	-0.46	0.08	-0.61	-0.34
γ_{22}	0.00	0.10	-0.19	0.19	-0.19	0.17	-0.50	0.06
elasticity p_{12}	0.01	0.19	-0.35	0.41	0.19	0.15	-0.10	0.38
elasticity p_{21}	0.01	0.15	-0.27	0.33	-0.31	0.31	-0.95	0.06
inflation mean, low	1.44							
inflation mean, high	7.91							
transition probabilities at indicator. mean					ergodic probabilities			
	low	high						
low	0.83	0.17				low	0.65	
high	0.32	0.68				high	0.35	

Table 10: Prior specification for the panel probit model

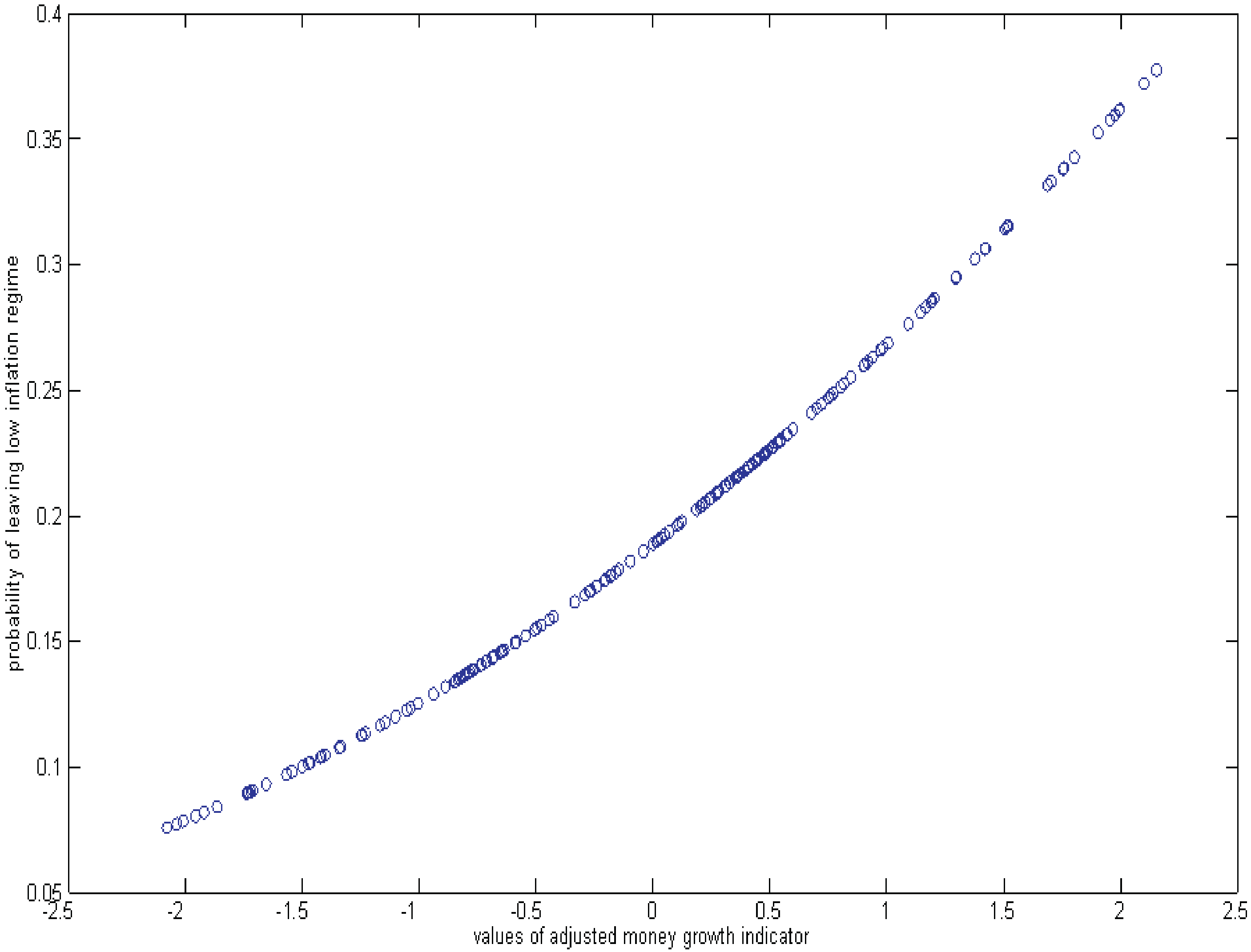
parameter	support	prior type	hp1	hp2	mean	std. dev.
γ_0	$(-\infty, +\infty)$	Gaussian	.0	1.0	.0	1.0
γ_1	$(-\infty, +\infty)$	Gaussian	.0	1.0	.0	1.0
γ_2	$(-\infty, +\infty)$	Gaussian	.0	1.0	.0	1.0
β_2	$(-\infty, +\infty)$	Gaussian	.0	1.0	.0	1.0
β_2	$(-\infty, +\infty)$	Gaussian	.0	1.0	.0	1.0
β_3	$(-\infty, +\infty)$	Gaussian	.0	1.0	.0	1.0
h_α	$[0, +\infty)$	Gamma	1	5	5.0	3.1

