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COLLATERAL AMPLIFICATION UNDER COMPLETE MARKETS

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In 2014 all ECB publications feature a motif taken from the €20 banknote.

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Abstract

This paper examines the robustness of the Kiyotaki-Moore collateral amplification mechanism to the existence of complete markets for aggregate risk. We show that, when borrowers can hedge against aggregate shocks at fair prices, the volatility of endogenous variables becomes identical to the first best in the absence of credit constraints. The collateral amplification mechanism disappears.

To motivate the limited use of contingent contracts, we introduce costs of issuing contingent debt and calibrate them to match the liquidity and safety premia the data. We find that realistic costs of state contingent market participation can rationalize the predominant use of uncontingent debt. Amplification is restored in such an environment.

JEL Classification: E32, D52.

Key Words: Collateral constraints, Amplification.

Non-technical summary

This paper introduces markets which help borrowers insure against aggregate shocks into the standard model of collateral amplification due to Kiyotaki and Moore (1997). Collateral amplification works because borrowers' assets are risky while their debt is risk-free. Consequently, negative aggregate shocks reduce borrowers' asset values and net worth, forcing them to deleverage. This exerts downward pressure on economic activity, amplifying the economic cycle.

My paper shows that costless access to insurance markets can completely neutralise this mechanism by helping to insulate the net worth of borrowers from asset price fluctuations. This presents a puzzle: if accessing insurance markets is so beneficial why don't most firms do it? In order to motivate the absence of hedging by many firms the paper introduces small costs to accessing insurance markets and calibrates these to match the safety premia found in the empirical literature. I find that even relatively small costs of accessing insurance markets can discourage their use sufficiently to restore the functioning of the collateral amplification mechanism.

The main policy implication of the paper is that, despite being often blamed for causing financial crises, some types of financial innovation can be helpful in improving economic stability. In particular, policies that reduce the costs of hedging risks for credit constrained firms (for example small and medium sized enterprises) could be very helpful in terms of reducing their contribution to business cycle volatility.

1 Introduction

The Kiyotaki and Moore (1997) model is one of the leading macro models of collateral amplification. This model crucially relies on the assumption of incomplete markets for aggregate and idiosyncratic risk. Such an assumption is consistent with the facts: most firm liabilities in the US are uncontingent. According to the US Flow of Funds, for non-financial corporate business, debt liabilities in 2012 were around half of all liabilities (equity plus debt). Non-financial, non-corporate business (with capital stock equal to 60% of the capital stock held by corporate business) funds itself entirely with debt liabilities.

The widespread use of uncontingent debt is a challenge for the micro-foundations of the collateral amplification model. While incomplete markets for idiosyncratic risk can be derived from first principles (Cole and Kocherlakota (2001)), the hedging of aggregate shocks is harder to rule out. A natural question then arises: what can explain the high degree of exposure to aggregate risk by US businesses?

This paper examines the causes and consequences of incomplete hedging against aggregate risk by introducing debt securities which are contingent on the aggregate state of the economy into the Kiyotaki (1998) version of the Kiyotaki and Moore (1997) model (hereafter referred to as the KM model). Crucially we assume that such securities may only be issued after paying a proportional transaction cost. In contrast, markets for simple debt can be accessed costlessly. We think of this transaction cost as a simple way to introduce the kinds of liquidity and safety premia identified in the empirical work of Krishnamurthy and Vissing-Jorgensen (2012). The idea is that issuing safe and standardized securities (such as default free corporate bonds) can be done cheaply while issuing more complex securities (such as high yield corporate bonds) is costly in line with the empirical evidence.[1]

[1] There may be many reasons why such liquidity and safety premia arise in reality. For example, liquidity premia on complex assets could exist due to the need to find buyers who understand such securities. Alternatively, such premia could be generated by assuming that only a small subset of households participate in risky asset markets.

All these modelling approaches ensure that hedging by issuing risky securities is a lot more costly compared to issuing riskless bonds only. Our approach (which is based on proportional transactions costs) models the high cost of contingent securities in a very simple and tractable way.

The paper asks two main questions. (i) How does the strength of the collateral amplification mechanism vary with the cost and availability of state contingent debt? (ii) Can empirically realistic costs of issuing risky and non-standard assets motivate the predominant use of simple debt by firms?

In line with the theoretical results of Krishnamurthy (2003), we find that costless hedging of aggregate shocks completely eliminates the collateral amplification mechanism. Our quantitative business cycle version of the collateral model allows us to analyze the way complete markets affect the model properties. Second moments become identical to those in a frictionless version of the model even if first moments remain distorted by the limited commitment problem which stops borrowers from issuing unlimited liabilities relative to the value of their collateral.

The model's second moments go to their 'first best' values because contingent debt securities insulate the net worth of borrowing agents from fluctuations in collateral values. Consequently the wealth distribution is constant over time and independent of aggregate shocks. When the interaction between asset prices and the wealth distribution disappears, the KM collateral amplification mechanism stops operating.

To answer the second question in our paper, we introduce costs of accessing state contingent debt markets and analyze how quickly the use of hedging instruments declines and the collateral amplification mechanism is restored as the cost of state contingency rises. Our approach is motivated by the growing finance literature (Longstaff, Mithal and Neis (2005), Krishnamurthy and Vissing-Jorgensen (2012) as well as others) which has shown that investors like liquid and safe securities and are willing to pay a substantial premium for them. To keep the analysis simple, we do not model liquidity or safety premia explicitly but introduce proportional transaction costs which are calibrated to match such premia in the data.

We find that empirically realistic transactions costs discourage firms sufficiently from hedging and aggregate shocks start to affect the wealth distribution. This brings the behavior of the model very close to that of the standard Kiyotaki (1998) model with only uncontingent debt.

The rest of the paper is organized as follows. Section 2 outlines the model environment.

Section 3 outlines the competitive equilibrium for our model economy. Section 4 outlines the baseline calibration. Section 5 examines the macroeconomic impact of different levels of the transaction cost needed in order to access financial markets. Finally, Section 6 concludes.

