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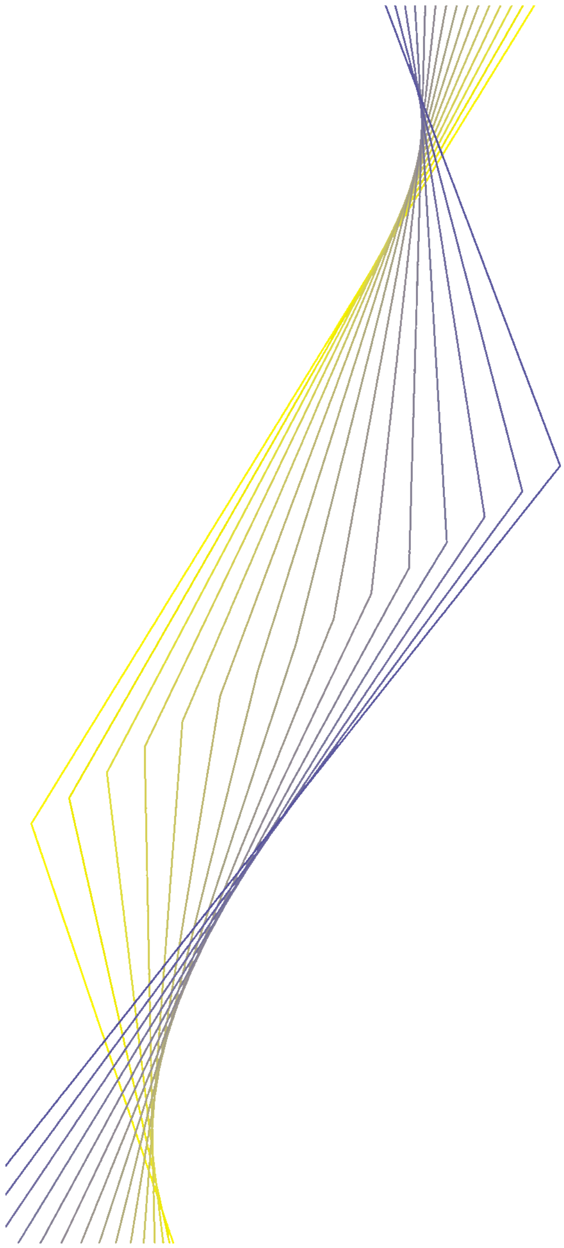

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EURO AREA PRODUCTION FUNCTION AND POTENTIAL OUTPUT: A SUPPLY SIDE SYSTEM APPROACH

BY ALPO WILLMAN*

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Abstract

In this paper, we present a three equation supply-side model based on aggregation across sectors with sector specific mark-ups and the technology parameters of the production function. The model has been applied to euro area data from the 1970s assuming that the underlying production function is either CES or Cobb-Douglas. Estimation results support the Cobb-Douglas case and the estimated supply-side model accounts satisfactorily for the stylised features of the data, i.e. the hump shape in the labour income share coupled with the relatively stable capital-to-labour income ratio and a noticeable change in profit margins and sectoral production shares. We also produce estimates of potential output and the output gap conditional on estimated production functions and examine the sensitivity of output gap estimates with respect to the alternative parameterisation of the production function.

JEL classifications: E23; E25.

Keywords: Production function, Supply-side, Factor shares, Potential output, Output gap.

Non-technical summary

In this paper, we model the euro area supply side with due consideration of the stylised features of the data, i.e. the hump shape in the labour income share coupled with the capital-to-labour income ratio, which remained relatively stable, except for the shift in the level in the late 1970s. We also produce estimates of potential output and the output gap conditional on the estimated production function, as contained in the supply side, and the equilibrium unemployment rate (or NAIRU), as estimated in the context of the ECB's area-wide model. In this context, our main interest is the sensitivity of output gap estimates to alternative parameters of the production function.

Our theoretical framework is a multi-sector model of imperfect competition, where output is produced by otherwise common technology, except for the sector-specific scale and technological progress parameters of the production function. By allowing price and income elasticity to differ across sectors, the aggregation of conditions of profit maximisation at the firm level implies that the aggregated level mark-up may develop secularly although the mark-up in each sector remains constant. The development of the aggregated level mark-up reflects changes in the production shares of sectors with high or low mark-ups and/or a fast or slow speed of technological progress. In line with this argument, we show that, throughout our sample period starting from 1970, the production share of the services sector has increased in the euro area. This growth in services will translate into a widening of the aggregate mark-up, if services are less competitive, on average, than other sectors or if they experience relatively lower technological progress. Further, the assumption of non-isoelastic demand curves implies time-variant sectoral mark-ups, as competing foreign prices also affect the pricing behaviour. This offers an avenue for explaining the hump-shaped development of the labour income share in the euro area.

In estimating the euro area supply side, we use the system approach. Compared with the single-equation approach, we believe that the system approach, with the implied cross-equation parameter constraints and the use of the wider range of data, markedly increases the efficiency of the estimation of the structural parameters of the system. This also facilitates the identification of the right functional form of the production function. We estimate the supply side system for the euro area conditional on two alternative functional forms of the production function, i.e. on the Cobb-Douglas and the CES cases. All estimations are carried out under the assumption of constant returns to scale with technological progress alternatively signified by a linear trend or HP-filter. The assumption of constant returns to scale is quite standard in empirical applications as it is coupled to the fact that, in estimation, it is very difficult to disentangle the rate-of-returns-to-scale effect from the effects of technological progress.

Estimation results based on the CES specification of the production function proved to be very sensitive with respect to the nature of technological progress. Estimation results were the most robust when technological progress was specified as Solow-neutral. In this case, the estimates of the elasticity-of-substitution parameter were close to unity, indicating that the Cobb-Douglas production function is a good approximation of the euro area production function. Hence, this implies that the hump-shaped labour income share would reflect variations in the mark-up, which are not transmitted in to the capital-to-labour income ratio. In order to be able to explain the level shift in the capital-to-labour income ratio, however, the inclusion of a level dummy was needed. We tentatively interpreted the dummy to comprise the effect of credit rationing on the user cost capital in period of the regulated financial markets of 1970s.

Estimation results with the Cobb-Douglas technology assumption were also very robust with respect to an alternative normalisation of estimated equations, to alternative ways of measuring productivity and, in addition, no simultaneity bias was found. Also the use of trend or the HP-filter smoothed total factor productivity as a technological progress variable had hardly any effects on the estimated parameters and gave quite similar estimates for the output gap. We also found that, at least in the euro area data, output gap estimates are quite insensitive with respect to alternative parameterisation and functional forms of the underlying production function. Hence, the principal source of the measurement error of the output gap, when the production function approach is applied, is the estimate of the NAIRU.

1 Introduction

The stability of capital and labour income shares is quite generally considered to be a stylised fact that describes the growth process in most economies rather well. Therefore, this assumption has also been a natural starting point in the bulk of both empirical and theoretical research. However, in the last 30 years, the development of income shares in quite a number of countries in continental Europe and, hence, in the aggregated euro area, has not been compatible with the above-mentioned stylised fact. After increasing strongly in the 1970s, the share of labour income in GDP in the euro area decreased continuously in the two subsequent decades. In this respect, developments in the United States or, more generally, in the "Anglo-Saxon" countries has differed, remaining broadly in line with the stylised fact of the stable labour income share.

Blanchard (1997) and Caballero and Hammour (1998) were among the first to take an interest in these differences in the US and European experiences. With help of differences in the labour markets and in wage formation in Europe and the United States, they linked different developments of income shares to another striking difference between the United States and Europe, namely to the steadily increasing unemployment rate in continental Europe, as opposed to the largely stable development of unemployment in the United States. Additional elements included in their efforts to explain the dynamics of the European development were the oil price shocks of 1970s, coupled with the slow adjustment of capital and low short-run, but high long-run substitutability of capital and labour.

As a possible source of the observed decrease in the labour income share in the 1980s and 1990s, Blanchard (1997) also mentions an increase in the mark-up. However, he finds this explanation implausible as "... the period since the early 1980s has been characterised by increased, not decreased competition – especially so in continental Europe, with the reduction of barriers to trade within the European Union." Bentolila and Saint-Paul (1998) introduce changes in the relative price of imported materials, in the skill mix, in the bargaining power of labour unions or in current and expected adjustment costs as possible factors affecting the development of the labour income share. Alcalá and Sancho (2000) find quite similar time profiles of European inflation and the labour income share and suggest inflation for a proxy for uncertainty in explaining the mark-up. Recently, de Serres, Scarpetta and de la Maisonneuve (2000) studied the possible role of aggregation bias due to sectorally differentiated wage shares and conclude that aggregation bias explains at least part of the decline in labour income share in many countries.

Our aim in this paper is to estimate the system determining the aggregated long-run supply of the economy of the euro area.[2] Hence, data congruent explanations for the development of the aggregated

[2] For recent examples of the estimation of the supply-side systems, see Allen (1997), Allen and Urga (1998) and Hall and Nixon (2000) with respect to the UK economy. In these three papers, the supply-side equations have been derived in terms of the cost function, instead of in terms of that of the production function as in this paper. As can be seen in e.g. Grilliches and Mairessw (1998) and Intriligator (1978), the simultaneous system approach has been used, since Marschac and Andrews (1947) first suggested it, in several studies on estimating the production function. The majority of these studies have used cross-section data, as in the case of e.g. Zellner, Kmenta and Dreze (1966). For an example of a time series application based on cost minimisation, see Bodkin and Klein (1967). McAdam and Willman (2002) have moreover applied a similar theoretical framework for estimating German long-run supply.

mark-up and the income shares of capital and labour are needed, in addition to the estimation of the parameters of the underlying production function. Our aim of explaining these developments simultaneously in a coherent estimation framework also differentiates our work from that presented in the aforementioned papers. On the other hand, the coherence of our framework limits the set of explanations that can be included in our framework. Hence, the views presented in this paper are complementary to, rather than substitutes for, those presented in the literature referred to.

In addition, we also produce estimates for potential output and the output gap conditional on the estimated production function and the equilibrium unemployment rate (or NAIRU), as estimated in the context of the ECB's area-wide model.[3] In this context, our main interest is in the sensitivity of output gap estimates to alternative parameterisations of the production function.

Our theoretical framework is a multi-sector model of imperfect competition, where output is produced by otherwise common technology, except for the sectorally differentiated scale and technological progress parameters of the production function. By allowing price and income elasticity to differ across sectors, the aggregation of the firm-level conditions of profit maximisation implies that the aggregated level mark-up may develop secularly, although the mark-up in each sector remains constant. The development of the aggregated level mark-up reflects changes in the production shares of sectors with the high or low mark-up and/or with a fast or slow speed of technological progress. In line with this argument, we show that, throughout our sample period as from 1970, the production share of the services sector has increased in the euro area, again, as opposed to the rather stable development in the United States. This growth in services will translate into a widening of the aggregate mark-up if services are less competitive on average than other sectors or if they experience relatively lower technological progress. The study by Oliveira Martins, Scarpetta and Pilat (1996), for instance, supports the former view, while the study by Gouyette and Perelman (1997) supports the latter view, at least partially. Furthermore, the assumption of non-isoelastic demand curves implies time-variant sectoral mark-ups, as competing foreign prices also affect the pricing behaviour. This offers an avenue for explaining the hump-shaped development of the labour income share in the euro area.

In estimating the euro area supply side we use the system approach. Compared with the single equation approach, we believe that system approach with the implied cross equation parameter constraints and the use of a wider range of data markedly increases the efficiency of the estimation of the structural parameters of the system. This also facilitates the identification of the right functional form of the production function. We estimate the supply side system for the euro area conditional on two alternative functional forms of the production function, namely the Cobb-Douglas and the CES production function.

The Cobb-Douglas specification for the economy-wide production function has been popular in both theoretical and empirical analyses for more than half a century. There are many reasons behind this popularity. Besides analytical simplicity, the Cobb-Douglas function has many attractive properties (e.g.

concavity, homogeneity and aggregation) and it can moreover be regarded as a special case of more general production functions (**C**onstant **E**lasticity of **S**ubstitution – CES – and translog production functions). It is also easy to estimate or calibrate and has quite often fitted, at least broadly, the data rather well. However, a strict restriction implied by the Cobb-Douglas production function is that it postulates the elasticity of substitution between factors of production to be equal to one. When coupled with a constant mark-up, this implies stable factor income shares. In this respect, the CES production function, the most widely used alternative to the Cobb-Douglas function, is markedly less restrictive since it allows the elasticity of substitution to be estimated from the data. Hence, factor income shares can also change. On the other hand, the non-linearity and larger number of estimated parameters of the CES function complicate the estimation, especially if the nature of technological progress is not fixed a priori. In practice, this difficulty may offset the advantage of greater theoretical generality of the CES function. However, despite its attractiveness, adherence only to the Cobb-Douglas form in estimating the production function entails the risk of committing a specification error of unknown magnitude and consequences. Rowthorn (1999) and Duffy and Papageorgiou (2000), for instance, have recently questioned the empirical relevance of the Cobb-Douglas function quite strongly.[4]

The importance of correctly specifying the production function is stressed if analytical frameworks containing empirical production functions are used by policy-making institutions like the ECB. This is the case, at least if macroeconometric models are used. In a well-specified macro-model, the production function plays the leading role in defining, in particular, the supply side of the model. It is also necessary if the production function approach is used as an alternative for measuring potential output and the output gap. The latter may play an important role, at least as a benchmark, in formulating monetary policy decisions, as suggested in recent years by the rapidly growing literature on policy rules originated by Taylor (1993). Orphanides (1998), Smets (1998), Orphanides and Norden (1999) and Orphanides et al. (2000), for instance, have shown that the measurement errors in output gap estimates lead to a significant deterioration of feasible policy outcomes and cause efficient policies to be less activist. In this paper we evaluate, by producing output gap estimates based on the quite broad range of alternative parameterisations of the production function, how important the source of the parameter uncertainty of the production function is for the measurement error of the output gap.

All estimations are carried out under the assumption of constant returns to scale. This corresponds to the practice, which has been adopted e.g. in the ECB's area-wide and the ESCB's multi-country-model framework (see Fagan, Henry and Mestre (2000)). Another argument is that, in estimation, it is very difficult to disentangle the rate-of-returns-to-scale effect from the effects of technological progress. In the case of the CES production function, technological progress is allowed to be either Hicks, Harrod or

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[3] As described in Fagan, Henry and Mestre (2000) and Fabiani and Mestre (2000).