Table 11: Posterior simulation dynamic panel probit model, CA, GE, UK, US

coeff.	regressor	post mean	post std	lower	median	upper	APE
γ_0	intercept	-0.37	0.45	-1.24	-0.38	0.56	
γ_1	lagged m	-.28	.11	-.51	-.28	-.06	-.03
γ_2	lagged state	2.48	.24	2.03	2.48	2.96	.26
α_1	intercept shift unit 1	-0.08	0.45	-1.02	-0.08	0.80	
α_2	intercept shift unit 2	-0.22	0.45	-1.16	-0.21	0.65	
α_3	intercept shift unit 3	0.06	0.46	-0.89	0.06	0.96	
α_4	intercept shift unit 4	-0.05	0.45	-0.98	-0.04	0.83	
h_α	precision α	1.91	1.33	0.26	1.61	5.26	
corr.actual-fitted	0.80						
corr. actual-forecast.	0.87						

Results based on 130,000 MCMC replications. First 30,000 are rejected.

Figure 1: Relationship between money growth indicator and probability of leaving low inflation state



Probabilities of leaving low inflation state, $p_{12,t}$ are computed using the posterior mean values of the parameters $\gamma_{11} = .93$ and $\gamma_{21} = -.44$ and the standardized in-sample values for the adjusted money growh indicator.

Figure 2: Correlations of inflation with lags of adjusted money growth indicator, different countries