2 The Model

2.1 The Economic Environment

2.1.1 Population and Production Technology

The economy is populated with a continuum of infinitely lived entrepreneurs and a continuum of infinitely lived workers - both of measure 1. Each entrepreneur is endowed with a constant returns to scale production function which uses land k_{t-1}, intermediate inputs x_{t-1} and labour h_{t-1} to produce output y_t.[2] The production function is like in Aoki, Benigno and Kiyotaki (2009b) where the timing of inputs reflects the fact that inputs must be paid for one period in advance of the realization of production.

$$y_t = a_t A_t \left(\frac{k_{t-1}}{\alpha}\right)^{\alpha} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \left(\frac{h_{t-1}}{1-\alpha-\eta}\right)^{1-\alpha-\eta} \tag{1}$$

a_t is the idiosyncratic component of productivity which is revealed to the entrepreneur one period in advance and can be high a^H or low a^L. Following Kiyotaki (1998), the idiosyncratic state evolves according to a Markov process with the following transition matrix.

$$\begin{array}{ccc} & a^H & a^L \\ a^H & 1-\delta & \delta \\ a^L & n\delta & 1-n\delta \end{array}$$

where $n\delta$ is the probability that a currently unproductive firm becomes productive and δ is the probability that a currently productive firm becomes unproductive. This implies that the steady state ratio of productive to unproductive firms is n.

[2] The inclusion of intermediate inputs into the production function is done for two reasons. First, such inputs are present in the data (see section 4 for more discussion). But, secondly, their inclusion helps the model fit the ratio of the value of tangible assets to GDP in the data.

A_t is the aggregate component of productivity which also evolves according to the following Markov process

$$\begin{array}{ccc} & A^H & A^L \\ A^H & \pi & 1-\pi \\ A^L & 1-\pi & \pi \end{array}$$

where π is the probability that the aggregate technology state persists in the following period. The realization of the aggregate state A_t occurs at the beginning of time t.

Land is fixed in aggregate supply and its total quantity is normalized to unity.

2.2 Entrepreneurs

2.2.1 Preferences

Entrepreneurs are ex-ante identical and have logarithmic utility over consumption streams

$$U^E = E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t \tag{2}$$

2.2.2 Flow of Funds

Entrepreneurs purchase consumption (c_t), land (k_t) at price q_t, intermediate inputs (x_t) and labour (h_t) at wage w_t. All inputs are chosen a period in advance. When the market for aggregate risk is complete, entrepreneurs borrow using Arrow-Debreu securities $b_t(A^s)$ whose payoffs are contingent on the realization of the aggregate state of TFP (denoted by A^s, $s = H, L$) at time $t+1$. These securities trade at price $p_t(A^s)$ in terms of goods today.

$$c_t + x_t + w_t h_t + q_t k_t - \sum_{A^s} p_t(A^s)\, b_t(A^s) = y_t + q_t k_{t-1} - b_{t-1} \equiv z_t \tag{3}$$

The current wealth of the entrepreneur (z_t) consists of the revenues from productive projects, the value of land holdings minus repayments of state contingent debt.

In the standard version of the Kiyotaki-Moore economy entrepreneurs borrow using simple debt securities:

$$c_t + x_t + w_t h_t + q_t k_t - \frac{b_t}{R_t} = y_t + q_t k_{t-1} - b_{t-1} \tag{4}$$

where R_t is the risk-free real interest rate. We assume an environment in which agents are anonymous and in which idiosyncratic shocks are private information. Therefore securities contingent on the realization of the idiosyncratic state do not trade in equilibrium.

2.2.3 Collateral constraints

We assume limited commitment in the credit market. Borrowers can refuse to repay their debts and the only punishment is that their land holdings can be seized by creditors. We also assume that entrepreneurs only have the opportunity to default before the aggregate shock has been realized.[3]

Hence the collateral constraint limits the ex ante value of an entrepreneur's debt to the ex ante value of collateral:

$$\sum_{A^s} p_t\left(A^s\right) b_t\left(A^s\right) \leqslant \sum_{A^s} p_t\left(A^s\right) q_{t+1}\left(A^s\right) k_t \tag{5}$$

where $q_{t+1}\left(A^s\right)$ is the price of land at time $t+1$ when the aggregate state of TFP is A^s. Both the assets and liabilities of the entrepreneur are evaluated at Arrow Debreu prices for the purposes of the collateral constraint[4].

In the standard version of the Kiyotaki-Moore economy, (5) becomes:

$$b_t \leqslant \sum_{A^s} \pi\left(A^s|A_t\right) q_{t+1}\left(A^s\right) k_t \tag{6}$$

where $\pi\left(A^s|A_t\right)$ is the probability that TFP state A^s realizes at $t+1$ conditional on the current value of TFP A_t.

[3] This is an assumption which is often made in the literature on collateral constraints. It allows the analysis of the effect of collateral constraints without the complications of characterising defaults. A full analysis of default in this framework would be an interesting avenue for future research.

[4] When entrepreneurs are risk neutral, the collateral constraint boils down to a condition whereby expected repayments cannot exceed the expected value of collateral.

2.3 Workers

2.3.1 Preferences

Workers have the following preferences:

$$U^W = E_0 \sum_{t=0}^{\infty} \beta^t \left(c_t^w - \varkappa \frac{(h_t^w)^{1+\omega}}{1+\omega} \right) \tag{7}$$

where c_t^w is household consumption and h_t^w is labour supply. ω is the inverse of the Frisch elasticity of labour supply while $\varkappa$ is a labour supply shift parameter.[5]

2.3.2 Flow of Funds

Workers do not have the opportunity to produce. They purchase consumption (c_t^w) and save using Arrow securities $b_t^w(A^s)$ at price $p_t(A^s)$. Their net worth consists of labour income ($w_t h_t^w$) and returns from Arrow security investments b_{t-1}^w.