[4] According to Rowthorn (1999), based on the evidence of cross-country estimations, the elasticity of substitution is well below unity. Estimates presented by Duffy and Papageourgiou (2000), which are based on the use of cross-country panel data, are well above unity, namely in the range of 1.3-3.

Solow-neutral. In a Cobb-Douglas world, these three alternative forms of neutral technological change cannot be distinguished from one another.

Technological progress is signified either by a linear trend or by a HP-filter. However, the use of HP-smoothed technological progress requires that total factor productivity (TFP) can be isolated as a separate component. That is always the case with the Cobb-Douglas production function, but in the case of the CES function it is possible only if technological change is Hicks-neutral. In estimating HP-filtered technological progress, an iterative approach is used.

One of our main findings is that, within the aggregated single good framework, implying a constant mark-up, neither the CES nor the Cobb-Douglas function is able to explain the developments observed in factor income shares in the euro area. However, the framework based on aggregation across sectors, which allows the mark-up measured at the aggregate level to vary in time, is capable of explaining the observed hump shape in the labour income share. The remaining problem in the data was an upward level shift in the ratio of capital to labour income in the late 1970s and early 1980s as a result of the negative real interest rate in most of the 1970s. As the relative factor prices were not able to explain that feature of the data within the CES production technology, our tentative answer to this issue is that, due to financial regulations, the real interest rate did not measure the marginal costs of financing in 1970s correctly. By allowing a level correction in the real interest rate, our system was able to explain satisfactorily and robustly the observed features of the data both over our whole sample period from the second quarter of 1971 to the fourth quarter of 1997 and the sub-sample period from the first quarter of 1983 to the fourth quarter of 1997.

Our estimation results based on the CES specification proved to be sensitive with respect to the nature of technological progress. Estimation results were the most robust when technological progress was specified to be Solow-neutral. In that case, the estimates of the elasticity of substitution parameter were close to unity, indicating that the Cobb-Douglas function is a good approximation for the euro area production function. Estimation results were also very robust with respect to alternative normalisations of estimated equations, to alternative ways of measuring productivity and, in addition, no simultaneity bias was found. We also found that, at least in the euro area data, output gap estimates are quite insensitive with respect to alternative parameterisations and functional forms of the underlying production function. Hence, the principal source of the measurement error of the output gap, when the production function approach is applied, is the estimate of the NAIRU.

The structure of the paper is as follows. In Section 2, the aggregated long-run supply side system is derived on the basis of the aggregation of the firm-level first-order conditions of profit maximisation, both with the Cobb-Douglas and with the CES production technology. In Section 3, the data and its stylised features are presented. In Section 4, estimation results are presented. The systems, based on the assumption of the CES production function, are estimated, first, to gain a better understanding of how legitimate the constraint of the unit elasticity of substitution implied by the Cobb-Douglas function is. Thereafter, the results based on the Cobb-Douglas technology are presented. In the third sub-section, the system with Hicks-neutral technological progress is estimated, when total factor productivity is defined in terms of HP filter-smoothed TFP. In the fourth sub-section of Section 4, the robustness of the estimation results is studied with respect to alternative normalisations of estimated equations. Likewise, the existence of possible simultaneity bias is studied by applying multivariate non-linear three-stage least squares as an alternative estimation method. Estimates for the output gap are presented in Section 5. Section 6 concludes the paper.

2 The theoretical framework

In this section, we first derive the aggregated supply relations based on the profit maximisation behaviour of firms. In Section 2.1, aggregation is done under the assumption that the firms face isoelastic demand curves. This assumption allows us to neglect the fact that at least some of the firms face foreign competition, in addition to domestic competition. In Section 2.2, the isoelasticity assumption is relaxed by assuming that the functional form of the export demand function is AIDS.[5] Now, instead of being constant, the price elasticity and, hence, the mark-up depends on the competing relative foreign price.

2.1 The aggregation of the supply side of the firm

Consider an economy with *m* production sectors. Firms in each sector produce differentiated goods, which are close substitutes within each sector, but they are not substitutes for one another across sectors. Except for the differences in the technological level and the growth rate of technological progress in each sector, all firms use the same production technology.

As the analysis in such a friction-free economy is static, the time index *t* in the context of variables is suppressed for clarity, unless necessary. Hence the production and demand of firm *i* in sector *j* are determined by the relations:

$$(1) \quad Y_i^j = A^j e^{\gamma^j t} F\left(K_i^j, N_i^j\right) = A^j e^{\gamma^j t} f\left(k_i^j\right) N_i^j$$

$$(2) \quad \frac{Y_i^j}{Y^j} = D^j\left(\frac{P_i^j}{P^j}\right); \qquad \text{with} \quad \frac{\partial \log D^j}{\partial \log P_i^j} = \varepsilon^j < -1$$

where Y_i^j, N_i^j K_i^j and k_i^j is output, labour, the capital stock and the capital-labour ratio, respectively, of firm *i* in sector *j*. Y^j is output of sector *j* and D^j is the demand function faced by firms in sector *j*. Parameter γ^j is technological change and ε^j is the price elasticity of demand in sector *j*. As, for the sake of simplicity, no cross-sector substitutability between goods is assumed, the aggregate demand for goods produced in sector *j* is determined by the demand system,

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[5] See Deaton and Muellbauer (1980).

(3) $$\frac{Y^j}{Y} = s_t^j = s_0^j + \xi^j \log\left(\frac{Y/Y_0}{N/N_0}\right) \text{ with } \sum_{j=1}^{m} s_t^j = 1,\ \sum_{j=1}^{m} \xi^j = 0$$

where s_t^j represents the output share of sector j in total output, $\frac{Y^j}{Y}$, and subscript *0* refers to the starting (or reference) period values of variables. Equation (3) expresses the demand system in per capita terms. Values of parameter $\xi^j > 0 \left(\xi^j < 0\right)$ imply greater (smaller) than unit income elasticity of demand. The economy-wide aggregates are determined by the identities:

(4) $$X \equiv \sum_{j=1}^{m} X^j \equiv \sum_{j=1}^{m}\sum_{i=1}^{n_j} X_i^j \quad \text{, with } X_i^j = Y_i^j, K_i^j \text{ and } N_i^j$$

(5) $$PY = \sum_{j=1}^{m} P^j Y^j = \sum_{j=1}^{m}\sum_{i=1}^{n_j} P_i^j Y_i^j$$

As factor markets are assumed to be competitive, each firm *i* in each sector *j* faces the same nominal wage rate *w* and the nominal user cost of capital *c* and, hence, maximises the profit function[6],

(6) $$\max_{Y_i^j, N_i^j, K_i^j} \Pi_i^j = P_i^j Y_i^j - wN_i^j - cK_i^j$$

subject to (1) and (2).

The first order conditions of profit maximisation imply the following 3-equation system, which determines the price of output and the demand for capital and labour conditional on the demand-determined output.

(7) $$P_i^j = \left(1+\mu^j\right)\left[\frac{w}{A^j e^{\gamma^j t}\left(f\left(k_i^j\right) - k_i^j f'\left(k_i^j\right)\right)}\right] \quad ;\ 1+\mu^j = \frac{\varepsilon^j}{\varepsilon^j + 1} \geq 1$$

(8) $$\frac{\partial Y_i^j / \partial N_i^j}{\partial Y_i^j / \partial K_i^j} = \frac{f\left(k_i^j\right) - k_i^j f'\left(k_i^j\right)}{f'\left(k_i^j\right)} = \frac{w}{c}$$

(9) $$\frac{Y_i^j}{N_i^j} = A^j e^{\gamma^j t} f\left(k_i^j\right)$$

As the relative factor price w/c in the right hand side of equation (8) is same for all firms that implies that also the capital-labour ratio is also the same across firms equalling the aggregate capital-labour rate: $k_i^j = k^j = k, \forall i, \forall j$. Hence aggregated production and the price in sector *j* are determined by relations:

[6] As we are interested in the long-run relations, the static profit maximisation framework can be used. In the dynamic framework, where the changes in the capital stock are associated with adjustment costs, the maximisation of the present value of the firm with respect to investment results in the first order condition, according to which the change in capital stock depends on the difference of the marginal product and the user cost of capital. However, the steady state of this relation implies the same long-run relation as that derived from the static profit maximisation, when implemented with respect to the capital stock.

(10) $\dfrac{Y^j}{N^j} = f(k) A^j e^{\gamma_j t}$

(11) $P^j = (1+\mu^j)\left[\dfrac{w}{A^j e^{\gamma^j t}\left(f(k) - kf^{'}(k)\right)}\right]$

Aggregation across sectors, as defined by identity (4), implies that the aggregate level supply-system, corresponding the firm level supply-system (7)-(9), can be written as:

(12) $\dfrac{Y}{N} = f(k)\left[\sum_{j=1}^{m} s_t^j A^{j^{-1}} e^{-\gamma^j t}\right]^{-1}$

(13) $\dfrac{c[f(k) - kf'(k)]}{wf'(k)} = 1$

(14) $P = \sum_{j=1}^{m} s_t^j P^j = \dfrac{w}{f(k) - kf^{'}(k)} \sum_{j=1}^{m} s_t^j (1+\mu^j) A^{j^{-1}} e^{-\gamma^j t}$

Equations (12) and (13) become more transparent after transforming them into logarithmic form and then linearising the logarithms of the summation terms around the values $s_t^j = s_0^j$ and *t=0*:

(15) $\log\dfrac{Y}{N} = \log f(k) + \log A + \gamma_A \cdot t - \sum_{j=1}^{m} AA^{j^{-1}}(s_t^j - s_0^j)$

(16) $$\log P = \log w - \underbrace{\left\{\log[f(k) - k \cdot f^{'}(k)] + \log A + \gamma_A \cdot t - \sum_{j=0}^{m} AA^{j^{-1}}(s_t^j - s_0^j)\right\}}_{\text{Log of the marginal product of labour}}$$

$$+ \underbrace{\log(1+\mu_A) + \sum_{j=1}^{m} \frac{AA^{j^{-1}}(\mu^j - \mu_A)}{1+\mu_A}(s_t^j - s_0^j) - \sum_{j=1}^{m} b^j(\gamma^j - \gamma_A)\cdot t}_{\text{Log of the mark-up}}$$

where $A = \left(\sum_{j=0}^{m} s_0^j A^{j^{-1}}\right)^{-1}$, $\mu_A = \sum_{j=1}^{m} AA^{j^{-1}} s_0^j \mu^j$ $b^j = \dfrac{AA^{j^{-1}} s_0^j (1+\mu^j)}{\sum_{j=1}^{m} AA^{j^{-1}} s_0^j (1+\mu^j)}$, $\sum_{j=1}^{m} b^j = 1$, and

$\gamma_A = \sum_{j=1}^{m} AA^{j^{-1}} s_0^j \gamma^j$

From the point of view of estimation, the multi-sector system, such as the one above, is problematic. There are two alternatives to circumvent the problems associated with a multi-sectoral approach. First, it is possible to retain the essential elements of this framework in a way that proxies sectoral shifts of the economy, if we redefine our multi-sector system in terms of just two sectors. Second, with help of the demand system (3), sectoral production shares can be reduced to trend, if income elasticities produced by each sector deviate from unity. This will be shown later in this section.

Besides leading to a convenient simplification, the two-sector system accounts for the stylised fact that the output share of the services industries has increased strongly in the euro area.

In the case of two sectors, equations (15) and (16) reduce to the forms:

$$(17)\qquad \log\frac{Y}{N} = \log f(k) + \log A + \gamma_A \cdot t + \left(\frac{A^1 - A^2}{s_0^1 A^2 + (1 - s_0^1)A^1}\right)(s_t^1 - s_0^1)$$

$$(18)\log P = \log w - \underbrace{\left\{\log[f(k) - k \cdot f^{`}(k)] + \log A + \gamma_A \cdot t + \left(\frac{A^1 - A^2}{s_0^1 A^2 + (1 - s_0^1)A^1}\right)(s_t^1 - s_0^1)\right\}}_{\text{Log of the marginal product of labour}}$$

$$+ \log(1 + \mu_A) + \frac{A^1 A^2 (\mu^1 - \mu^2)}{[s_0^1 A^2 + (1 - s_0^1)A^1][s_0^1 A^2 (1 + \mu^1) + (1 - s_0^1)A^1(1 + \mu^2)]}(s_t^1 - s_0^1)$$

$$- \frac{A^1 A^2 s_0^1 (1 - s_0^1)(\mu^1 - \mu^2)(\gamma^1 - \gamma^2)}{(A^2 s_0^1 + A^1(1 - s_0^1))[A^2 s_0^1(1 + \mu^1) + A^1(1 - s_0^1)(1 + \mu^2)]} \cdot t$$

Let us examine more closely the implications of equations (15) – (16) and (17) – (18). If $A^i \neq A^j$ $(i \neq j)$, then the aggregated change of technological development in (15) depends, in addition to the weighted average of the sectoral growth rates of technological progress γ_A, on the changes of sectoral production shares. It is straightforward to see that the growth of production shares of sectors j with $A^j > A$ ($A^j < A$) contributes positively (negatively) to the growth of productivity measured on the aggregate level. From equation (16), it can be seen, in turn, that the constancy (stationarity) of sectoral mark-ups μ^j does not imply the constancy (stationarity) of the mark-up on the aggregate level. The necessary and sufficient condition for the constancy (stationarity) of the mark-up at the aggregate level is that the mark-up in each sector j is the same.[7] If this is not true, then both the changes in the production shares and differences in growth rates of technological progress across sectors are transmitted to the aggregate mark-up. The effects of these two developments can strengthen or compensate each other in the aggregate level mark-up. From (16) it is straightforward to see also that the mark-up measured at the aggregate level increases if the production share of the sector *j* with $\mu^i > \mu_A$ grows. The examination of the two-sector case (see equation (18)) also indicates that if, in addition, the higher mark-up is coupled

[7] If $\mu^j = \mu \; \forall j$ then $b^j = \dfrac{AA^{j^{-1}} s_0^j}{\sum_{j=0}^{m} AA^{j^{-1}} s_0^j}$. After also reducing A and γ_A to their arguments, it is now simple to show that the term $\sum_{j=1}^{m} b^j (\gamma^j - \gamma_A)$ in (16) equals zero.

with the slower growth of technological progress, then the increase of the mark-up, measured at the aggregate level, is strengthened.