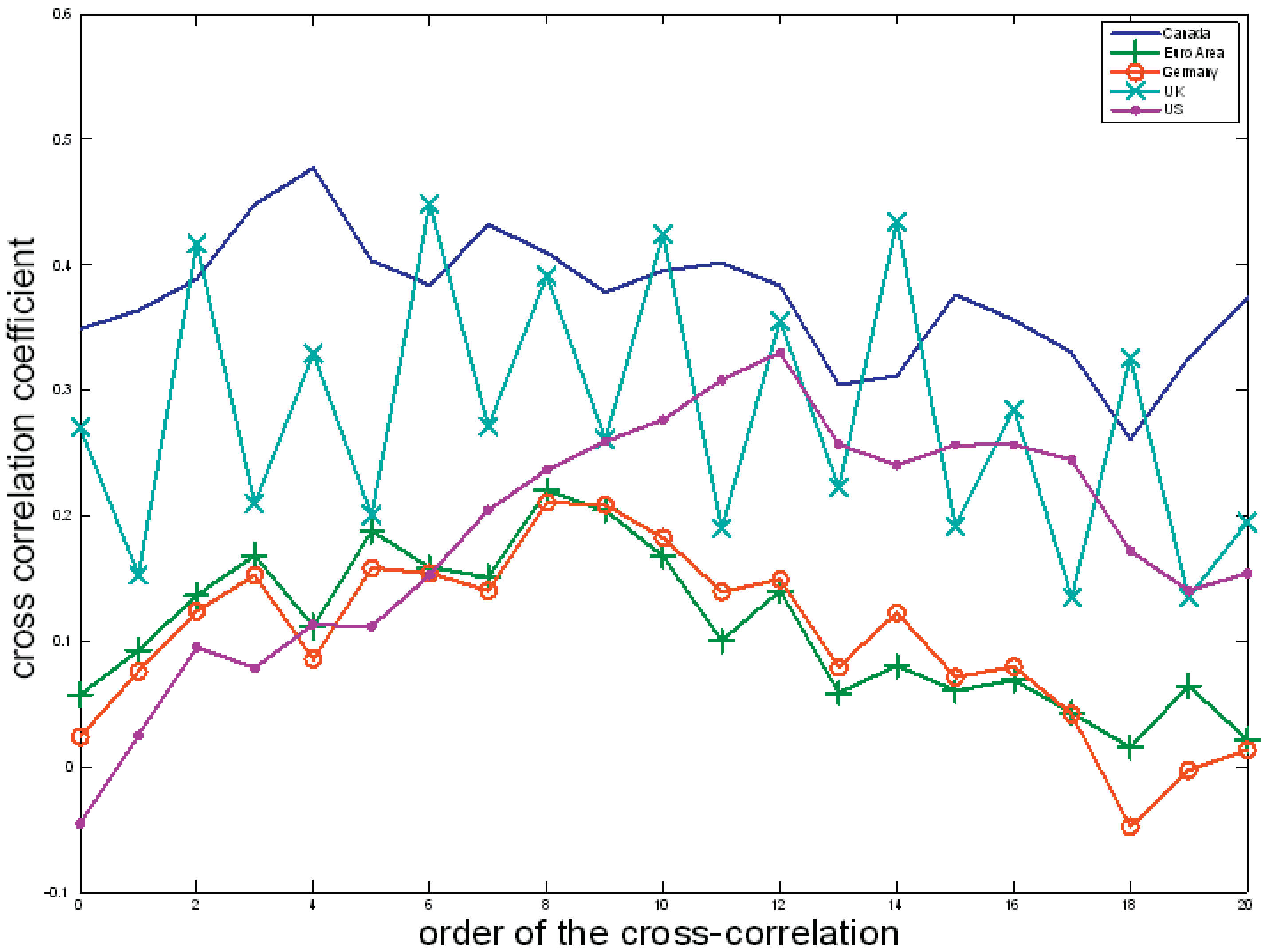


Figure 3: Early warning model results, Canada

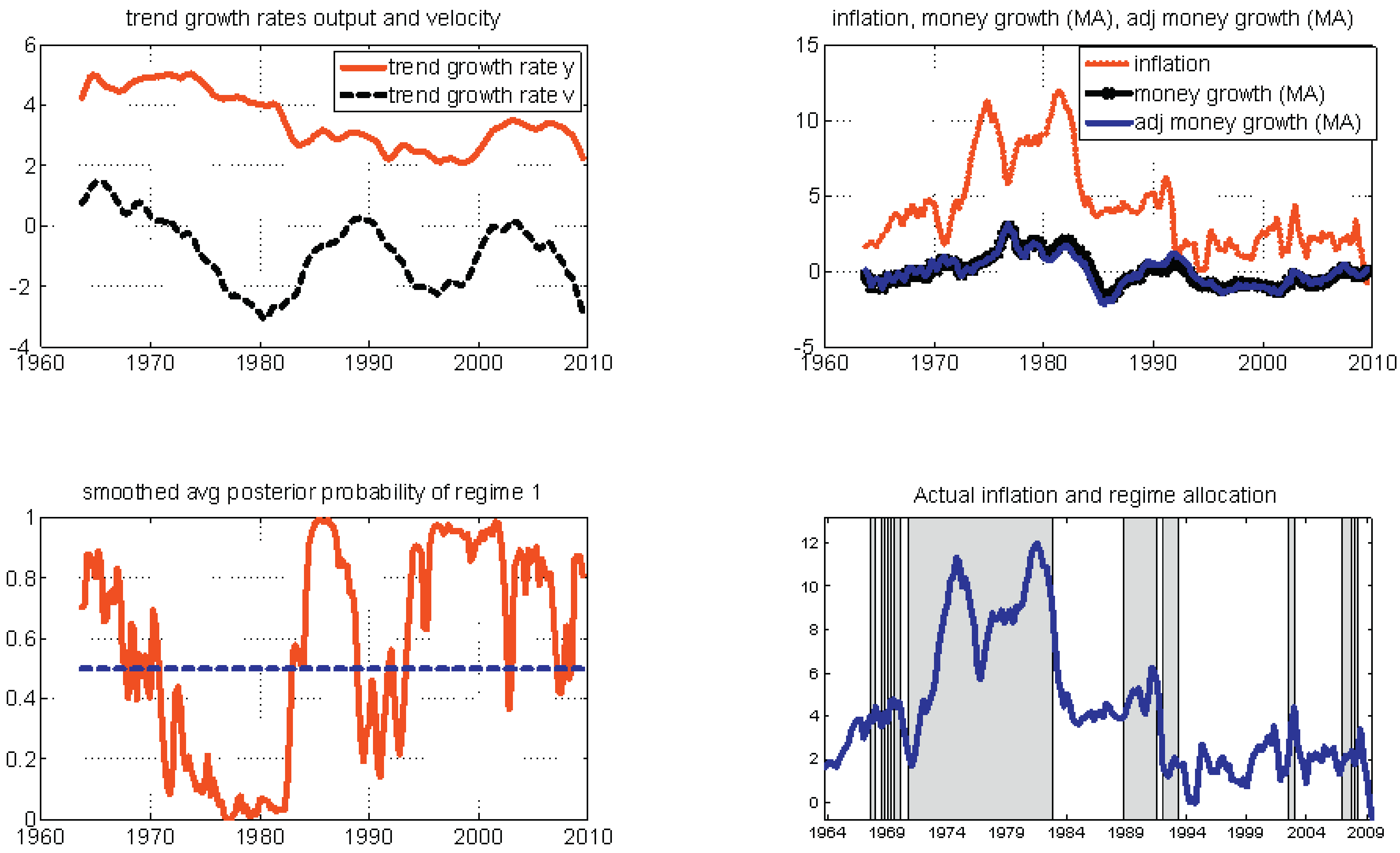