$$c_t^w + \sum_{A^s} p_t(A^s)\, b_t^w(A^s) = w_t h_t^w + b_{t-1}^w \tag{8}$$

When debt is uncontingent, the flow of funds becomes:

$$c_t^w + \frac{b_t^w}{R_t} = w_t h_t^w + b_{t-1}^w \tag{9}$$

2.3.3 Collateral constraints

Workers cannot short-sell any of the Arrow securities:

$$b_t^w(A^s) \geqslant 0 \tag{10}$$

3 Competitive Equilibrium

3.1 Entrepreneurial behavior

Kiyotaki and Moore (1997) show that the entrepreneur's idiosyncratic productivity is a key state variable for individual choices. Therefore we will adopt a notation which makes this

[5] Hereafter, we index workers' choices using a w superscript.

explicit from the outset. For example, let c_t^H and c_t^L denote, respectively, the current consumption choices of high and low productivity entrepreneurs. In the case of future dated stochastic variables we also add an index of the future realization of the stochastic aggregate technology state A^s $(s = H, L)$. For example, this means that $c_{t+1}^i(A^s)$ denotes the consumption choice at time $t+1$ of an entrepreneur of type i $(i = H, L)$ when the aggregate state is A^s.[6]

3.1.1 Optimal consumption

Following Sargent (1987) we can express the problem of entrepreneurs as a consumption problem with uncertain returns. Consequently, the log utility assumption ensures that consumption is always a fixed fraction of wealth that depends upon the discount factor.

$$c_t^i = (1-\beta) z_t^i \tag{11}$$

Entrepreneurs choose how much to invest in land, intermediate inputs, labour input and debt under the presence of the collateral constraint (5).

3.1.2 Optimal production

The first order condition for land k_t^i is:

$$\begin{aligned} \lambda_t^i q_t &= \beta \sum_{A^s} \pi(A^s) \left[\left(\frac{\alpha y_{t+1}^i(A^s)}{k_t^i} + q_{t+1}(A^s) \right) \lambda_{t+1}^i(A^s) \right] && (12) \\ & + \mu_t^i \sum_{A^s} p_t(A^s) q_{t+1}(A^s) && (13) \end{aligned}$$

where $\lambda_t^i = 1/c_t^i$ is the lagrange multiplier on the flow of funds constraint while μ_t^i is the lagrange multiplier on the collateral constraint.

The first order condition for intermediate inputs is:

$$\lambda_t^i = \beta \sum_{A^s} \pi(A^s) \left(\frac{\eta y_{t+1}^i(A^s)}{x_t^i} \lambda_{t+1}^i(A^s) \right) \tag{14}$$

[6]Of course, t-dated realizations of choice variables also depend on the t-dated realization of the aggregate technology state A_t. However we omit the A_t index in order to save on notation.

The first order condition for labour input is:

$$\lambda_t^i w_t = \beta \sum_{A^s} \pi\left(A^s\right)\left(\frac{(1-\alpha-\eta)\, y_{t+1}^i\left(A^s\right)}{h_t^i}\lambda_{t+1}^i\left(A^s\right)\right) \tag{15}$$

Finally, the first order condition for borrowing using the Arrow security in state A^s is:

$$\lambda_t^i p_t\left(A^s\right) - \beta\pi\left(A^s\right)\lambda_{t+1}^i\left(A^s\right) + p_t\left(A^s\right)\mu_t^i = 0 \tag{16}$$

where $\pi\left(A^s\right)$ is the conditional probability of aggregate technology state A^s realizing in the next period ($s = H, L$).

Combining (12), (15) and (16) we get an expression for the optimal mix between land and labour:

$$\frac{k_t^i}{h_t^i} = \frac{\alpha}{1-\alpha-\eta}\frac{w_t}{u_t} \tag{17}$$

where

$$u_t = q_t - \sum_{A^s} p_t\left(A^s\right) q_{t+1}\left(A^s\right) \tag{18}$$

denotes the user cost of land in the economy with complete markets for aggregate risk. Combining (12), (14) and (16) we get the optimal mix between capital and intermediate inputs:

$$\frac{k_t^i}{x_t^i} = \frac{\alpha}{\eta}\frac{1}{u_t} \tag{19}$$

Using (17) and (19) we can derive the unit cost of investment

$$\nu_t = u_t^{\alpha} w_t^{1-\alpha-\eta} \tag{20}$$

which depends on the user cost of land and the real wage rate. Then the return on production for the two types of entrepreneurs is given by:

$$\frac{a^i A^s}{\nu_t}$$

Clearly, since $a^H > a^L$, high productivity entrepreneurs will have a higher return on productive investments and will want to borrow. When the value of a unit of productive investment (evaluated at Arrow security prices) for high productivity entrepreneurs exceeds the cost of investment, borrowing constraints bind and productive agents borrow up to the limit given by (5).

$$a^H \sum_{A^s} p_t\left(A^s\right) A^s > \nu_t \tag{21}$$

The low productivity agents will want to save and in the first best equilibrium without borrowing constraints, they will be inactive in production. However, when credit constraints are sufficiently tight, low productivity entrepreneurs are active in production when the following condition is satisfied:

$$a^L \sum_{A^s} p_t\left(A^s\right) A^s = \nu_t \tag{22}$$

3.1.3 Optimal debt structure

Our assumed collateral constraint (5) restricts the total value of debt not to exceed the value of land. However, the constraint leaves the composition of debt (in terms of the two Arrow securities) free. In other words, borrowers are free to choose which securities to borrow with. Manipulating the first order conditions for the two Arrow securities (16) and using the fact that

$$\frac{\lambda_{t+1}^i\left(A^s\right)}{\lambda_t^i} = \frac{c_t^i}{c_{t+1}^i\left(A^s\right)} \tag{23}$$

we get the following condition which must hold at the optimal debt mix:

$$\pi^H \frac{1/c_{t+1}^H\left(A^H\right)}{p_t\left(A^H\right)} = \pi^L \frac{1/c_{t+1}^H\left(A^L\right)}{p_t\left(A^L\right)} \tag{24}$$

where $c_{t+1}^H\left(A^s\right)$ denotes the consumption of high productivity agents in state A^s.