We show in the next step that changes in the sectoral production shares are reduced to trend if the income elasticities produced by each sector deviate from unity. It must be assumed that, in the equilibrium growth path, output and capital grow at a constant rate g. Equation (3) implies that, in this path, the production shares $\bar{s}_t^j$ are determined as,

(19) $$\bar{s}_t^j = \bar{s}_0^j + \xi^j g \cdot t$$

Using (19), equation (3) can be written as

(20) $$s_t^j - s_0^j = \xi^j g \cdot t + \xi^j \left[\log\left(\frac{Y/Y_0}{N/N_0} \right) - g \cdot t \right] = \xi^j g \cdot t + \xi^j u_t$$

where u_t can be assumed to be stationary around zero. After substituting (20) in (15) and (16), we end up with:

(21) $$\log \frac{Y}{N} = \log f(k) + \log A + \left[\underbrace{\gamma_A - g \sum_{j=1}^{m} AA^{j-1} \xi^j}_{\Gamma} \right] \cdot t - \left(\sum_{j=1}^{m} AA^{j-1} \xi^j \right) u_t$$

(22) $$\log P = \log w - \underbrace{\left\{ \log\left[f(k) - k \cdot f^{'}(k) \right] + \log A + \Gamma \cdot t \right\}}_{\text{Log of the marginal product of labour}}$$

$$+ \underbrace{\log(1+\mu_A) + \left[g \sum_{j=1}^{m} \frac{AA^{j-1}\left(\mu^j - \mu_A\right)}{1+\mu_A} \xi^j - \sum_{j=1}^{m} b_j \left(\gamma^j - \gamma_A\right) \right] \cdot t}_{\text{Log of the mark-up}}$$

$$+ \left[\sum_{j=1}^{m} \frac{AA^{j-1}\left(\mu^j - \mu_A\right)}{1+\mu_A} \xi^j - \sum_{j=1}^{m} AA^{j-1} \xi^j \right] u_t$$

Since u_t is stationary, it is absorbed by the residual of the estimated long-run aggregated supply system. An important difference of the above-mentioned multi-sector approach, as compared with the one-sector approach, is that there is no cross-equation parameter constraint concerning trend in the production function equation (21) and the price equation (22).

2.2 The AIDS demand function and the mark-up

Although the demand function (3) is written in a general form, the implicit assumption has been that the price elasticities $\varepsilon^j \ \forall j$ are constant. In this section, we relax that assumption and, instead, assume the AIDS demand function. We show that, in the open sectors of the economy, this assumption implies that the mark-up also depends on foreign competitiveness, i.e. the ratio of competing foreign to open sector prices. The following analysis is presented in terms the export sector, which for simplicity is treated as a single aggregate, although similar argument applies to import competing sectors as well.

Let us approximate the AIDS export demand function as follows:[8]

$$(23)\quad v=\frac{P^x\cdot X}{P_f\cdot D_f}=a+\theta\cdot\log\left(\frac{P_f}{P^x}\right)\qquad ;\theta>0$$

where v is the market share of export value, X is export volume, P^x is export price, P_f is the deflator of world exports and D_f is the volume of world exports.

Equation (23) implies the following price elasticity and the mark-up in the export sector:

$$(24)\qquad \varepsilon^x=-1-\frac{\theta}{v}=-1-\frac{\theta}{a+\theta\log\left(P_f/P^x\right)}$$

$$(25)\qquad 1+\mu^x=\frac{\varepsilon^x}{\varepsilon^x+1}=1+\frac{a}{\theta}+\log\left(\frac{P_f}{P^x}\right)$$

Equation (24) states that, with $\theta>0$, the price elasticity of exports is $\varepsilon^x<-1$. We also see that the price elasticity is not constant, but depends on the relative price P_f/P^x. This implies, as shown by (25), that the mark-up of the export sector depends positively on the competing world market prices. For estimation purposes, it is useful to linearise the logarithm of (25). Linearising it around the point $\log\left(P^x/P_f\right)=0$ results in:

$$(26)\quad \log\left(1+\mu^x\right)\cong\log\left(1+\overline{\mu}^x\right)+\frac{1}{1+\overline{\mu}^x}\log\left(\frac{P_f}{P^x}\right)\ ;\quad \overline{\mu}^x=\frac{a}{\theta}$$

The fact that the mark-up of the export sector depends on the competitive pressure of foreign prices allows us to write the economy mark-up as follows:

$$(27)\quad \log\left(1+\mu_A\right)=\log\left(1+\overline{\mu}_A\right)+\frac{s_0^x}{1+\overline{\mu}^x}\log\left(\frac{P_f}{P^x}\right)\ ,$$

where $\overline{\mu}_A$ is the aggregate mark-up, $\overline{\mu}^x$ is the export sector (or more generally the open sector) mark-up and s_0^x is production share of the export (or open) sector in the base (reference) period.

[8] In terms of the AIDS expenditure system, the share of country i exports in world imports (at current prices) is:

$$v_i=\frac{P_i^x\cdot X_i}{P_f\cdot D_f}=a_i-\theta_{ii}\cdot\log P_i^x+\sum_j\theta_{ij}\log P_{ij}^x\qquad ,$$

where $\sum_i a_i=1$, $\theta_{ii}=\sum_j\theta_{ij}$ and P_f is a weighted index of export prices P_j^x (see Deaton and Muellbauer (1980)).

2.3 The specification of the aggregated supply side system with the Cobb-Douglas and the CES technology

Before being able to estimate the aggregated supply side system, the functional form of the underlying technology must be specified. In the following, the supply side system is specified alternatively in terms of the Cobb-Douglas and the CES production function. However, due to a co-linearity between the time trend and the GDP share of services, it is apparent that the estimation of separate parameters for these two variables both in output and price equation will not succeed. Therefore, in specifying the estimated form of the supply system on the basis of the aggregation of two sectors, we abstract possible sectoral differences in the growth rate of technological progress, i.e. in equations (17) to (18), parameters $\gamma^1 = \gamma^2$. In addition, in the price and production function equations, we adopt the approximation, which our demand system also supports: $\gamma_A \cdot t + \frac{A^1 - A^2}{s_0^1 A^2 + (1 - s_0^1) A^1}\left(s^1 - s_0^1\right) \approx \Gamma \cdot t$

This implies that the GDP share of the services sector explains the trend in the aggregate mark-up and the time trend explains technological progress. However, these simplifying assumptions, which make it easier to identify the parameters in the context of two-sector aggregation, are not needed when the sectorally differentiated income elasticities of the demand system are utilised in aggregating the multi-sector supply system, as was done in deriving equations (21) to (22).

In case of *the Cobb-Douglas production function,* we obtain:

$$f(k) = \left(\frac{K}{N}\right)^{\beta} \text{ and}$$

$$f(k) - k \cdot f'(k) = (1 - \beta)\left(\frac{K}{N}\right)^{\beta}.$$

When the aggregation is based on two differentiated sectors (e.g. services and non-services) in line with equations (17) to (18), the aggregated supply system can be written as follows:

$$\text{(28a)} \quad \log \frac{Y}{N} = \beta \log\left(\frac{K}{N}\right) + \log A + \Gamma \cdot t$$

$$\text{(29)} \quad \frac{cK}{wN} = \left(\frac{\beta}{1 - \beta}\right)$$

$$\text{(30a)} \quad \log P = \log w - \underbrace{\left\{\log(1 - \beta) + \beta \log\left(\frac{K}{N}\right) + \log A + \Gamma \cdot t\right\}}_{\log(\text{MPL}) = \text{Log of the marginal product of labour}}$$

$$\underbrace{+ \log(1 + \overline{\mu}_A) + \chi \log\left(\frac{P_f}{P^x}\right) + \Phi\left(s_t^1 - s_0^1\right)}_{\textit{Log of the mark-up}}$$

where $\chi = \frac{s_0^x}{1+\bar{\mu}^x}$; $\Phi = \frac{A^1 A^2 \left(\mu^1 - \mu^2\right)}{\left[s_0^1 A^2 + \left(1-s_0^1\right)A^1\right]\left[s_0^1 A^2\left(1+\mu^1\right) + \left(1-s_0^1\right)A^1\left(1+\mu^2\right)\right]}$

It should be noted that relation (27) is also utilised in defining the mark-up behaviour.

When, in turn, as implied by (21) to (22), changes in sectoral production shares are reduced to trend, as implied by differences in income elasticities of goods and services produced in the different sectors of the economy, equations (21) to (22) imply that equations (28a) and (30a) are replaced by (28b) and (30b). However, equation (29) does not change.

(28b) $\log\frac{Y}{N} = \beta\log\left(\frac{K}{N}\right) + \log A + \Gamma\cdot t$

(30b) $\log P = \log w - \underbrace{\left\{\log(1-\beta) + \beta\log\left(\frac{K}{N}\right) + \log A + \Gamma\cdot t\right\}}_{\text{Log (MPL)=Log of the marginal product of labour}} + \underbrace{\log\left(1+\bar{\mu}_A\right) + \chi\log\left(\frac{P_f}{P^x}\right) + \eta\cdot t}_{\text{Log of the mark-up}}$

where $\eta = g\sum_{j=1}^{m}\frac{AA^{j-1}\left(\mu^j - \mu_A\right)}{1+\mu_A}\xi^j - \sum_{j=1}^{m} b_j\left(\gamma^j - \gamma_A\right)$

The aggregate Cobb-Douglas production function (28a), or equally (28b), implies that the marginal product of labour in equation (30a) and (30b) can be expressed alternatively in the form $MPL = (1-\beta)\cdot(Y/N)$. That implies that equation (30a), and equally (30b), can also be written in that form, where the (inverted) income share is the left-hand variable:

(30c) $\log\left(\frac{pY}{wN}\right) = -\log(1-\beta) + \log of\ the\ mark-up$

Equation (30c) shows that the aggregated labour income share $\frac{wN}{pY}$ is inversely related to the changes in the mark-up and, hence, a non-stationarity of the total economy mark-up also implies the non-stationarity of the aggregated labour income share, even on the assumption of the Cobb-Douglas production function. However, simultaneously equation (29) states that the relative factor income, $\frac{cK}{wN}$, has to be constant (stationary) for the assumption of the aggregate Cobb-Douglas production function to be true. That implies that the GDP share of capital income, excluding profits, is also inversely related to the aggregate mark up, which may be time-variant, although sectoral mark-ups would be constant.

In the case of *the CES production function* we can write:[9]

$$f(k)=\left[be^{(\gamma_N-\gamma_K)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho}+(1-b)\right]^{-\frac{1}{\rho}} \quad \text{and}$$

$$f(k)-k\cdot f'(k)=(1-b)\left[be^{(\gamma_N-\gamma_K)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho}+(1-b)\right]^{-\frac{1+\rho}{\rho}}$$

where the "substitution" parameter ρ and the distribution parameter b satisfy the conditions: $\rho \geq -1$ and $0 \leq b \leq 1$. The elasticity of substitution between labour and capital σ is defined as follows by the "substitution" parameter ρ: $\sigma = 1/(1+\rho)$. In the special case where $\rho \to 0$ or, identically, $\sigma = 1$, the CES function converges to the Cobb-Douglas case with $b=\beta$. In the relation above, technological development is defined quite generally, but in estimation we restrict ourselves to the cases of Harrod-neutral ($\gamma_K = 0$), Hicks-neutral ($\gamma_K = \gamma_N$) and Solow-neutral ($\gamma_N = 0$) technological change. Now, in the case of two-sector aggregation, the aggregated supply side system can be expressed in the form:

(31a) $$\log\frac{Y}{N}=-\frac{1}{\rho}\log\left[be^{(\gamma_N-\gamma_K)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho}+(1-b)\right]+\log A+\Gamma\cdot t$$

(32) $$\frac{cK}{wN}=\left(\frac{b}{1-b}\right)e^{(\gamma_n-\gamma_k)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho}$$

(33a) $$\log P=\log w-\underbrace{\left\{\log(1-b)-\frac{1+\rho}{\rho}\log\left[be^{(\gamma_N-\gamma_K)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho}+(1-b)\right]+\log A+\Gamma\cdot t\right\}}_{\text{Log (MPL)=Log of the marginal product of labour}}$$

$$+\underbrace{\log(1+\bar{\mu}_A)+\chi\log\left(\frac{P_f}{P^x}\right)+\Phi\left(s_t^1-s_0^1\right)}_{\textit{Log of the mark-up}}$$

Equation (32) implies a constant (stationary) factor income share only, if substitution parameter $\rho = 0$, i.e. the CES production function reduces to the Cobb-Douglas function. Otherwise changes in the capital-labour ratio or in the factor augmentation in technological change are transmitted to the capital-to-labour income ratio. It is worth noting, however, that the constancy (stationarity) of the aggregated mark-up does

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9 Hence, the production function in sector *j* is $Y^j = A^j e^{(\gamma^j-\gamma_N)t}\left[b\left(e^{\gamma_K t}K^j\right)^{-\rho}+(1-b)\left(e^{\gamma_N t}N^j\right)^{-\rho}\right]^{-\frac{1}{\rho}}$

$= A^j e^{\gamma^j t}\left[be^{(\gamma_N-\gamma_K)\rho t}k^{-\rho}+(1-b)\right]^{-\frac{1}{\rho}}N^j$

not imply a constancy (stationarity) of the labour or the capital income share, when the production function is CES. What is required is that the sum of factor income shares is constant (stationary).

Again, as in the case of Cobb-Douglas technology, equations (31a) and (33a) can be replaced by equations (31b), and (33b), where changes in sectoral production shares are reduced to trend on the basis of the demand system with sectorally differentiated income elasticities.

$$\text{(31b)} \quad \log\frac{Y}{N} = -\frac{1}{\rho}\log\left[be^{(\gamma_N-\gamma_K)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho} + (1-b)\right] + \log A + \Gamma\cdot t$$

$$\text{(33b)} \quad \log P = \log w - \underbrace{\left\{\log(1-b) - \frac{1+\rho}{\rho}\log\left[be^{(\gamma_N-\gamma_K)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho} + (1-b)\right] + \log A + \Gamma\cdot t\right\}}_{\log(\text{MPL})=\text{Log of the marginal product of labour}}$$

$$+ \underbrace{\log(1+\bar{\mu}_A) + \chi\log\left(\frac{P_f}{P^x}\right) + \eta\cdot t}_{\text{Log of the mark-up}}$$

3 The data and some stylised facts

In this section, we first present the data that we use in estimating the euro area supply side system derived in Section 2 and, in the light of selected charts, examine qualitatively the stylised features of the data.