Figure 4: Early warning model results, Euro Area

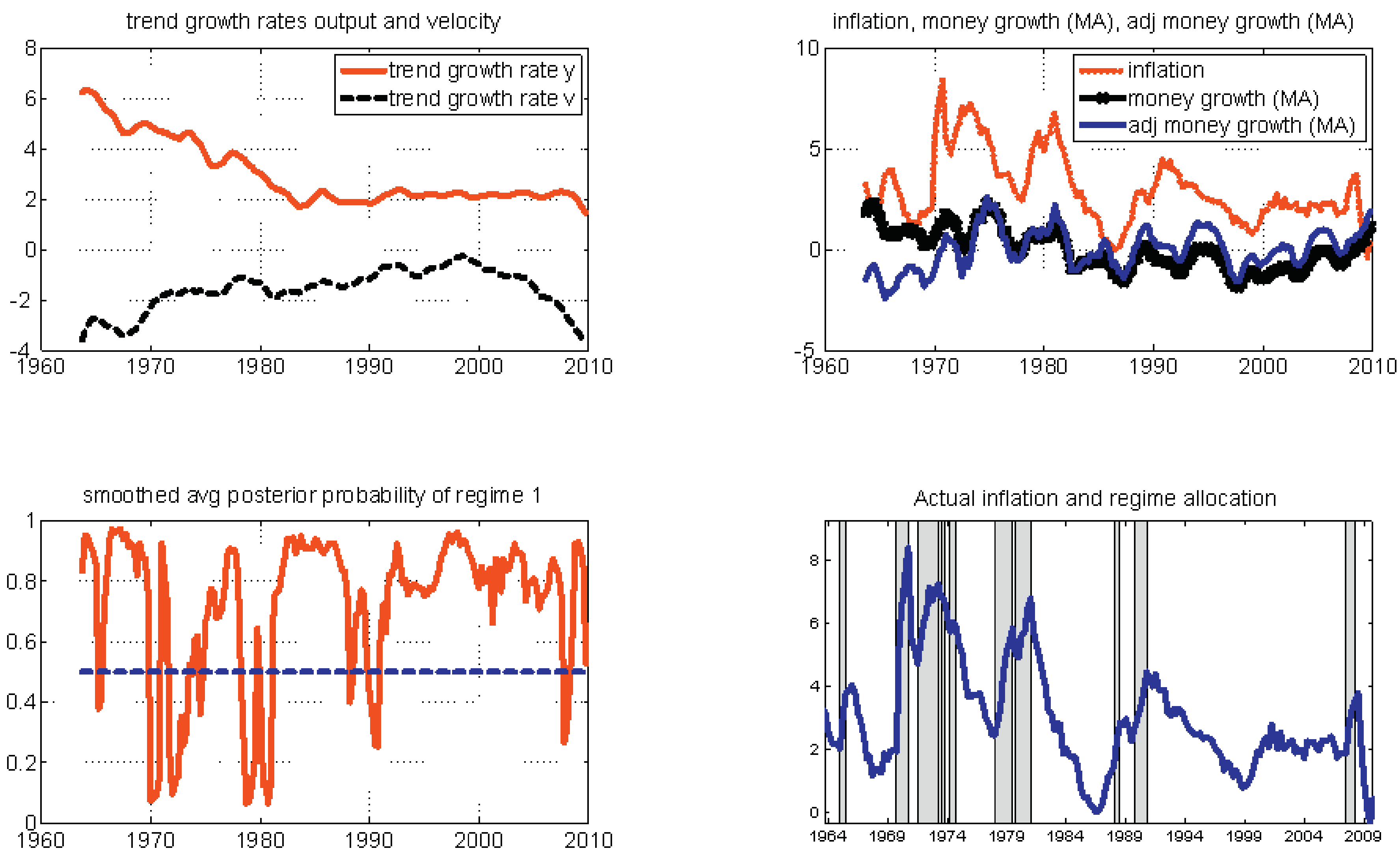


Figure 5: Early warning