On the margin, borrowers equalize the ratio of the (expected) marginal utilities in different states of nature $\left(1/c_{t+1}^H\left(A^s\right)\right)$ to the state contingent debt prices $\left(p_t\left(A^s\right)\right)$ they are charged in the market. Using the first order condition for Arrow securities investment for the low productivity savers and the fact that consumption is always $1-\beta$ fraction of own wealth, we can transform (24) into a statement about the state contingent evolution of the wealth distribution:

$$\frac{Z_{t+1}^L\left(A^H\right)}{Z_{t+1}^H\left(A^H\right)} = \frac{Z_{t+1}^L\left(A^L\right)}{Z_{t+1}^H\left(A^L\right)} \tag{25}$$

where $Z_{t+1}^i\left(A^s\right)$ denotes the net worth in state A^s, $s=H,L$ at time $t+1$ of entrepreneurs who have idiosyncratic productivity a^i, $i=H,L$ at time t.

Under log utility all agents save a β fraction of wealth. The evolution of the wealth distribution then depends exclusively on different households' rate of return on wealth. Let

$R^i_{t+1}(A^s)$ denote the rate of return on wealth in state A^s at time $t+1$ of agents in idiosyncratic productivity state i, $i = H, L$ at time t. These rates of return are given below:

$$R^H_{t+1}(A^s) = \left\{ \left(\frac{a^H A^s/\alpha}{u_t^{\alpha-1} w_t^{1-\alpha-\eta}} + q_{t+1}(A^s) \right) k_t^H - b_t^H(A^s) \right\} / \left(\beta z_t^H \right) \tag{26}$$

$$R^L_{t+1}(A^s) = \left\{ \left(\frac{a^L A^s/\alpha}{u_t^{\alpha-1} w_t^{1-\alpha-\eta}} + q_{t+1}(A^s) \right) k_t^L + b_t^L(A^s) \right\} / \left(\beta z_t^L \right) \tag{27}$$

Wealth in state A^s at time $t+1$ is given by the returns from productive investments for entrepreneurs of type i ($\left(\frac{a^i A^s/\alpha}{u_t^{\alpha-1} w_t^{1-\alpha-\eta}} + q_{t+1}(A^s) \right) k_t^i$) net of debt repayments $b_t^i(A^s)$. The rate of return on invested wealth is obtained by dividing this by total saving (β fraction of current wealth z_t^i).

Equation (25) states that the ratio of the aggregate wealth of high and low productivity entrepreneurs is equalized across states of nature. The addition of complete markets stabilizes the wealth distribution over the business cycle. Under logarithmic utility, this implies that, at the optimal debt mix, the rates of return for high and low productivity entrepreneurs co-move perfectly as the economy gets hit by aggregate shocks. In other words:

$$\frac{R^L_{t+1}(A^H)}{R^H_{t+1}(A^H)} = \frac{R^L_{t+1}(A^L)}{R^H_{t+1}(A^L)} \tag{28}$$

Borrowers commit to make larger repayments in the good state of the world ($b_t^H(A^H) > b_t^H(A^L)$) because the value of land is high in this state ($q_{t+1}(A^H) > q_{t+1}(A^L)$). Because, in equilibrium, debt repayments become effectively indexed to the value of land, savers also share in the ups and downs of asset prices. As a result, the wealth distribution no longer fluctuates over the economic cycle.

Note that although complete markets for aggregate risk stabilize the wealth shares of the high and low productivity groups of agents, the wealth of individual agents still fluctuates over individual productivity realizations due to the absence of markets for hedging idiosyncratic shocks. Even though (28) implies that the rates of return on wealth of different agents co-move perfectly in response to aggregate shocks, there is a large difference in rates of return over productivity spells. $R^H_{t+1}(A^s) > R^L_{t+1}(A^s)$ when credit constraints are binding and this ensures that the wealth of individual entrepreneurs grows during high productivity states and declines during low productivity states just like in the Kiyotaki (1998) model.

3.2 Behavior of Workers

The first order conditions for the workers are given by:

$$w_t = \varkappa \left(h_t^w\right)^{\omega} \tag{29}$$

$$b_t^w \left(A^s\right) \left(p_t \left(A^s\right) - \beta\pi \left(A^s\right)\right) = 0 \tag{30}$$

In equilibrium, workers will not buy any of the Arrow securities as long as low-productivity entrepreneurs experience negative consumption growth in every state. To see that this is the case, recall that low productivity demand for Arrow securities implies that:

$$p_t \left(A^s\right) = \beta\pi \left(A^s\right) \frac{c_t^L}{c_{t+1}^L \left(A^s\right)} \tag{31}$$

Substituting this expression into (30) we see that when $\frac{c_{t+1}^L(A^s)}{c_t^L} < 1$, workers find the Arrow security too expensive and choose $b_t^w \left(A^s\right) = 0$ for all A^s. We verify that this is the case throughout any simulation path.

3.3 Aggregation and Market Clearing

We complete the characterization of the competitive equilibrium of our model economy by specifying the evolution equations for the endogenous state variables well as the market clearing conditions.

The Arrow Debreu market clearing condition in state A^s is given by:

$$B_t^H \left(A^s\right) = B_t^L \left(A^s\right) + B_t^W \left(A^s\right) \tag{32}$$

where $B_t^H \left(A^s\right)$ is aggregate borrowing by high productivity entrepreneurs using the Arrow security which pays out in state while $B_t^L \left(A^s\right)$ and $B_t^W \left(A^s\right)$ are, respectively, the aggregate investment by low productivity entrepreneurs and workers in the same security.[7]

The land market clearing stipulates that land demand is equal to total land supply (assumed to be fixed and normalized to unity)[8]

$$K_t^H + K_t^L = 1 \tag{33}$$

[7] Workers will have zero demand for Arrow securities in equilibrium under the calibration we use. Nevertheless we include their demands in the market clearing condition for completeness.