3.1 The data

The principal source for the euro area data we use is the data of the ECB area-wide model.[10] The empirical counterparts offered by this data set for production, labour input, capital input and prices are:

$Y = YFR$ = real GDP at factor cost

$N = LNN$ = total employment

$K = KSR$ = the gross capital stock

$p = YFD$ = GDP deflator at factor cost

Factor prices are not directly available from the data of the area-wide model and, hence, some additions to the original data must be done. In the case of labour income, the problem is that, at the area-wide level, no data on the income of self-employed workers are available. The use of compensation to employees as a proxy for total labour income results in a serious underestimation of both the labour income share and the average compensation rate of total employment. To correct the bias, the employment shares of self-employed workers are used as a proxy for the income share of self-employed.

[10] See Fagan, Henry and Mestre (2000).

Table 1 Self-employed divided into unpaid family workers and paid self-employed as ratios to the number of employees				
	1977	1980	1987	1997
Self-employed to employees ratio, %	25.0	23.9	23.3	20.5
Unpaid family workers to employees ratio, %[11]	5.2	4.7	3.4	2.1
Paid self-employed to employees ratio, %	19.8	19.2	19.6	18.4

Source: Labour Force Statistics 1977-1997, OECD 1998.

Table 1 shows that there has been a downward trend in the employment share of self-employed in the period from 1977 to 1997. However, as can also be seen from the second row of Table 1, this is mainly due to a decrease in the number of unpaid family members. Instead, the labour share of paid self-employed has remained quite stable through the sample period. Therefore, also as the output of unpaid family workers is not measured national accounting statistics, we use the sample average of the labour share of paid self-employed and the average compensation per employee in calculating the imputed income of self-employed individuals.[12] The rest of the entrepreneurial income of self-employed is interpreted as belonging to capital income. Hence, average labour income per employed person is calculated as follows:

(34) $w = 1.193 * WIN/LNN$, where

WIN = compensation for employees

In calculating the capital income component cK, we need, in addition to the capital stock series K, an operational counterpart for the user cost of capital c. However, regarding the capital stock, there are two different capital stock concepts available, i.e. gross and net capital stocks. The gross capital stock can be described as a capacity concept, i.e. it measures the potential volume of capital services which can be produced by the existing capital stock at a given point of time (e.g. Biorn and Olsen (1989) and OECD (1992)). The net capital stock can be described as a wealth concept; capital has a value, which is derived from its ability to produce capital services today and in the future. Typically, the expected future services of a unit of capital of an old vintage are smaller than the expected services of a unit of capital of the new vintage. In general, the gross capital stock is more appropriate for the estimation of a production function,

[11] Figures on unpaid family workers are not available in France and Luxembourg. In addition, figures for Germany cover the period from 1980 to 1997 and those for the Netherlands the period from 1987 to 1997. In Table 1, the area-wide estimate of the employment share of unpaid family workers for the year 1977 is assumed to be same as the employment share of unpaid family workers in Germany in 1980 and in the Netherlands in 1987. The 1987 figure for the Netherlands is also used in calculating the area-wide estimate for the year 1980.

[12] This practice is also followed by Blanchard (1997).

whereas the net capital stock is more suitable for the definition of production costs. Accordingly, the recommended practice in calculating the consumption of capital in national accounting statistics is to use the net capital stock.

The above argument supports the view that the net capital stock and the respective depreciation rate should be used in calculating the capital income component, while the gross capital stock should be used in the production function. To reconcile these views, we resort to the fact that, in practice, the ratio of net to gross capital stock is quite stable and, in the equilibrium growth path, this ratio should equal to the ratio of the respective depreciation rates. Hence, on the basis of the steady state condition $\frac{K_{net}}{K_{gross}} = \frac{\delta_{gross}}{\delta_{net}} < 1$ we can write:

$$(35) \qquad cK = P_I(r + \delta_{net})K_{net} = P_I(r \cdot \frac{\delta_{gross}}{\delta_{net}} + \delta_{gross})K_{gross}$$

where P_I = investment deflator and K = gross capital stock.

Typically estimates of the net to gross capital stock ratio lie within a quite narrow range of 0.5 to 0.7 (see e.g. Steele, 1980).[13] According to OECD statistics, for instance, the ratio of net to gross capital stock in 1990 was 0.64 in Germany, 0.58 in France, 0.69 in Italy 0.64 in Belgium and 0.63 in Finland (OECD 1996).[14] In the following we use value 0.64 as a "median" estimate for the euro area ratio of net to gross capital stock. With the annual depreciation rate of 4% in the data of the ECB area-wide model, the estimate for the capital income is defined as follows:

$$(36) \qquad cK = \frac{P_I\left[(i - 4 \cdot \pi^e) \cdot 0.64 + 4\right]}{400} \cdot KSR$$

P_I = investment deflator

i = long-term interest rate

π^e = inflation expectations = the HP-filter (λ=100) fit for one period P_I-inflation

In constructing a series for the price competitiveness of open sector production, the deflator of euro area exports proxies the price of the total open sector production as constructed in the area-wide model data.

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13 Steele's (1980) simulation experiments suggest that the relevant range could be even narrower.

14 For other euro area countries, data were not available. The French figure does not include the housing stock as an estimate for the gross housing stock was not available. However, assuming that the ratio of net to gross housing stock is around 0.7, as is the case very uniformly in other countries, this would imply that in France, too, the ratio of (total) net to (total) gross capital stock would be around 0.64.

As a measure for the competing foreign price of the open sector, we use the import price of non-primary goods. It is constructed utilising the following quasi-identity:[15]

(37) $$MTD = P_f^{\,1-m_t}\left(EEN * COMPR\right)^{m_t}$$

where MTD = deflator of the euro area imports

COMPR = commodity prices (HWWA index), in US dollars

EEN = nominal effective exchange rate

m_t = elasticity estimate (the share of the primary goods imports of total euro area imports)

The GDP share of service sector output is a natural candidate for a measure of the change in the production structure in the euro area. As shown in Table 2, the share of service sector output in GDP, in volume terms, has increased strongly in the euro area, especially in the two largest countries, i.e. in Germany and France. In this respect, the US development is very different. While the GDP share of services output increases by 9.8 percentage points in the euro area, the US share remains quite stable, actually decreasing by 1.7 percentage points in the period from 1970 to 1998.

Table 2. Services sector output as a percentage share of GDP at constant prices (1971=100)

	1970	1980	1990	1998	Change 70-98
Germany	49.2	55.4	61.1	68.4	19.2
France	55.0	61.9	67.2	67.4	12.2
Italy	50.9	52.2	55.4	56.1	5.2
Spain	49.5	52.7	54.9	53.8	4.3
Euro area	51.5	55.9	59.5	61.3	9.8
United States	64.7	63.8	63.3	63.0	-1.7

Source: OECD Historical Statistics 1970-1999.

In the euro area, the growth in the services share will translate into a widening of the aggregate mark-up if service industries, on average, are less competitive or if they experience lower technological progress than other sectors of the economy. The empirical evidence supporting the former feature is quite strong and well documented. For instance, Oliveira Martins, Scarpetta and Pilat (1996) show that, across the main industrialised countries, mark-ups measured for the service industries typically exceed those for

[15] The area-wide model data contain the variables MTD, EEN and COMPR. The series for the import share of primary goods is calculated by the Directorate Statistics of the ECB on the basis of Eurostat data. This series covers the period from 1980/1981. In the 1970s, m_t is assumed to be constant, equalling to the value of 1980/1981.

manufacturing. The evidence for slower technological change among service industries, however, is more mixed. On average, and in most countries, this would appear to hold true (e.g., Gouyette and Perelman, 1997).

3.2 Some stylised facts of the data

In this section, we undertake a graphical examination on the stylised facts relevant from the point of view of the euro area supply side. Figures 1 and 2 document the development of the share of labour income in GDP and the ratio of capital-to-labour income, respectively, in our sample period from 1970 to 1997. Already a simple "eyeball inspection" shows that the development of the labour income share cannot be stationary. During the first half of the 1970s, the labour income share rose, but it started to declining in the early 1980s and reverted to around the same level as in 1970 by the end of the sample period. Moreover, the augmented Dickey-Fuller (ADF) t-test statistics (without a trend) is only -0.65, implying non-stationarity.

Likewise, from Figure 2, we observe that, at least over the full sample period, the capital-to-labour income ratio cannot be stationary. The time profile of the capital-labour income ratio is, however, quite different from that of the labour income share. We can observe two regimes in the capital-to-labour income share; a very low level covering the most of 1970s and a shift in the late 1970s and early 1980s to a markedly higher level covering the rest of the rest of the sample period.

Can our supply side system explain these developments? To answer this question, it is useful to write the following relation implied by the supply side system (31)-(33).

$$(38)\quad \frac{pY}{wN}=\left\{1+\left(\frac{b}{1-b}\right)e^{(\gamma_n-\gamma_k)\rho\cdot t}\left(\frac{K}{N}\right)^{-\rho}\right\}\times mark-up \;=\left\{1+\frac{cK}{wN}\right\}\times mark-up$$

Equation (38) states that there should be a close (inverse) relationship between the time profiles of the labour income share and the capital-to-labour income ratio, if the mark-up is not excessively volatile. That is not what we see in Figures 1 and 2; we see no level shift in the labour income share corresponding to the level shift in the capital-to-labour ratio at the end of 1970s and in the early 1980s.

This leads us to consider whether there could be other explanations for the level shift in the capital-to-labour income ratio, linked e.g. to the way capital income is constructed. Our capital income cK is an imputed concept. It is very sensitive with respect to variations in the measured user cost of capital and, hence, to variations in the real interest rate. In fact, as Figure 3 shows, the variation of the capital-to-labour income ratio, including the regime shift, very closely follows the profile of the real interest rate. Although there is nothing to prevent the ex post real interest rate from being temporarily negative, a precarious feature in the observed development of the real interest rate is that it was negative for most of the 1970s. The use of a persistently negative real interest rate as an operational counterpart for the expected real interest rate used in the optimisation framework seems contradictory.

One possible explanation of the negative character of the measured real interest rate in the 1970s and of the upward level shift in the late 1970s and early 1980s would be that financial markets were highly regulated in Europe during the most of 1970s. In the late 1970s and early 1980s, the regulated system broke up, at first perhaps partly due to leakage caused by financial innovations, and later due to the formal removal of regulations.[16] Under this explanation, the long-term government bond rate i, measuring the regulated interest rate, does not measure the marginal cost of financing correctly. Figure 5 gives, perhaps, indirect support to this view. It presents the development of ex post long-term real interest

Figure 1. Labour income share

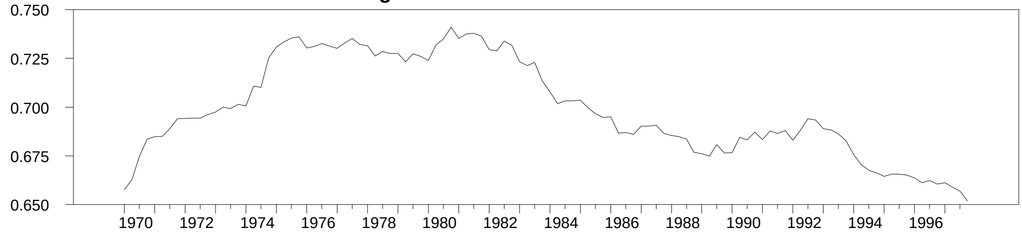


Figure 2. Capital to labour income ratio

Figure 3. Long-term real interest rate

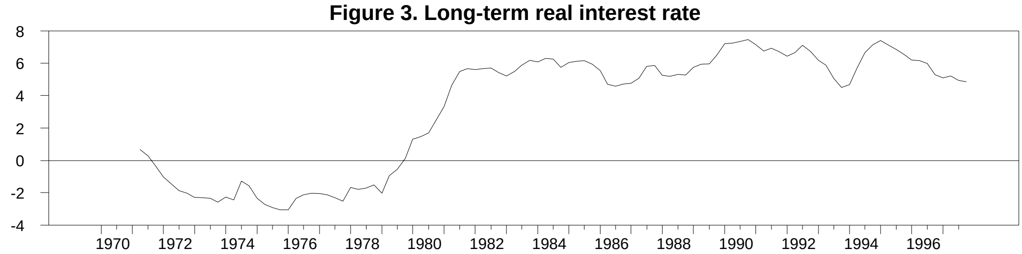
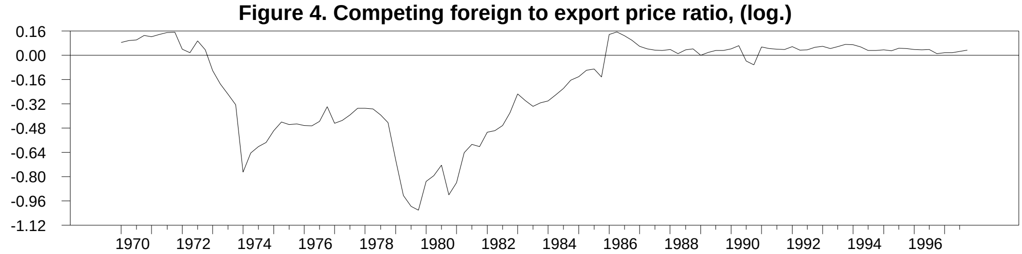


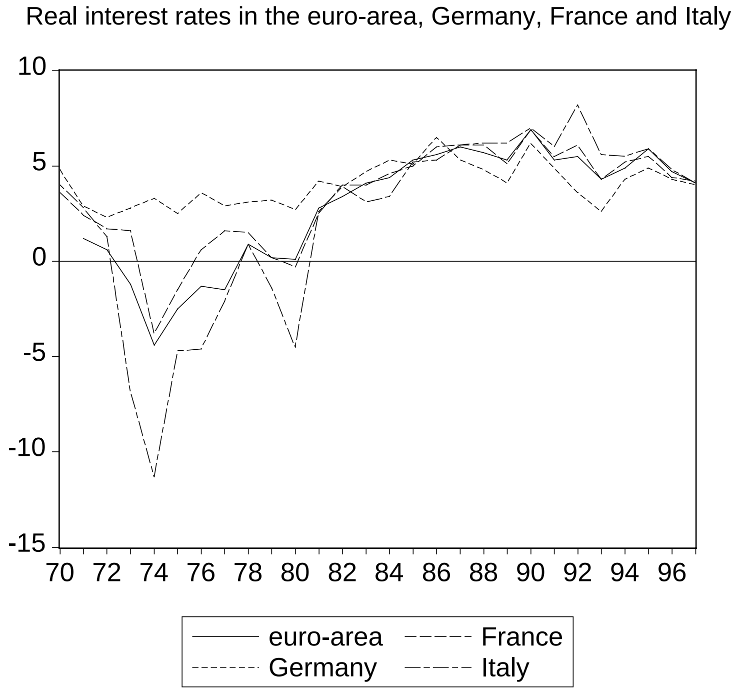
Figure 4. Competing foreign to export price ratio, (log.)