model results, Germany

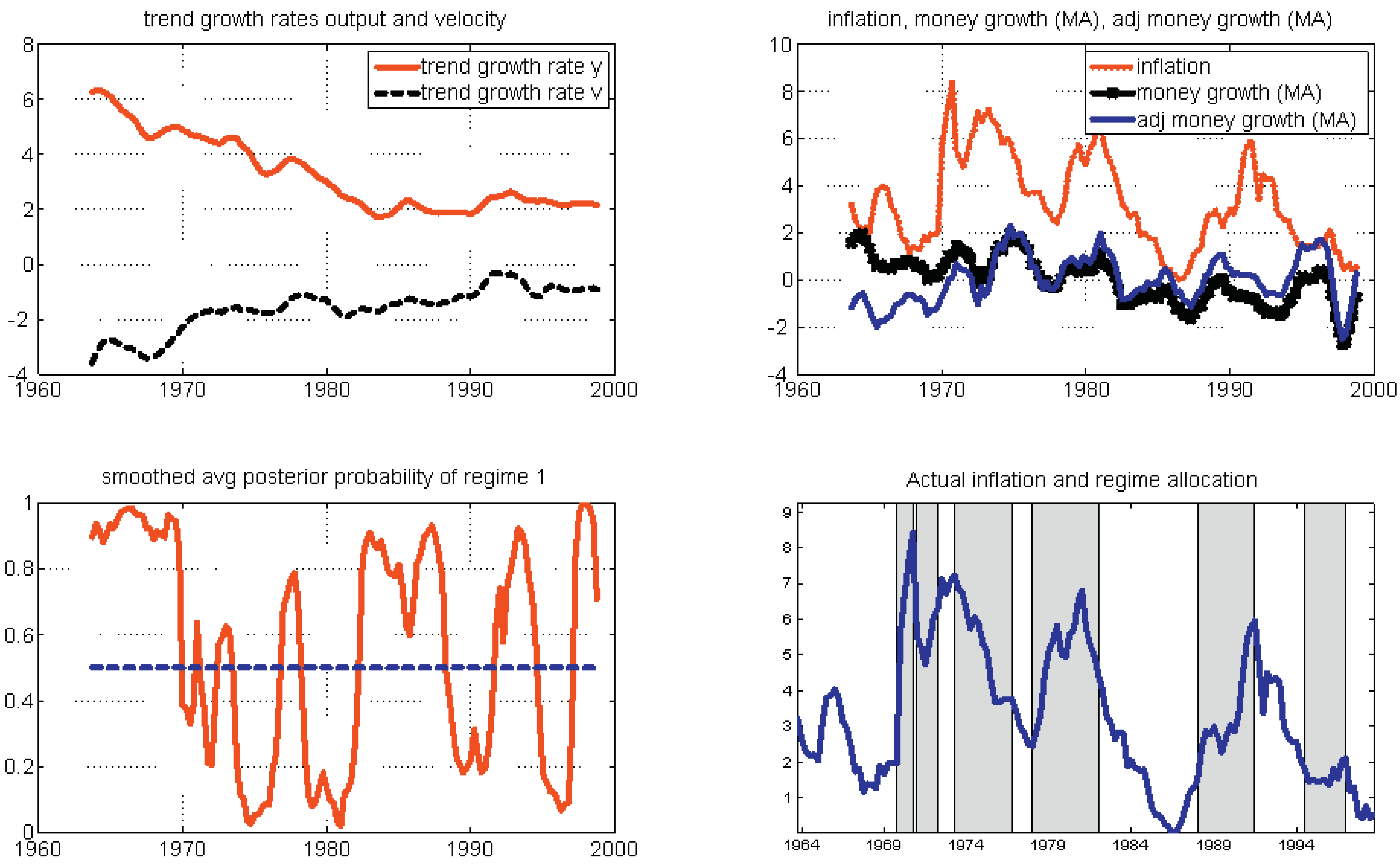


Figure 6: Early warning model results, UK