[8] Workers have zero demand for land because they cannot use it for production.

where K_t^H is the land demand of productive agents and K_t^L is the land demand of unproductive ones. Goods markets clearing necessitates that the goods demands of entrepreneurs (both high and low productivity) for consumption $(C_t^H + C_t^L)$ and investment purposes $(X_t^H + X_t^L)$ plus the consumption demands of workers (C_t^W) is equal to the total output in the economy $(Y_t^H + Y_t^L)$.

$$C_t^H + C_t^L + C_t^W + X_t^H + X_t^L = Y_t^H + Y_t^L \tag{34}$$

Labour market clearing implies that demand for labour from the two groups of entrepreneurs equals workers' labour supply:

$$H_t^H + H_t^L = (w_t/\varkappa)^{1/\omega} \tag{35}$$

Finally the economy's endogenous state variables evolve according to the following transition law. Total economy-wide wealth $(Z_{t+1}(A^s))$ evolves in state A^s according to a process that depends on the share of wealth held by productive agents (d_t) and the returns on wealth for the two types of agents $(R_{t+1}^i(A^s))$.

$$Z_{t+1}(A^s) = \left[d_t R_{t+1}^H(A^s) + (1-d_t) R_{t+1}^L(A^s)\right] \beta Z_t \tag{36}$$

The share of wealth of the high productivity entrepreneurs in state A^s $(d_{t+1}(A^s))$ is determined by the portfolio returns on the two groups $(R_{t+1}^i(A^s))$ as well as the (exogenous) transition probabilities between the two idiosyncratic productivity states which are determined by the values of n and δ.

$$d_{t+1}(A^s) = \frac{(1-\delta) d_t R_{t+1}^H(A^s) + n\delta (1-d_t) R_{t+1}^L(A^s)}{d_t R_{t+1}^H(A^s) + (1-d_t) R_{t+1}^L(A^s)} \tag{37}$$

3.4 Equilibrium Definition

Recursive competitive equilibrium of our model economy is a price system w_t, u_t, q_t, R_t, $p_t(A^s)$, entrepreneur decision rules k_t^i, x_t^i, $b_t^i(A^s)$, h_t^i and c_t^i, $(i = L, H)$, worker decision rules $b_t^w(A^s)$, h_t^w and c_t^w, and equilibrium laws of motion for the endogenous state variables (36) and (37) such that

(i) The decision rules k_t^i, x_t^i, $b_t^i\,(A^s)$, h_t^i and c_t^i, $(i = L, H)$ solve the entrepreneur's decision problem conditional upon the price system and the decision rules $b_t^w\,(A^s)$, h_t^w and c_t^w solve the worker's decision problem conditional upon the price system.

(ii) The process governing the transition of the aggregate productivity and the household decision rules k_t^i, x_t^i, $b_t^i\,(A^s)$, h_t^i, c_t^i, $(i = L, H)$, $b_t^w\,(A^s)$, h_t^w and c_t^w induce a transition process for the aggregate state given by (36) and (37).

(iii) All markets clear

4 Calibration and Solution Method

4.1 Baseline Calibration

In this section we outline the main features of the baseline calibration. We calibrate η (the share of of intermediate goods in gross output) at 0.45 in line with evidence in the BEA Industrial Accounts. We calibrate the share of land α in gross output to 0.2. This corresponds to a tangible asset share in value added of 0.36.[9] The technology process at the firm level consists of an aggregate and an idiosyncratic component. The high (low) realizations of the aggregate TFP shock are 1.0% above (below) the steady state TFP level. The probability that the economy remains in the same aggregate state it is today is equal to 0.8.

Calibrating the cross-sectional dispersion of TFP is important because it does affect significantly the quantitative properties of the model. Bernard et. al. (2003) report an enormous cross-sectional variance of plant level value added per worker using data from the 1992 US Census of Manufactures. The standard deviation of the log of value added per worker is 0.75 in the data while their model is able to account for only around half this number. The authors argue that imperfect competition and data measurement issues can account for much of this discrepancy between model and data. In addition, the study assumes fixed labour share across plants so any departures from this assumption would lead

[9] The inclusion of intermediate inputs helps to model combine a realistic tangible asset share in value added (0.36) with a realistic tangible asset to GDP ratio (which depends strongly on the value of α).

to more variations in the measured dispersion of labour productivity.

In a comprehensive review article on the literature on cross-sectional productivity differences, Syverson (2009) documents that the top decile of firms has a level of TFP which is almost twice as high as the bottom decile. He finds that unobserved inputs such as the human capital of the labour force, the quality of management and plant level 'learning by doing' can account for much of the observed cross-sectional variation in TFP.

This model does not have intangible assets of the sort discussed in Syverson (2009) and consequently calibrating the model using the enormous productivity differentials identified in the productivity literature would overestimate the true degree of TFP differences. Aoki et. al. (2009a) also consider these issues in their calibration of a small open economy version of Kiyotaki and Moore (1997). They argue that a ratio of the productivities of the two groups of 1.15 is broadly consistent with the empirical evidence and we choose this number for the baseline case.

Moving on to the parameters that govern consumer preferences, we set labour supply we set ω^{-1} (the Frisch elasticity of labour supply) to 3. This is higher than micro-data estimates but is consistent with choices made in the macro literature. We pick $\varkappa$, a parameter governing the disutility of labour to get a value of labour supply as a fraction of workers' time endowment which is equal to 0.33.

The discount factor β, the probability that a highly productive entrepreneur switches to low productivity δ, and the ratio of high to low productivity entrepreneurs n are parameters we pick in order to match three calibration targets - the ratio of tangible assets to GDP, aggregate leverage and the leverage of the most indebted decile of firms. we use data on tangible assets and GDP from the BEA National Accounts in the 1952-2011 period. The concept of tangible assets includes Business and Household Equipment and Software, Inventories, Business and Household Structures and Consumer Durables. GDP excludes government value added so it is a private sector output measure.