[16] The level-shift in the real interest rate seems somewhat to precede the formal removal of financial regulations in many countries. It is quite possible, however, that financial innovations caused the regulated system to start to leak long before the formal removal of regulations.

rates (in terms of private consumption deflator) in Germany, France and Italy as well as in the euro area.[17] We see that the real interest rate in France and, especially, Italy was strongly negative throughout most of the 1970s. The real interest rate in France mimics the euro area real interest rate rather well, but the real interest rate in Germany is clearly positive through the whole sample. The German case is interesting as, according to Issing (1997), Germany took the lead in financial liberalisation and all direct controls had been removed before 1974, i.e. by the point of time when real interest rates in the two other countries turned negative.

Figure 5



To take into account the possibility that our data for the euro area real interest rate do not measure correctly the marginal cost of financing in the 1970s, a level shift dummy was constructed to correct the interest rate upwards in the 1970s.[18] The dummy-corrected interest rate could be interpreted as a shadow

[17] Figures for Germany, France and Italy are from "The EU Economy: 1999 Review, European Commission, Directorate-General for Economic and Financial affairs".

[18] This is in line with the findings of Coenen and Wieland (2000). They found a strong and significant negative dependence of euro area aggregated demand on the German real interest rate, whilst the dependency of the weighted average of the euro area real interest rate was markedly weaker and statistically insignificant. Following Coenen and Wieland (2000) we also could have used the German real interest as a proxy for the real interest rate of the euro area. However, the drawback of this approach would be that we loose the information contained by the euro area real interest rate in the latter part of the sample, when we think that the euro area real interest rate measures reasonably well the real marginal cost of financing in the euro area. In addition, the size of the correction to the real interest rate implied by estimated parameter for the dummy may also serve as evidence or counter-evidence of our hypothesis.

interest rate $\left(i^n\right)$, measuring the marginal cost of financing.[19] As shown below, the dummy takes a hyperbolic form:

$$(39) \quad i^n = i + h \cdot DUM = LTN + h \cdot \left(1 - \frac{1}{1 + \exp(2 - 0.3(time - 35))}\right)$$

Variable DUM is 1 in the early 1970s and starts deviating from unity in around 1976 and converging to zero in around 1983, after which i^n in practice equals to the observable long-term interest rate i. Now, after replacing i by i^n in relation (35) defining capital income *cK*, we can rewrite (35):

$$(40) \quad cK = P_I \left\{\left(i - 4\pi^e\right) \cdot 0.64 + 4\right\} \cdot KSR/400 + h \cdot 0.64 \cdot P_I \left(KSR/400\right) \cdot DUM \ ,$$

where parameter *h* can be estimated jointly with the other parameters of the system.

The inclusion of the level shift dummy in the equation of the capital-to-labour income ratio allows the removal of the regime shift in the capital-to-labour income ratio, implying a rather stable evolution of the ratio. This, in turn, would be consistent with a close-to-unity elasticity of substitution between capital and labour. Therefore, the non-stationarity of the share of labour income in GDP could be explained mainly by the evolution of the mark-up. Since the early 1980s, the downward trend can be explained by the observed increase in the output share of the services sector, coupled with a higher mark-up than in the manufacturing sector. Regarding the development of the labour income share in the 1970s, we can resort to the widely approved assessment of Bruno and Sacks (1985), which is also shared by Blanchard (1997) and by Caballero and Hammour (1998). According to this assessment, countries in continental Europe were affected by large adverse shifts in the "labour" supply in the 1970s. These shifts came from the failure of wages to adjust to the productivity slowdown and from the adverse supply shocks of the 1970s, i.e. the sharp oil price increases and, to a lesser extent, also increases in other commodity prices. By contrast, the "Anglo-Saxon" countries appear to have been largely shielded from adverse labour supply shifts in the 1970s.

However, as can be seen in Figure 4, these effects have been transmitted to our measure of the ratio of competing foreign prices to export prices, i.e. the price competitiveness of the open sector. Two oil price shocks and a weakening US dollar in the 1970s caused the losses of competitiveness in the euro area. However, as Figure 4 shows, there was a recovery in price competitiveness during the first half of 1980s, which reflects the appreciation of the US dollar and a gradual decrease in oil prices which ended up in the collapse of the OPEC cartel in 1986. Thereafter, until the end of our sample period, our measure of price competitiveness remained remarkably stable.

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[19] This would, of course, presuppose the existence of a rather well functioning "grey" financial market. Then, when regulation is binding, the marginal cost of financing can be markedly above the average cost of financing, which the interest rate LTN measures. After financial deregulation, under the Modigliani-Miller theorem, as our user cost definition assumes, the marginal and average costs of financing are equal. For a more detailed analysis of a credit-rationed economy, see e.g. Willman (1981) and Tarkka (1985).

4 Estimation results

Our estimation strategy is the following. We start with the CES production function, which is more general than the Cobb-Douglas function. The motivation behind the order is the interest in evaluating to what extent estimation results based on the CES function justify the restrictions contained by the Cobb-Douglas function.

In the CES case, the three-equation supply system is estimated assuming that technological progress is alternatively Harrod, Hicks or Solow-neutral. To alleviate the empirical identification of parameters, we imposed a further constraint, in the context of Harrod and Solow-neutral technological progress, by assuming that in (31a) to (33a) $\gamma_N = \gamma_A$ and in (31b) to (33b) $\gamma_N = \Gamma$. Effectively, this implies that the speed of Harrod or Solow-neutral technological progress is assumed to be equal across sectors. However, this constraint is not needed, when technological progress is defined as Hicks-neutral.

Since, in principle, the CES function is able to explain non-stationarity in the labour income share and in the factor income ratio, the supply system was first estimated assuming a constant mark-up. Thereafter, that constraint was relaxed by explaining the changing mark-up by the GDP share of services or, alternatively, by reducing the GDP share to trend and by the competing foreign to export price ratio. This procedure helps us to evaluate the importance of the variables explaining the non-constant mark-up from the point of view of their explanatory power, on the one hand, and from the point of view of the co-integration properties of the estimated systems, on the other. Due to the non-linearities of the estimated systems, co-integration is studied simply in the light of the stationarity properties of the residuals of the estimated systems. To study the parameter stability of the estimates supply systems, they are estimated both over the full sample covering the period from the second quarter of 1971 to the fourth quarter of 1997 and over the sub-sample period from the first quarter of 1983 to the fourth quarter of 1997. In the latter period, the dummy explaining the observed level shift in the real interest rate is not needed.

The method of estimation is multi-variate non-linear least squares, although in section 4.4, where the existence of possible simultaneity bias is studied, the three-equation system is also estimated with the method of multi-variate non-linear three-stage least squares.

4.1 The CES case

The estimation results of the CES production function are presented in Tables 3.1 to 3.3. All estimations were carried out under alternative assumptions concerning the nature of technological development, i.e. Harrod-neutral ($\gamma_N \neq 0$ and $\gamma_K = 0$ in Table 3.1) Hicks-neutral ($\gamma_N = \gamma_K$ in Table 3.2), and Solow-neutral ($\gamma_N = 0$ and $\gamma_K \neq 0$ in Table 3.3). All the full-sample estimation results presented in Tables 3.1 to 3.3 contain the dummy variable scaling up the real interest rate in the 1970s. The dummy proved to be necessary for us to be able to explain satisfactorily, as we anticipated in Section 3, the upward shift in the ratio of capital to labour income in the late 1970s and early 1980s.

Tables 3.1 to 3.3 show that the parameter estimates of the production function are sensitive with respect to the neutrality assumption of technological progress. Parameter estimates associated both with the mark-up ($1+\overline{\mu}_A, \Phi, \eta$ and χ) and with the dummy scaling up the real interest rate (h) in the 1970s are, in turn, quite stable with respect to the alternative neutrality assumptions of the technological progress and with respect to alternative estimation periods. We also see in many cases that specifications with the variable mark-up, when compared with the constant mark-up specification, more than halve the standard error of estimate of price equation (SEE_P), with only marginal effects on the residuals of the production function and the equation of the factor income ratio. Specifications, where – through the demand system – changes in the sectoral production shares are reduced to trend, work marginally better than the corresponding specifications with the GDP share of services as an explanatory variable. We also see that the ratio of competing foreign prices to export prices is important in explaining the mark-up. That is the case especially in the context of full-sample estimations, but this variable has explanatory power also in the sub-sample period, although its development in the sub-sample period from the first quarter of 1983 to the fourth quarter of 1997 is markedly smoother than in the 1970s and early 1980s (see Figure 4 in Section 3). That smoothness might, at least potentially, impede the identification of the effect of the ratio of competing foreign prices to export prices on the mark-up in the sub-sample and is a plausible explanation for smaller parameter estimates in the sub-sample than in the full sample.

Of the three alternative neutrality assumptions of technological progress, the results associated with Harrod-neutrality are the least robust. This is reflected by the instability of the parameter estimates of the production function, when the full-sample results are compared with the sub-sample results. In the full sample, Harrod-neutral estimates for the elasticity of substitution σ are in the range of 0.4 to 0.7, while the sub-sample results imply a very high elasticity of substitution. In some cases, the results support the value of minus one for substitution parameter ρ, implying infinity for the elasticity of substitution. Variation in the distribution parameter b is also very high. Typically, the full sample estimates are only slightly below unity, while the sub-sample estimates are only slightly above zero, as associated with statistically insignificant t-test values.

The Hicks-neutral case also suffers from parameter instability, although it is not as severe as in the Harrod-neutral case. Full sample estimates for the elasticity of substitution σ vary in the range of 0.7 to 0.9, except in the case where the mark-up is constrained to be constant. In that case, the estimate for the elasticity of substitution is 1.4. Again, the sub sample estimates are uniformly above unity, although markedly less so than in the case of Harrod-neutral technological progress. In the sub-sample, estimates for the elasticity of substitution are in the range of 1.2 to 1.5 with non-constant mark-ups and equal 2.2 with the constant mark-up. In the context of the assumption of the non-constant mark-up, the full sample estimates of distribution parameter b are in the range of 0.4 to 0.8 and the sub-sample estimates in the range of 0.1 to 0.2. Hence, variation is still high, but within a more reasonable range than in the context of Harrod-neutral technological progress.

With Solow-neutral technological development, the estimation results are most stable irrespective of the combination of the explanatory variables or the estimation period. Estimates for the elasticity of substitution σ are 1.1 and 1.2 in the full sample and the sub-sample, respectively, when the constraint of a constant mark up is imposed. However, the estimates with the time-varying mark-up imply almost uniformly the unit elasticity of substitution. The exception is the full-sample estimation, when the change in the mark-up is explained by trend (parameter η)and the ratio of competing foreign prices to export prices (parameter χ) (see Table 3.3, column 5, the full sample case). However, even in this case, the deviation from unity is not large, with the elasticity-of-substitution estimate σ equalling 0.9. Also the range for the estimated values of the distribution parameter b is considerably narrower than with Harrod and Hicks-neutral progress. In the context of the assumption of the non-constant mark-up, the full sample estimates are in the range of 0.3 to 0.4 and the sub-sample estimates in the range of 0.2 to 0.3.

Regarding the co-integration properties of the estimated systems, as implied by the ADF t-test measures, we see that estimation alternatives based on the CES production function, together with a constant mark-up, do not reject the non-stationarity of residuals. Price equation is the problematic equation.[20] However, in estimation alternatives covering the full sample the problem is removed – or at least markedly alleviated – when estimated specifications account for the change in the aggregated level mark-up.[21] We also see that this simple co-integration analysis supports Hicks and Solow-neutral technological progress over the Harrod-neutral alternative. With Harrod-neutral technological progress, the ADF t-test measures remain systematically below the level required by the rejection of the non-stationarity of the residuals at 5% significance level. The same problem is encountered with the sub-sample residuals irrespective of the underlying neutrality assumption. However, we are inclined to interpret that rather more as a small-sample peculiarity than a serious co-integration problem. Same test results are also obtained for the residuals based on the full-sample estimation, and the augmented Dickey-Fuller test is applied only to the residual values of the sub-period from the first quarter of 1983 to the fourth quarter of 1997.

We can conclude that the results obtained with the Solow-neutral technological development assumption are the most reasonable, especially from the point of view of parameter stability. In this case, estimates for the elasticity of substitution between labour and capital are quite close to one and, in most cases, the deviations from unity are not statistically significant. Hence, the CES results based on the assumption of Solow-neutral technological development are quite supportive of the unit substitution elasticity constraint

[20] Without the dummy scaling up of the real interest rate in the 1970s, the residuals of the equation for the capital-to-labour income ratio would also be non-stationary.