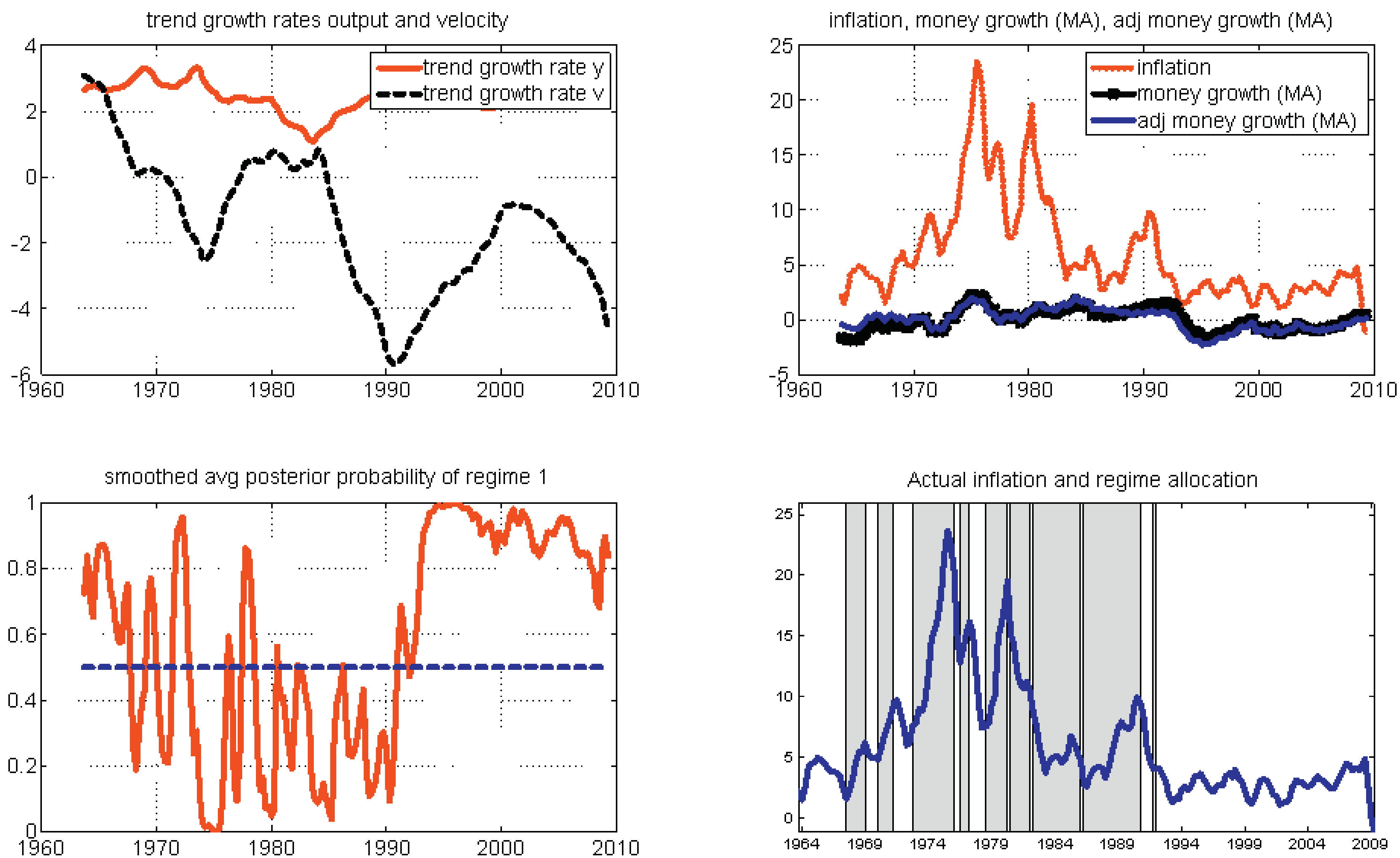


Figure 7: Early warning model results, US

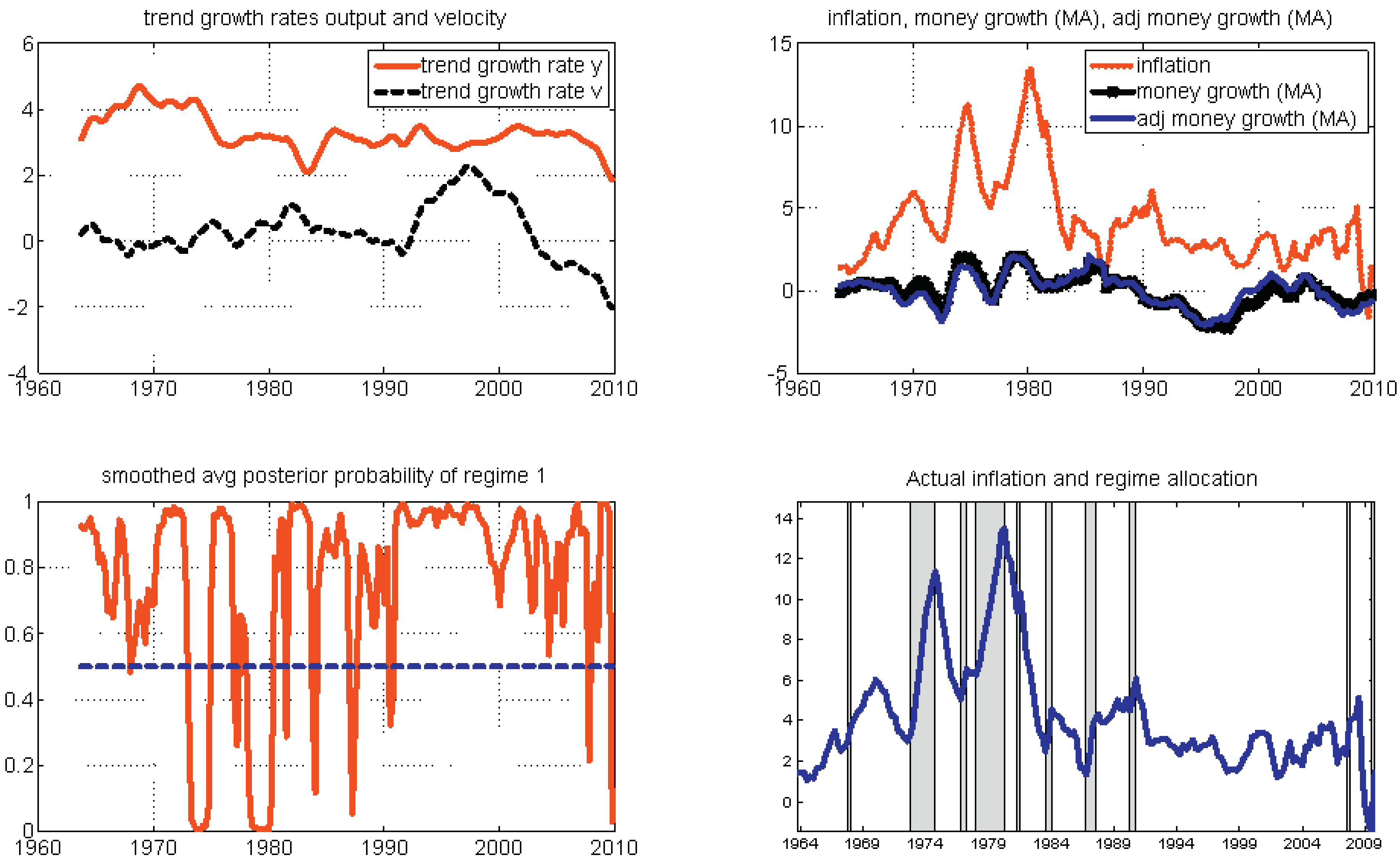


Figure 8: Predictive comparison between EW-MS model and MS model using conditional predictive likelihood differential, Euro Area and US