Aggregate leverage is defined as the average ratio of the value of the debt liabilities of the non-financial corporate sector to the total value of assets. Leverage measures can be obtained from a number of sources. In the US Flow of Funds, aggregate leverage is approximately equal to 0.5 for the 1948-2011 period. This is broadly consistent with the findings of den

Haan and Covas (2007) who calculate an average leverage ratio of 0.587 in Compustat data from 1971 to 2004. Den Haan and Covas (2007a) also examine the leverage of large firms and find that it is slightly higher than the average in the Compustat data set. Firms in the top 5% in terms of size have leverage of around 0.6. Den Haan and Covas (2007b) have similar findings in a panel of Canadian firms. There the top 5% of firms have leverage of 0.7-0.75 compared to an average of 0.66 for the whole sample. High productivity entrepreneurs in our economy run larger firms so differences in productivity and therefore leverage could be one reason for the findings of Den Haan and Covas (2007a and 2007b). But the perfect correlation of firm size and leverage that holds in our model will not hold in the data. So if we are interested in the distribution of firm leverage, the numbers in Den Haan and Covas will be an underestimate. This is why we pick a target for the average leverage of the top 10% most indebted firms to be equal to 0.75. This number is broadly consistent with the findings in Den Haan and Covas.

Table 1 below summarizes the calibration targets we match while Table 2 summarizes the baseline parameter values used in the paper.

Table 1: Calibration targets

Target	Value	Source
Tangible Assets to GDP $= qK/\left(Y^H + Y^L - X^H - X^L\right)$	3.49	BEA National Accounts
Aggregate Leverage $=L^A = B/\left(qK + Y^H + Y^L\right)$	0.50	Flow of Funds
Leverage of indebted firms $=L^H = B/\left(qK + Y^H\right)$	0.75	Den Haan-Covas (2007a)
Share of intermediate inputs in gross output $= \eta$	0.45	BEA National Accounts
Share of capital in GDP $= \alpha/\left(1-\eta\right)$	0.36	BEA National Accounts
Cross sectional productivity dispersion $= a^H/a^L$	1.15	Aoki et. al. (2009a)

Table 2: Summary of baseline model calibration

Parameter Name	Parameter Description	Parameter Value
β	Discount factor	0.896
δ	Probability that a high productivity spell ends	0.145
n	Ratio of high to low productivity entrepreneurs	0.084
α	Capital share in gross output	0.200
η	Share of intermediate inputs in gross output	0.450
ω	Inverse of the Frisch elasticity of labour supply	0.330
$\varkappa$	Disutility of labour parameter	2.290
π	Probability that the aggregate state persists	0.800
A^H	TFP in the high aggregate state	1.010
A^L	TFP in the low aggregate state	0.990
a^H/a^L	Ratio of productivities of high and low productivity firms	1.150

4.2 Solution Method

The model is solved by the Parameterized Expectations method of den Haan and Marcet (1990). The exogenous aggregate state at time t consists of the aggregate TFP realization A_t. Similarly to the Kiyotaki (1998) model, the endogenous aggregate state variables at time t are aggregate wealth Z_t and the share of wealth belonging to high productivity entrepreneurs d_t. We parameterize the state contingent land price realization when the aggregate TFP state tomorrow is A_{t+1} when the aggregate TFP state is A_t today by a log-linear function:

$$\ln q_{t+1}\left(A_{t+1}|A_t, Z_t, d_t\right) = \psi_c\left(A_{t+1}|A_t\right) + \psi_z\left(A_{t+1}|A_t\right)\ln Z_t + \psi_d\left(A_{t+1}|A_t\right)\ln d_t \tag{38}$$

The solution procedure takes a starting guess of the coefficients of the four[10] land price functions (38) and then simulates the model as follows:

1. Conditional upon the value of the aggregate state (A_t, Z_t, d_t)[11] and conditional upon

[10] We have a separate land price function for every possible combination of successive TFP realisations: (A^H, A^H), (A^H, A^L), (A^L, A^H) and (A^L, A^L).

[11] Under complete markets, the wealth distribution is constant over time and the share of wealth of productive entrepreneurs d_t is no longer part of the state vector we use to parameterise the land price function.

the state contingent future realizations of the price of land $q_{t+1}(A_{t+1}|A_t, Z_t, d_t)$, the model first order conditions and market clearing conditions define a non-linear system of equations. We solve this system using the inbuilt Matlab function fsolve.m

2. In the following period we update the endogenous state variables Z_t and d_t using the transition equations (36) and (37).

3. We simulate a long time series of realizations under the guessed land price coefficients. We use linear regression to update the coefficients on the land price function.

4. We repeat steps 1 - 3 above until successive coefficient guesses have converged within a tolerance limit.

5. We check that the maximum prediction error in each of the four land price functions is less than 0.1%.

5 The Economic Impact of Market Completeness

In this section we answer the central questions of the paper: (i) How does the strength of the collateral amplification mechanism vary with the cost and availability of state contingent debt? (ii) Can empirically realistic costs of issuing risky and non-standard assets motivate the predominant use of simple debt by firms?

We begin by examining the first and second moments of the economy change as we introduce state-contingent debt. We then introduce realistic costs of accessing state contingent markets in order to see whether such costs can restore the collateral amplification mechanism.

5.1 Complete markets for aggregate risk and the impact of collateral constraints

In the steady state, the 'state contingent debt' economy is identical to the standard Kiyotaki-Moore economy. Hence, the first moments of this economy are distorted by the presence of the collateral constraint on productive agents. As Table 3 below shows, output, the land price and TFP are all lower than the first best because the most productive agents are unable to absorb the entire national saving and this necessitates inefficient production in equilibrium.

Only the consumption of productive agents increases in the economies in which there are no markets for hedging idiosyncratic shocks (columns CM and KM) in Table 3. This is because the absence of these markets implies that wealth and hence consumption become volatile over idiosyncratic productivity episodes. When entrepreneurs are productive they make large returns on invested wealth (higher than the rate of time preference) and their net worth grows rapidly. When they are unproductive, they earn a low return (lower than the rate of time preference) and their net worth declines over time. The result is a more dispersed wealth distribution with some rich entrepreneurs (those who have had a long productive spell) and some poor entrepreneurs (those who have had a long unproductive spell). But the really key message from Table 3 is this: the economy with complete markets for aggregate risk has first moments which are identical to that in the Kiyotaki-Moore economy with only simple debt. The important difference between these two economies will appear in the second moments, which we turn to next.