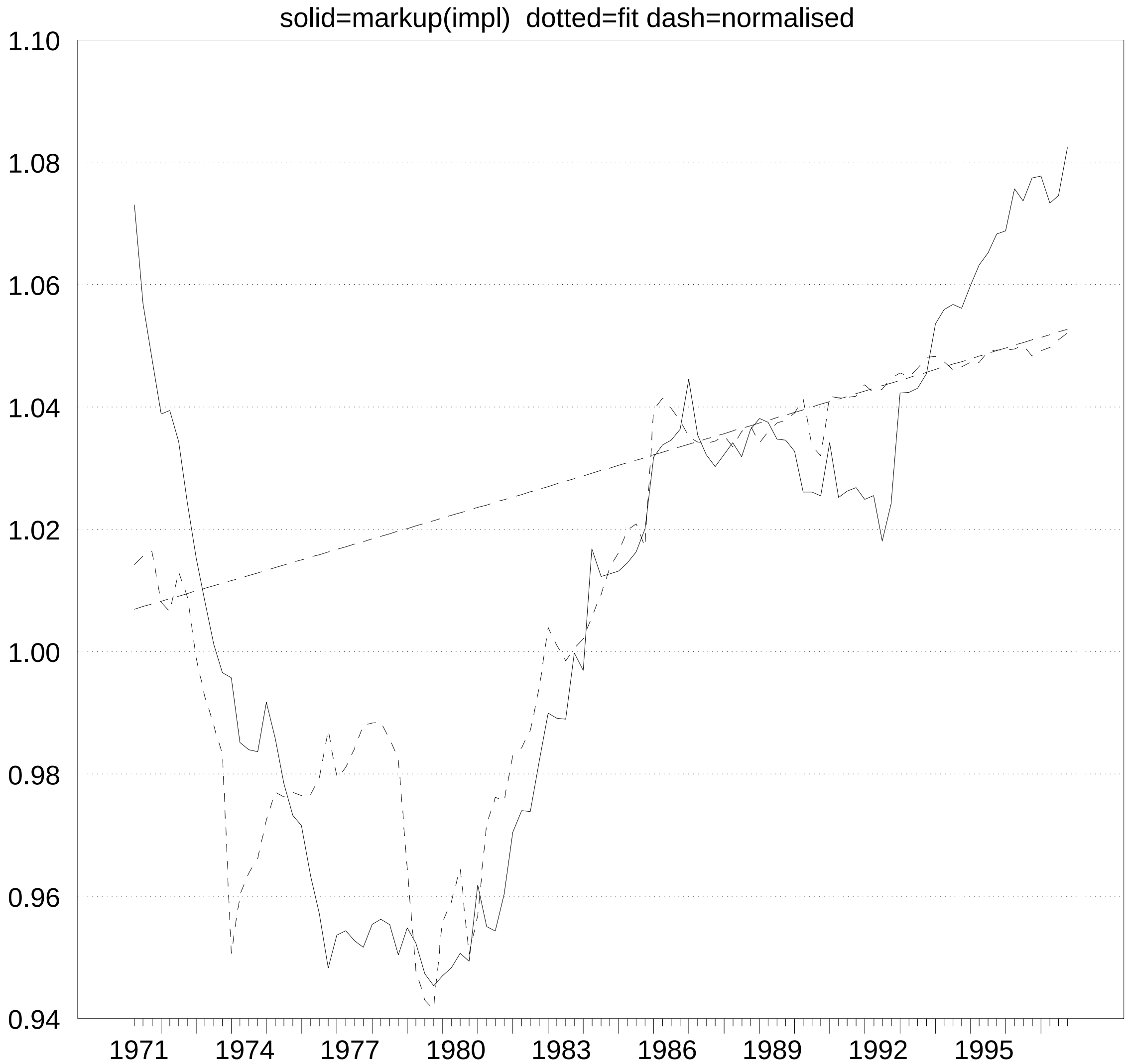
[21] A problem in the context of system estimation is that the critical values of the DF or the ADF tests are not tabulated. However, one would expect that, in the context of system estimation, with across equation parameter constraints, the bias towards stationary residuals is smaller than in the context of single-equation estimation. This implies that the critical values for the residuals of system estimation may be below those required from single-equation residuals.

of the Cobb-Douglas production function. With Hicks and Harrod-neutral technological progress, the full sample results imply elasticity of substitution below unity and the sub-sample results one above unity.

4.2 The Cobb-Douglas case

The estimation results with the Cobb-Douglas production function are presented in Table 4. We see that estimation results are very similar in both the full-sample and the sub-sample cases. The output-capital elasticity β is practically identical (0.29) in all cases, corresponding very closely to the estimated values of distribution parameter b of the CES function when technological progress is defined as Solow-neutral. That is what one would expect, as also the elasticity of substitution estimates are close to unity when the CES production function is estimated on the basis of the assumption of Solow-neutral technological progress. The technological development parameter γ is also very close to 0.0035 in all cases, implying an

Figure 6. Implied mark-up and its fits



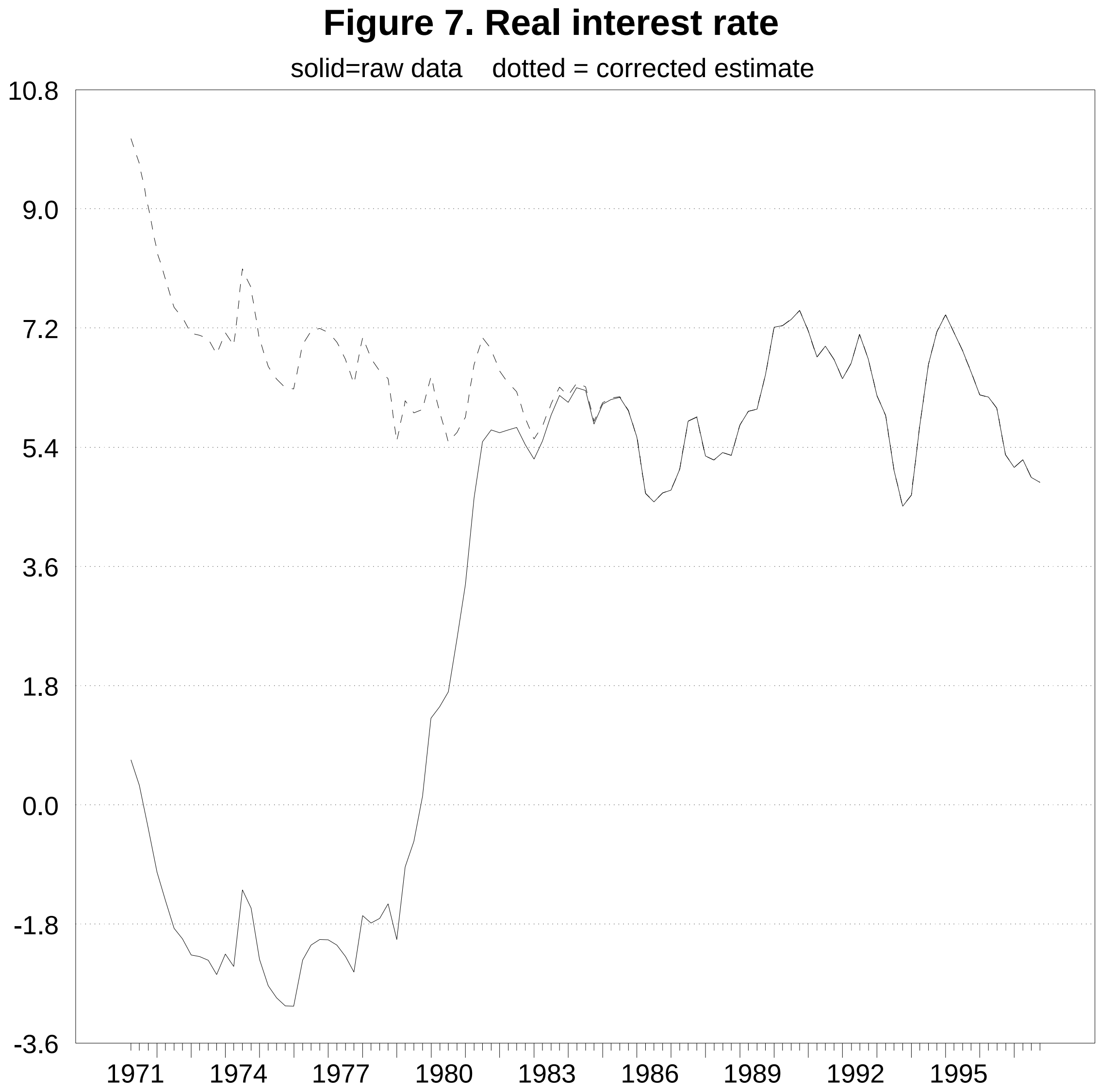
average annual growth of 1.4% in total factor productivity in the estimation period.

We also see that the ADF-test results are very similar as in the case of the CES production function with Solow-neutral technological progress. Again, regarding the stationarity of residuals, the price equation remains the borderline case.

As in the case of the CES production function, we also see that the stationarity of the residuals of estimated equations is no problem at least in the context of full-sample estimations. In the sub-sample, the non-stationarity of the residuals of price equation cannot be rejected at 5% significance level, as is also the case in the context of CES estimations. Here we are inclined to interpret that result as a small-sample peculiarity rather than as a serious co-integration problem.

Figure 6 presents the development of the mark-up – including the residual – implied by equation (30b) with parameter estimates shown in Table 4 (column 5, the full sample case). Figure 6 also shows the fit of the mark-up and the explanation of the mark-up with the ratio of export prices to competing foreign prices fixed to equal the average (1.043) of the period from the first quarter of 1986 to the fourth quarter of 1997. We see that, although there are still problems in explaining the development of the mark-up, especially in the 1970s, estimated model traces the general profile of the mark-up satisfactorily. It is interesting to see that, although the implied mark-up and its fit are below unity for most of the 1970s, implying prices below marginal costs, the estimate of the mark-up with the fixed ratio of export prices to competing foreign prices remains above unity throughout the sample period.

Figure 7, in turn, presents the real interest rate and the "dummy-corrected" level representing an estimate of the shadow marginal cost of financing. Our estimate would imply that the shadow marginal cost of financing would have been around on the same level in the "regulated" period as in the "deregulated" period starting in 1983. Quite high estimates for the "dummy-corrected" real interest rate at the beginning of the 1970s indicate, however, that the simple level-shift correction is too rough to give realistic estimates for the whole pre-liberalisation period.



4.3 System estimation with technology defined by the HP-filter-smoothed total factor productivity

An alternative to the use of a deterministic linear trend as a determinant of technological progress is to apply the HP filter to the total factor productivity component of the production function, i.e. the Solow-residual of the production function. This is easy to calculate if the production function is Cobb-Douglas and if the average labour income share is used as an estimate for the output-labour elasticity parameter of the production function.[22] The situation is somewhat more complicated if the parameters of the production function are estimated simultaneously from the system of equations. However, by using an iterative approach, this can be done if the total factor productivity, TFP, is a separable term in the

[22] See e.g. OECD (1994).

production function. With the Cobb-Douglas production function, this is always the case, but if the production function is CES, then the separability of TFP requires that technological progress is Hicks-neutral.

In calculating the HP filter-smoothed total factor productivity, which is consistent with the parameter estimates given by the whole system, the following iterative approach was used. First the Solow-residual was calculated by using the system estimation results with trend as a technological progress variable. Hence, in the Cobb-Douglas case, the Solow-residuals are calculated as follows:

$$(41)\quad \text{Solow\_res} = \log Y - \hat{\beta}\cdot \log K - \left(1-\hat{\beta}\right)\log N$$

where $\hat{\beta}$ is the estimate obtained, when trend is used as a technological progress variable. After applying the HP-filter to the Solow_res and denoting the fit by *TFP*, the technology component $\log A + \Gamma \cdot t$, in the context of the system (28b), (29) and (30b), can be replaced by *TFP*. Now the system (28b), (29) and (30b) can be written in the form:

$$(42)\quad \log\frac{Y}{N} = \beta \log\left(\frac{K}{N}\right) + TFP$$

$$(43)\quad \frac{cK}{wN} = \left(\frac{\beta}{1-\beta}\right)$$

$$(44)\quad \log P = \log w - \left\{\log(1-\beta) + \beta\log\left(\frac{K}{N}\right) + TFP\right\} + \left\{\log(1+\overline{\mu}_A) + \chi\log\left(\frac{P_f}{P^x}\right) + \eta\cdot t\right\}$$

By estimating the system (42) to (44), new estimates for β, h, $1+\overline{\mu}_A$, χ and η are obtained. If re-estimated parameter estimates are not close enough to the first round estimates, the procedure can be repeated until convergence is attained.

When applying this approach to equations (42) to (44), only two iterations were needed. Already after the first iteration round, parameters β, h and $1+\overline{\mu}_A$ had converged, equalling the initial estimates (see Table 4) and parameters χ and η deviated only marginally from the initial estimates. After the second round, they had again converged. Hence, at least in this case, the role of the re-estimation of the system (24) to (26) was mainly to work as a consistency check. When applied to the CES production function with Hicks-neutral technological development, our experience was otherwise the same, but now a few additional iteration rounds were needed. However, differences between the re-iterated and initial estimates were insignificant and without any practical importance.

Although the use of HP filter-smoothed TFP did not change the parameter estimates of the system from those estimated with the linear trend, the stationarity properties of residual may change. This was not, however, the case in our experiments. Test results concerning the rejection of the non-stationarity of residuals remained qualitatively the same as those presented in Table 4.

4.4 The robustness of results: some additional experiments

As we know (see footnote 5), the marginal product of labour can be expressed alternatively in terms of the capital-labour ratio, the capital-output ratio or the output-labour ratio. To study the sensitivity of the estimation results with respect to alternative ways of constructing the marginal product, all these productivity measures were tried in the price equation. These estimation results are not reported in a separate table, because they were practically identical with those presented in Tables 3 and 4.

It is also well known that estimation results are quite often sensitive with respect to normalisation. In addition, output, price, labour and the capital stock depend simultaneously on one another. The main concern of early proponents of the system approach for estimating the production function was mostly the simultaneity bias.[23] To address the importance of this problem, the Cobb-Douglas system, (28b) to (30b), was transformed into the following form:

$$(45)\quad \log\left(\frac{Y}{N}\right) = \log A + \Gamma \cdot t + \beta \log\left(\frac{K}{N}\right)$$

$$(46)\quad \log\left(\frac{K}{N}\right) = \log\left(\frac{1-\beta}{\beta}\right) - \log\left(\frac{c}{w}\right)$$

$$(47)\quad \log\left(\frac{p}{w}\right) = \log(1+\bar{\mu}_A) + \chi \log\left(\frac{P_f}{P^x}\right) + (\eta - \Gamma)\cdot t - \log(1-\beta) - \log A - \beta \log\left(\frac{K}{N}\right)$$

The main difference is that (46) is normalised with respect to the logarithm of the capital-labour ratio, whereas (29) (in Section 2) is normalised in terms of the factor income ratio *cK/wN* without taking logarithms. From the decision-making point of view of the firm, the normalisation of equation (46) looks more natural. To take into account possible simultaneity bias, the endogenous variables of the system (*Y/N, K/N* and *p/w*) were also instrumented with their lagged values and with the exogenous variables of the system (*w/c*, P_f/P^x and *t*), and thereafter the system was estimated with the method of multivariate non-linear three-stage least squares. However, no simultaneous equation bias was found. The use of instrument variables had practically no effect on the parameter estimates.[24]

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23 See the literature mentioned in footnote 2.

24 Nor did the instrumentation of variables *w/c* and P_f/P^x have any impact of practical importance on the parameter estimates. These results are in line with experience with linear models, where a frequent finding is that, even though least square estimators are not consistent, they usually turn out to be quite close to consistent estimators in practice (see Christ (1994)).