Table 3: The impact of market completeness on the level of economic activity

	FB	CM	KM
Output relative to first best	1.00	0.72	0.72
Land Price relative to first best	1.00	0.92	0.92
Productive Wealth Share relative to first best	1.00	2.80	2.80
Productives' Consumption relative to first best	1.00	2.40	2.40
Unproductives' Consumption relative to first best	1.00	0.73	0.73
TFP relative to first best	1.00	0.96	0.96

Note: FB = First Best, CM: Complete Markets for Aggregate Risk, KM = Baseline Kiyotaki-Moore economy with uncontingent debt

Table 4 below presents the second moments for the baseline calibration. The first column represents the first best economy with complete markets for both idiosyncratic and aggregate risk. In this economy, collateral constraints do not bind and only high productivity agents are active in production. The second column is the model with complete markets for aggregate risk but incomplete markets for idiosyncratic risk. The final column is the standard Kiyotaki-Moore model with only non-contingent debt.

Table 4: The impact of market completeness on aggregate volatility

	FB	CM	KM
St. Dev. of logged Output	2.19	2.19	2.93
St. Dev. of logged Capital Price	2.19	2.19	4.04
St. Dev. of Productive Wealth Share	0.00	0.00	1.07
St. Dev. of logged Productives' Consumption	2.19	2.19	8.90
St. Dev. of logged Unproductives' Consumption	2.19	2.19	2.38
St. Dev. of logged TFP	1.00	1.00	1.52

Note: FB = First Best, CM: Complete Markets for Aggregate Risk, KM = Baseline Kiyotaki-Moore economy with uncontingent debt

It is immediately apparent from the table that the second moments of the economy with complete markets for only aggregate risk are identical to those of the first best economy (complete markets for all types of risk). This finding is intuitive and it is consistent with the theoretical results of Krishnamurthy (2003). Completing the market for aggregate risk kills off the collateral amplification mechanism and it does this by stabilizing the wealth distribution, which stops being a state variable in the economy. Table 4 shows this very clearly in the third row which shows the standard deviation of the share of wealth in the hands of high productivity entrepreneurs. In the first best and in the economy with complete markets for aggregate risk, this standard deviation is zero. In other words, the wealth distribution is stable through time and does not respond to the aggregate technology shocks hitting the economy. In contrast, the final column (the Kiyotaki-Moore economy) shows that the wealth distribution does move over time when debt is uncontingent.

Looking at the consumption volatility of different groups, we can see how the presence or absence of complete markets affects individual types of entrepreneurs. When we assume uncontingent debt contracts, leveraged high productivity agents experience the most volatile consumption path while unproductive entrepreneurs who are largely invested in risk-free debt, experience the least volatile consumption path. The reason for this difference in the consumption volatility is straightforward. Being highly leveraged (and using uncontingent debt) exposes the net worth of borrowers to the fluctuations in the value of land. Borrowers experience large ex post returns on invested wealth when favorable shocks occur and the price of land increases and low returns when negative aggregate technology shocks occur. In contrast, low productivity agents hold unleveraged productive projects and safe debt. As a result, their wealth (and hence consumption) is relatively unaffected by aggregate shocks.

The resulting movements in the wealth distribution over the economic cycle further add to volatility. During positive technology shocks, high productivity entrepreneurs gain and are able to absorb a larger share of productive resources. This endogenously increases TFP, further expanding economic activity and boosting asset prices. This is the Kiyotaki-Moore amplification mechanism in action. This mechanism adds to the volatility of output, asset prices and TFP but it requires incomplete hedging of aggregate risk in order to operate.

When borrowers are able to fully hedge their exposure to aggregate risk, we can see from Table 4 that the productive wealth share is constant (its standard deviation is zero) and the consumption volatility of all groups in the economy is equalized. Because we no longer have fluctuations in the relative wealth of the most productive agents, aggregate TFP becomes completely exogenous and its standard deviation is equal to 1% (the standard deviation of the exogenous technology shocks). The collateral amplification mechanism disappears and, in terms of its second moments, the economy replicates the first best.

5.2 Numerical results under costly state-contingent contracts

5.2.1 Introducing hedging transaction costs

In this section we examine the possibility that writing complex contracts is costly and this introduces a cost of hedging using state contingent debt. In particular, we assume that the Arrow-Debreu security price paid by savers differs from the amount received by borrowers by a wedge which reflects the cost of writing Arrow-Debreu contracts:

$$\widetilde{p}_t\left(A^s\right) = \left(1-\gamma\right) p_t\left(A^s\right) \tag{39}$$

where $\widetilde{p}_t\left(A^s\right)$ is the price the borrower receives when he sells an Arrow security while $p_t\left(A^s\right)$ is the price the saver pays for the same security. $\gamma > 0$ is the cost of writing the contract. This cost includes the cost of brokerage and advisory services but could also involve a time cost involved in evaluating and understanding complex financial products. When we come to calibrate this cost, we will choose it to match the safety premium found by Krishnamurthy and Vissing-Jorgensen (2012). In contrast, we assume that simple debt contracts do not have such a cost.

Next we ask two questions. What level of the cost can rationalize the use of debt contracts in equilibrium? What level of the cost restores the ability of the collateral amplification mechanism to amplify the impact of small technology shocks on the macroeconomy?

Note that once we introduce simple debt into the model, the use of the low state Arrow security is redundant and we remove it from the model[12]. In equilibrium, firms borrow using uncontingent debt which is payable in both states of the world as well as using Arrow securities which are payable only in the high aggregate productivity state of the world.

We assume that the collateral constraint (5) is specified in terms of the prices received by borrowers for Arrow securities[13]. Expressed in terms of the 'high state' Arrow security ($b_t\left(A^H\right)$) and the uncontingent debt security (b_t), the collateral constraint becomes:

$$\widetilde{p}_t\left(A^H\right) b_t\left(A^H\right)+\frac{b_t}{R_t} \leqslant \sum_{A^s} \widetilde{p}_t\left(A^s\right) q_{t+1}\left(A^s\right) k_t \tag{40}$$

In other words, the value of liabilities (evaluated at market prices) cannot exceed the value of the entrepreneur's land (again evaluated at Arrow security prices)[14].