5 Potential output and the output gap

In this section, we present output gap estimates based on an alternative parameterisation of the production function. The aim is to gain a feeling for the sensitivity of output gap estimates with respect to the functional form and alternative parameterisations of the production function. Also the importance of the HP filter-smoothed TFP for output gap calculations is illustrated. For these comparisons, the quality of the estimate of the equilibrium unemployment rate (or NAIRU) does not play any crucial role, because it is the same in all alternatives. However, it is needed to be able to calculate potential output and the output gap estimates. We use the NAIRU estimate of the ECB area-wide model (see Fabiani and Mestre (2000)).

Our estimates of potential output *YPOT* are based on the following estimated production functions presented in Tables 3.1 to 3.3 and 4.

Cobb-Douglas:

(48) $$YPOT = 1.52e^{0.0034 \cdot t} K^{0.29} \left((1 - NAIRU) LFN \right)^{1-0.29} \qquad ; \sigma = 1$$

CES with Harrod-neutral technological development:

(49) $$YPOT = 0.143 \cdot \left[0.998 \left(e^{0.0048 \cdot t} K \right)^{-1.67} + (1 - 0.998) \left((1 - NAIRU) LFN \right)^{-1.67} \right]^{-\frac{1}{1.67}} \quad ; \sigma = 0.37$$

CES with Hicks-neutral technological development:

(50) $$YPOT = 2.55 \cdot e^{0.0034 \cdot t} \left[0.098 \ (K)^{0.28} + (1 - 0.098) \left((1 - NAIRU)(LFN)^{0.28} \right) \right]^{\frac{1}{0.28}} \quad ; \sigma = 1.39$$

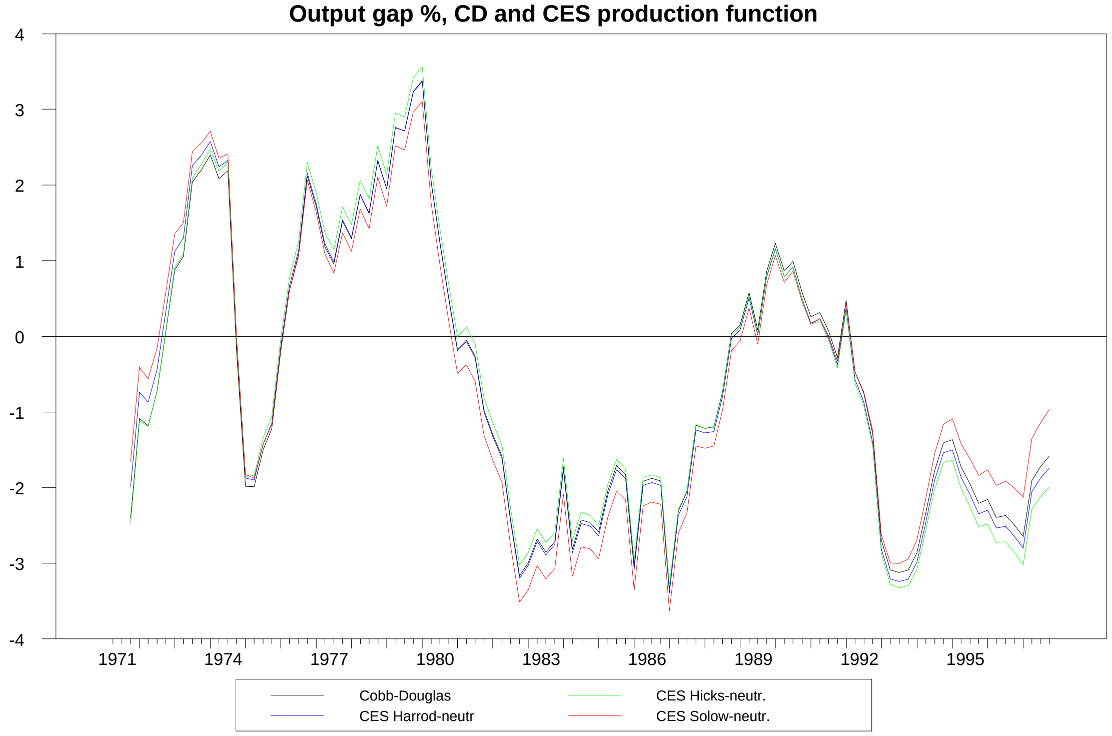
CES with Solow-neutral technological development:

(51) $$YPOT = 1.02 \cdot \left[0.44 (K)^{-0.12} + (1 - 0.44) \left(e^{0.0111 \cdot t} (1 - NAIRU) LFN \right)^{-0.12} \right]^{-\frac{1}{0.12}} \quad ; \sigma = 0.90$$

where LFN is the labour force, equalling to the sum of employed and unemployed persons. Besides the elasticity of substitution varying in quite wide range (from 0.37 to 1.39), the distribution parameter *b* of the CES function also varies from 0.098 in the Hicks-neutral case to 0.998 in the Harrod-neutral case. Hence, we may conclude that these production functions cover quite large part of the reasonable range for parameterisation of the empirical production functions.

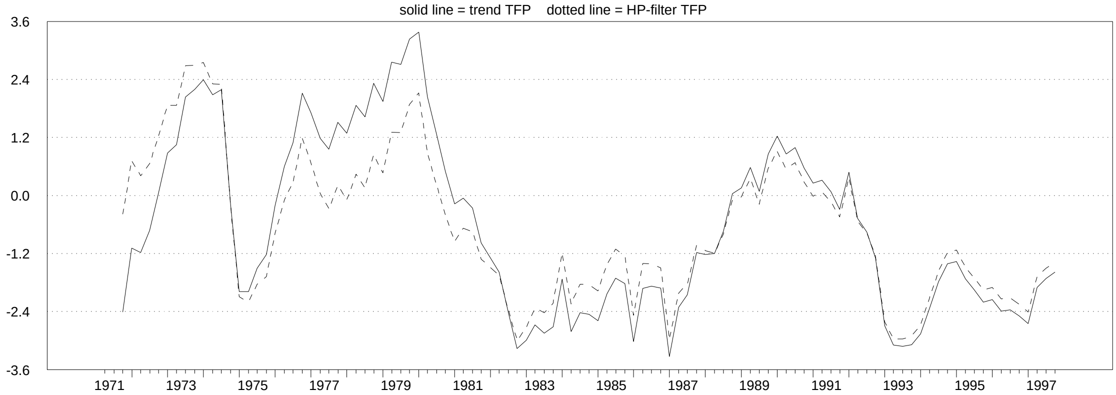
The main conclusion, based on Figure 8, is that the capacity utilisation estimates are quite insensitive with respect to the parameterisation of the production function. At the end of the sample period, where differences are widest, all output gap estimates are within the range of 1 percentage point. A counterintuitive result is that the output gap estimate based on the CES production function with the Solow-neutral technological development deviates most from the output gap estimate based on the Cobb-Douglas production function. This is surprising because, with a close-to-unit estimate for the elasticity of substitution, it was closest to the Cobb-Douglas specification and, hence, one would expect that it would also give the output gap estimate closest to that of the Cobb-Douglas function. The output gap estimate of the Harrod-neutral CES function, in turn, the parameterisation of which deviates most from the Cobb-Douglas function, is closest to the Cobb-Douglas output gap estimate and practically coincides with it.

Figure 8



In Figure 9, the estimates of the output gap are compared, when technological progress is explained by trend or by HP filter-smoothed TFP. As expected, the use of the HP filter has the effect of smoothing somewhat the variation in the estimates of the output gap. Otherwise these estimates provide very similar paths for the estimated gaps.

Figure 9. Output gap %, CD-production function



6 Conclusions

In this paper we estimated the three-equation system determining the long-run aggregated supply of the euro area economy. We also produced estimates for potential output and the output gap conditional on alternative estimated production functions and the equilibrium unemployment rate (or NAIRU). In this context, our main interest was in the sensitivity of output gap estimates to alternative parameterisations of the production function.

We found that CES-technology coupled with the constant mark-up was not able to explain two stylised facts in the aggregated euro area data, i.e. the hump-shaped profile of the labour income share and the level shift in the ratio of capital to labour income in the late 1970s and early 1980s. However, after modifying the conventional single-good framework to fit the aggregated multi-sector framework and after allowing sectorally differentiated income elasticities and time-variant price elasticities, as implied by the AIDS demand function, we could explain the hump-shape in the labour income share by factors determining the mark-up. We showed that high income-elasticity coupled with low price elasticity and/or low technological progress, characteristics of the services sector, introduces a positive trend into the aggregated level mark-up. In line with this reasoning, we showed that the output share of the services sector in the euro area increased steadily through the whole sample period from 1970 to 1997.

Defining the functional form of demand as that of AIDS, in turn, allowed us to introduce the ratio of export prices to competing foreign prices to explain the mark-up behaviour in the open sectors of the economy. In the 1970s, as a response to the oil price shocks, that price ratio rose significantly, implying – at the aggregate level – a decrease in the estimated mark-up and an increase in the labour income share.

However, this effect was not permanent as, by the middle of the 1980s, the ratio of export prices to competing foreign prices returned back to “normal”, equalling the average of the period from the first quarter of 1986 to the fourth quarter of 1997.

The level shift in the capital-to-labour income ratio reflected the corresponding level shift in the real interest rate of the euro area. We tentatively argued that, due to the regulated financial markets and credit rationing, the real interest rate did not correctly measure the effective marginal costs of financing in the 1970s. Along these lines of thinking, a level shift dummy was included by our estimated systems to scale up of the real interest rate in the 1970s. Nonetheless, the inclusion of the dummy proved to be necessary for the robustness of the estimation results.

Estimation results based on the CES specification of the production function proved to be very sensitive with respect to the nature of technological progress. Estimation results were the most robust when technological progress was specified as Solow-neutral. In this case, the estimates of the elasticity of substitution parameter were close to unity, indicating that the Cobb-Douglas function is a good approximation for the euro area production function. Hence, the hump-shaped labour income share reflects variations in the mark-up, which are not transmitted to the capital-to-labour income ratio.

Cobb-Douglas estimation results were also very robust with respect to alternative normalisation of estimated equations and to alternative ways of measuring productivity. In addition, no simultaneity bias was found. Also the use of trend or HP filter-smoothed total factor productivity as a technological progress variable had hardly any effects on the estimated parameters and they gave quite similar estimates for the output gap. We also found that, at least in the euro area data, output gap estimates are quite insensitive with respect to alternative parameterisations and functional forms of the underlying production function. Hence, the principal source of the measurement error of the output gap, when the production function approach is applied, is the estimate of the NAIRU.

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TABLE 3.1 – PARAMETER ESTIMATES OF THE SUPPLY SYSTEM: The CES production function and Harrod-neutral technological progress ($\gamma_N = \gamma_A$ or $\gamma_N = \Gamma$ and $\gamma_K = 0$)										
	1		2		3		4		5	
	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4
A	**0.566** (1.5)	**4.01** (12.7)	**0.143** (9.5)	**3.18** (5.3)	**0.142** (10.5)	**2.78** (4.7)	**0.196** (4.6)	**3.98** (12.8)	**0.177** (6.5)	**3.68** (10.7)
γ_N	**0.0051** (52.2)	**0.0045** (78.2)	**0.0048** (46.5)	**0.0048** (47.5)	**0.0048** (44.5)	**0.0049** (56.1)	**0.0048** (43.4)	**0.0048** (51.2)	**0.0048** (43.7)	**0.0049** (60.7)
b	**0.736** (2.8)	**0.004** (0.9)	**0.998** (582.9)	**0.031** (0.9)	**0.998** (678.0)	**0.060** (1.1)	**0.985** (58.9)	**0.003** (0.8)	**0.991** (121.5)	**0.009** (1.0)
ρ	**0.44** (1.4)	**-1.04** (-3.9)	**1.67** (6.9)	**-0.59** (-2.2)	**1.68** (7.5)	**-0.43** (-1.9)	**1.15** (4.5)	**-1.10** (-3.8)	**1.29** (5.9)	**-0.88** (-3.7)
σ	**0.69**	**Infinite[1]**	**0.37**	**2.43**	**0.37**	**1.75**	**0.46**	**Infinite[1]**	**0.44**	**8.6**
h	**0.024** (28.6)		**0.029** (23.5)		**0.029** (22.3)		**0.027** (29.0)		**0.027** (25.7)	
$1+\bar{\mu}_A$	**1.009** (89.4)	**1.035** (117.5)	**0.997** (132.4)	**1.004** (89.9)	**0.992** (140.9)	**1.004** (114.4)	**1.014** (183.1)	**1.015** (123.6)	**1.006** (171.4)	**1.012** (146.1)
Φ			**1.46** (7.0)	**1.41** (3.4)			**0.92** (4.7)	**0.92** (2.4)		
η					**0.0015** (9.5)	**0.0012** (6.4)			**0.0011** (6.3)	**0.0009** (5.6)
χ							**0.064** (7.2)	**0.071** (4.4)	**0.046** (6.0)	**0.057** (3.6)
SEE_Y / **ADF_Y**	**0.014** / **-3.0**	**0.009** / **-2.6**	**0.012** / **-3.8**	**0.007** / **-3.2**	**0.012** / **-3.9**	**0.007** / **-3.2**	**0.012** / **-3.9**	**0.007** / **-3.2**	**0.012** / **-3.9**	**0.007** / **-3.2**
$SEE_{c/w}$ / **$ADF_{c/w}$**	**0.039** / **-4.7**	**0.043** / **-3.5**	**0.041** / **-3.9**	**0.041** / **-3.6**	**0.041** / **-3.8**	**0.041** / **-3.6**	**0.038** / **-4.4**	**0.043** / **-3.2**	**0.039** / **-4.3**	**0.042** / **-3.5**
SEE_P / **ADF_P**	**0.045** / **-0.6**	**0.016** / **-2.3**	**0.023** / **-2.1**	**0.013** / **-1.7**	**0.021** / **-2.7**	**0.010** / **-2.1**	**0.017** / **-2.2**	**0.011** / **-2.6**	**0.016** / **-2.7**	**0.009** / **-2.2**

1. Infinite, if the estimates -1.04 and -1.1 for ρ are interpreted to support ρ = -1. Otherwise indeterminate, as the values of ρ <-1 are not in the legitimate regime.

TABLE 3.2 – PARAMETER ESTIMATES OF THE SUPPLY SYSTEM: The CES production function and Hicks-neutral technological progress ($\gamma_N = \gamma_K$)										
	1		2		3		4		5	
	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4
A	**2.55** (5.9)	**3.42** (19.6)	**1.06** (2.5)	**2.45** (4.6)	**0.614** (2.2)	**1.91** (3.7)	**1.08** (2.5)	**2.78** (5.5)	**0.408** (3.2)	**2.05** (3.8)
γ_K, Γ	**0.0035** (43.9)	**0.0035** (66.1)	**0.0034** (47.1)	**0.0035** (58.1)	**0.0033** (50.5)	**0.0035** (57.1)	**0.0034** (47.1)	**0.0035** (59.8)	**0.0033** (47.1)	**0.0035** (54.7)
b	**0.098** (2.0)	**0.029** (3.1)	**0.448** (2.5)	**0.110** (1.6)	**0.678** (3.9)	**0.196** (1.9)	**0.442** (2.5)	**0.074** (1.5)	**0.820** (8.7)	**0.171** (1.8)
ρ	**-0.28** (2.4)	**-0.55** (-7.8)	**0.14** (0.9)	**-0.25** (-1.8)	**0.34** (2.1)	**-0.11** (-0.8)	**0.14** (0.9)	**-0.35** (-2.4)	**0.50** (3.7)	**-0.16** (-1.0)
σ	**1.39**	**2.23**	**0.88**	**1.34**	**0.75**	**1.12**	**0.88**	**1.53**	**0.66**	**1.17**
h	**0.020** (20.0)		**0.025** (14.5)		**0.028** (12.9)		**0.025** (14.4)		**0.031** (15.2)	
$1+\overline{\mu}_A$	**1.022** (101.3)	**1.036** (117.1)	**1.006** (107.2)	**1.013** (70.5)	**0.994** (106.2)	**1.008** (92.9)	**1.023** (137.3)	**1.024** (82.4)	**1.000** (120.4)	**1.013** (99.3)
Φ			**1.01** (3.6)	**0.97** (1.7)			**0.56** (1.8)	**0.52** (0.9)		
η					**0.0014** (5.8)	**0.0010** (3.3)			**0.0014** (5.3)	**0.0008** (2.6)
χ							**0.087** (8.5)	**0.049** (3.2)	**0.066** (7.6)	**0.036** (2.0)
SEE_Y ADF_Y	**0.013** **-3.7**	**0.007** **-3.1**	**0.012** **-3.9**	**0.007** **-3.2**	**0.012** **-3.9**	**0.007** **-3.2**	**0.012** **-3.7**	**0.007** **-3.1**	**0.012** **-3.9**	**0.007** **-3.2**
$SEE_{c/w}$ $ADF_{c/w}$	**0.046** **-4.3**	**0.049** **-2.8**	**0.040** **-4.7**	**0.041** **-3.7**	**0.042** **-4.2**	**0.040** **-3.8**	**0.040** **-4.5**	**0.043** **-3.4**	**0.045** **-3.7**	**0.040** **-3.8**
SEE_P ADF_P	**0.035** **-2.4**	**0.012** **-2.0**	**0.032** **-3.1**	**0.012** **-1.7**	**0.027** **-3.6**	**0.011** **-2.0**	**0.018** **-2.6**	**0.010** **-1.6**	**0.015** **-3.2**	**0.010** **-1.9**

TABLE 3.3 – PARAMETER ESTIMATES OF THE SUPPLY SYSTEM: The CES production function and Solow-neutral technological progress ($\gamma_N = \gamma_A = 0$ or $\gamma_N = \Gamma = 0$)										
	1		2		3		4		5	
	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4
A	**2.00** (11.0)	**2.43** (19.7)	**1.45** (7.7)	**1.76** (7.1)	**1.28** (6.5)	**1.54** (6.9)	**1.32** (6.5)	**1.77** (7.0)	**1.02** (5.7)	**1.53** (6.9)
γ_K	**0.0123** (42.9)	**0.0119** (53.9)	**0.0116** (41.5)	**0.0119** (43.0)	**0.0114** (41.5)	**0.0119** (40.0)	**0.0113** (42.6)	**0.0119** (44.2)	**0.0111** (36.3)	**0.0118** (40.0)
b	**0.195** (7.0)	**0.139** (9.5)	**0.308** (6.5)	**0.240** (5.1)	**0.354** (6.2)	**0.285** (5.6)	**0.345** (6.0)	**0.237** (5.1)	**0.444** (6.7)	**0.287** (5.6)
ρ	**-0.09** (-.3.0)	**-0.163** (-7.6)	**0.01** (0.3)	**-0.05** (-1.1)	**0.05** (1.1)	**-0.01** (-0.1)	**0.04** (0.9)	**-0.05** (-1.1)	**0.17** (2.4)	**-0.00** (-0.1)
σ	**1.10**	**1.19**	**0.99**	**1.05**	**0.95**	**1.00**	**0.96**	**1.05**	**0.90**	**1.00**
h	**0.020** (25.1)		**0.024** (19.0)		**0.025** (17.1)		**0.025** (17.3)		**0.028** (15.7)	
$1+\overline{\mu}_A$	**1.021** (105.7)	**1.036** (117.9)	**1.010** (116.8)	**1.008** (72.2)	**1.002** (115.3)	**1.004** (90.2)	**1.025** (163.5)	**1.014** (84.0)	**1.011** (135.8)	**1.008** (96.3)
Φ			**0.83** (3.1)	**1.20** (2.1)			**0.52** (1.9)	**0.97** (1.8)		
η					**0.0011** (4.8)	**0.0012** (4.0)			**0.0010** (4.1)	**0.0011** (3.6)
χ							**0.089** (8.7)	**0.041** (2.6)	**0.077** (8.2)	**0.032** (2.0)
SEE$_Y$ **ADF$_Y$**	**0.014** **-3.5**	**0.008** **-3.0**	**0.012** **-3.9**	**0.007** **-3.1**	**0.012** **-3.9**	**0.007** **-3.2**	**0.012** **-3.9**	**0.007** **-3.2**	**0.012** **-3.7**	**0.007** **-3.2**
SEE$_{c/w}$ **ADF$_{c/w}$**	**0.045** **-4.3**	**0.048** **-2.9**	**0.041** **-4.8**	**0.040** **-3.8**	**0.041** **-4.7**	**0.040** **-3.7**	**0.041** **-4.7**	**0.040** **-3.8**	**0.043** **-4.2**	**0.040** **-3.7**
SEE$_P$ **ADF$_P$**	**0.035** **-2.2**	**0.013** **-1.7**	**0.032** **-3.3**	**0.012** **-1.7**	**0.029** **-3.9**	**0.011** **-2.0**	**0.018** **-2.7**	**0.011** **-1.3**	**0.015** **-3.3**	**0.010** **-1.8**

TABLE 4 – PARAMETER ESTIMATES OF THE SUPPLY SYSTEM: The Cobb-Douglas production function										
	1		2		3		4		5	
	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4	71:2-97:4	83:1-97:4
A	**1.52** (65.7)	**1.55** (80.5)	**1.52** (67.5)	**1.51** (73.4)	**1.52** (67.4)	**1.51** (75.3)	**1.52** (67.8)	**1.51** (73.0)	**1.52** (67.9)	**1.51** (75.2)
γ_A, Γ	**0.0035** (54.3)	**0.0031** (43.5)	**0.0034** (41.3)	**0.0034** (42.5)	**0.0034** (41.0)	**0.0035** (44.4)	**0.0034** (40.1)	**0.0034** (42.6)	**0.0034** (41.3)	**0.0035** (49.3)
β	**0.291** (98.4)	**0.292** (95.4)	**0.292** (100.8)	**0.292** (95.4)	**0.293** (100.7)	**0.292** (95.4)	**0.293** (102.9)	**0.292** (95.1)	**0.293** (102.5)	**0.292** (95.0)
h	**0.023** (28.8)		**0.023** (26.8)		**0.023** (26.9)		**0.023** (27.3)		**0.023** (27.4)	
$1+\overline{\mu}_A$	**1.013** (100.4)	**1.035** (117.4)	**1.012** (121.5)	**1.001** (103.2)	**1.008** (123.1)	**1.003** (121.0)	**1.029** (194.9)	**1.006** (120.4)	**1.025** (182.6)	**1.007** (128.7)
Φ			**0.76** (2.9)	**1.55** (4.4)			**0.30** (2.1)	**1.31** (3.7)		
η					**0.0008** (3.7)	**0.0012** (7.9)			**0.0004** (2.8)	**0.0011** (7.2)
χ							**0.087** (8.0)	**0.042** (2.5)	**0.077** (6.5)	**0.033** (2.0)
SEE_Y **ADF_Y**	**0.013** **-3.4**	**0.009** **-2.5**	**0.012** **-3.9**	**0.007** **-3.2**	**0.012** **-3.6**	**0.007** **-3.2**	**0.012** **-3.9**	**0.007** **-3.2**	**0.012** **-3.9**	**0.007** **-3.2**
$SEE_{c/w}$ **$ADF_{c/w}$**	**0.042** **-4.6**	**0.040** **-3.7**	**0.041** **-4.8**	**0.040** **-3.7**	**0.041** **-4.8**	**0.040** **-3.7**	**0.041** **-4.8**	**0.040** **-3.7**	**0.041** **-4.8**	**0.040** **-3.7**
SEE_P **ADF_P**	**0.042** **-0.8**	**0.018** **-1.3**	**0.032** **-3.3**	**0.013** **-1.8**	**0.030** **-4.1**	**0.011** **-2.0**	**0.019** **-2.8**	**0.012** **-1.6**	**0.018** **-3.2**	**0.010** **-1.8**

Appendix One: Proof of equations (12) to (15)

PROOF OF (12):

Equation (1) and $f\left(k^{j}\right)=f(k)$ imply:

(A12.1) $N^{j}=Y^{j}\left[A^{j}e^{\gamma^{j}\cdot t}f(k)\right]^{-1}$ $\Rightarrow$ (A12.2) $N=\sum_{j=1}^{m}N^{j}=\left[f(k)\right]^{-1}\sum_{j=1}^{m}Y^{j}A^{j^{-1}}e^{-\gamma^{j}t}$

Now

(A12.3) $$\frac{Y}{N}=\frac{f(k)Y}{\sum_{j=1}^{m}Y^{j}A^{j^{-1}}e^{-\gamma^{j}\cdot t}}=f(k)\left[\sum_{j=1}^{m}\frac{Y^{j}}{Y}A^{j^{-1}}e^{-\gamma^{j}\cdot t}\right]^{-1}\ \blacksquare$$

PROOF OF (13):

Equation (13) follows straightforwardly from (4) and (11). ■

PROOF OF (14):

In (12) denote,

(A14.1) $$\log\left[\left(\sum_{j=1}^{m}s_{t}^{j}A^{j^{-1}}e^{-\gamma^{j}\cdot t}\right)^{-1}\right]=h\left(s_{t}^{j},t\right)$$

$$\approx h\left(s_{0}^{j},0\right)+\frac{\partial h\left(s_{0}^{j},0\right)}{\partial t}t+\sum_{j=1}^{m}\frac{\partial h\left(s_{0}^{j},0\right)}{\partial s^{j}}\left(s_{t}^{j}-s_{0}^{j}\right)$$

where

(A14.2) $$h\left(s_{0}^{j},0\right)=\log\left[\left(\sum_{j=0}^{m}s_{0}^{j}A^{j^{-1}}\right)^{-1}\right]=\log A$$

(A14.3) $$\frac{\partial h\left(s_{0}^{j},0\right)}{\partial t}t=\sum_{j=1}^{m}\frac{s_{0}^{j}A^{j^{-1}}}{\sum_{j=1}^{m}s_{0}^{j}A^{j^{-1}}}\gamma^{j}t=\sum_{j=1}^{m}AA^{j^{-1}}s_{0}^{j}\gamma^{j}=\gamma_{A}\cdot t\ \blacksquare$$

(A14.4) $$\sum\frac{\partial h\left(s_{0}^{j},0\right)}{\partial s^{j}}\left(s_{t}^{j}-s_{0}^{j}\right)=\frac{-\sum_{j=1}^{m}A^{j^{-1}}\left(s_{t}^{j}-s_{0}^{j}\right)}{\sum_{j=1}^{m}s_{0}^{j}A^{j^{-1}}}=-\sum_{j=1}^{m}AA^{j^{-1}}\left(s_{t}^{j}-s_{0}^{j}\right)$$

PROOF OF (15):

In (13) denote

(A15.1) $\log\left(\sum_{j=1}^{m} s_t^j A^{j-1}\left(1+\mu^j\right)e^{-\gamma^j t}\right) = u\left(s_t^j, t\right)$

$$\approx u\left(s_0^j,0\right)+\frac{\partial u\left(s_0^j,0\right)}{\partial t}+\sum_{j=1}^{m}\frac{\partial u\left(s_0^j,0\right)}{\partial s^j}\left(s_t^j-s_0^j\right)t$$

where

(A15.2) $u\left(s_0^j,0\right)=\log\left(\sum_{j=1}^{m} A^{j-1}s_0^j\left(1+\mu^j\right)\right)=\log\left(\underbrace{\sum_{j=1}^{m} AA^{j-1}s_0^j\left(1+\mu^j\right)}_{=1+\mu_A}\right)-\log A$

(A15.3) $\frac{\partial u\left(s_0^j,0\right)}{\partial t}t=\sum_{j=1}^{m}\frac{AA^{j-1}s_0^j\left(1+\mu^j\right)\left(-\gamma^j\right)}{\sum_{j=1}^{m} AA^{j-1}s_0^j\left(1+\mu^j\right)}t=-\left(\sum_{j=1}^{m} b_j\gamma^j\right)\cdot t=-\left(\gamma_A+\sum_{j=1}^{m} b_j\left(\gamma^j-\gamma_A\right)\right)\cdot t$

where $\sum_j b_j = 1$

(A15.4) $\sum_{j=1}^{m}\frac{\partial u\left(s_0^j,0\right)}{\partial s^j}\left(s_t^j-s_0^j\right)=\frac{\sum_{j=1}^{m} AA^{j-1}\left(1+\mu^j\right)\left(s_t^j-s_0^j\right)}{\underbrace{\sum_{j=1}^{m} AA^{j-1}s_0^j\left(1+\mu^j\right)}_{1+\mu_A}}$

$$=\sum_{j=1}^{m} AA^{j-1}\left(s_t^j-s_0^j\right)+\sum_{j=1}^{m}\frac{AA^{j-1}\left(\mu^j-\mu_A\right)}{1+\mu_A}\left(s_t^j-s_0^j\right)$$

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