The first order conditions for the 'high state' Arrow security for, respectively, savers and

[12] We have two aggregate states and need two assets to span these two aggregate states. A 'high' state Arrow security and an uncontingent bond can do this just as well as a 'high' and a 'low' state Arrow security. Given that we assume that issuing the uncontingent security has no costs, the efficient market structure would be the one that uses uncontingent securities as much as possible.

[13] Assuming that the constraint holds in terms of the prices paid by savers actually makes state contingent assets disappear faster from use as γ grows. Hence, the collateral amplification mechanism is restored more quickly. Quantitatively, however, the difference is small.

[14] Even though a 'low state' Arrow security does not trade in equilibrium, its price can be easily computed from the prices of the 'high state' Arrow security and the price of the debt security:

$$p_t\left(A^L\right)=\frac{1}{R_t}-p_t\left(A^H\right)$$

borrowers are given by:

$$p_t\left(A^H\right) - \beta\pi\left(A^H\right)\frac{\lambda_{t+1}^L\left(A^H\right)}{\lambda_t^L} = 0 \tag{41}$$

$$p_t\left(A^H\right)(1-\gamma) - \beta\pi\left(A^H\right)\frac{\lambda_{t+1}^H\left(A^H\right)}{\lambda_t^H} + (1-\gamma)\,p_t\left(A^H\right)\frac{\mu_t^H}{\lambda_t^H} = 0 \tag{42}$$

Again, $\lambda_{t+1}^i(A^s) = 1/c_{t+1}^i(A^s)$ is the shadow value of wealth of entrepreneur of type i in state A^s at time $t+1$ and μ_t^i is the Lagrange multiplier on the borrowing constraint of an entrepreneur of type i. The first order condition for the debt security is given by:

$$\frac{1}{R_t} - \beta\sum_{A^s}\pi(A^s)\frac{\lambda_{t+1}^i(A^s)}{\lambda_t^i} + \frac{\mu_t^i}{\lambda_t^i}\frac{1}{R_t} = 0 \tag{43}$$

Using the fact that with log utility, state valuations are proportional to the rate of return on wealth

$$\beta\frac{\lambda_{t+1}^i(A^s)}{\lambda_t^i} = \frac{1}{R_{t+1}^i(A^s)} \tag{44}$$

and combining (41), (42) and (43) we get the condition which pins down the mix between borrowing using the Arrow security and the debt security is:

$$\left\{\frac{R_{t+1}^L\left(A^H\right)}{R_{t+1}^H\left(A^H\right)} - \left[\left(\sum_{A^s}\frac{\pi(A^s)}{R_{t+1}^H(A^s)}\right)/\left(\sum_{A^s}\frac{\pi(A^s)}{R_{t+1}^L(A^s)}\right)\right](1-\gamma)\right\}b_t^H\left(A^H\right) = 0 \tag{45}$$

The transaction cost γ introduces a wedge between the state contingent rates of return of productive and unproductive agents in different states of the world. (45) implies that, when $\gamma = 0$, the ratio of marginal utilities of the two types of entrepreneurs in the high state $\left(R_{t+1}^L\left(A^H\right)/R_{t+1}^H\left(A^H\right)\right)$ is equal to the ratio of ex ante expected marginal utilities $\left(\left(\sum_{A^s}\frac{\pi(A^s)}{R_{t+1}^H(A^s)}\right)/\left(\sum_{A^s}\frac{\pi(A^s)}{R_{t+1}^L(A^s)}\right)\right)$. Given that we have two aggregate states, this implies that the ratio of marginal utilities is constant across states. This is the complete markets benchmark: the wealth distribution is constant and there is full risk-sharing of aggregate (though not idiosyncratic) shocks.

Raising γ introduces variation in the evolution of relative rates of return over the business cycle. We can see from (45) that when $\gamma > 0$, $R_{t+1}^L\left(A^H\right)/R_{t+1}^H\left(A^H\right)$ is lower than average in the high aggregate state. In other words, high productivity agents experience a higher rate of return than low productivity agents and the wealth distribution shifts in their favour. In the low aggregate state the opposite happens.

These fluctuations in the wealth distribution over the business cycle bring the collateral amplification mechanism back into action. As γ increases, high productivity agents switch from costly contingent liabilities to cheaper uncontingent liabilities. They sacrifice insurance in order to obtain their funding more cheaply and maintain a high level of investment in their own productive projects. At some critical value of γ

$$\frac{R_{t+1}^{L}\left(A^{H}\right)}{R_{t+1}^{H}\left(A^{H}\right)} < \left[\left(\sum_{A^{s}} \frac{\pi\left(A^{s}\right)}{R_{t+1}^{H}\left(A^{s}\right)}\right) / \left(\sum_{A^{s}} \frac{\pi\left(A^{s}\right)}{R_{t+1}^{L}\left(A^{s}\right)}\right)\right](1-\gamma) \tag{46}$$

holds and $b_{t}^{H}\left(A^{H}\right)=0$. But even before this point, high productivity entrepreneurs increase their exposure to aggregate shocks and this starts to restore the Kiyotaki-Moore feedbacks between the wealth distribution and the aggregate equilibrium. The quantitative question we ask now is how quickly state contingent contracts diminish in importance and the collateral amplification mechanism returns in full force.

5.2.2 Calibrating state contingent market access costs

We calibrate γ by matching the empirical evidence on safety premia found in the data. Krishnamurthy and Vissing-Jorgensen (2012) show that the market values ultra safe assets. For example Treasuries yield 20 basis points less than the Aaa-rated corporate bonds even after controlling for the (very low) probability of default of the corporate bonds. In other words, investors demand a significant premium for bearing even the slightest default risk. In our model, uncontingent bonds are ultra safe (they never default) while state contingent bonds pay a return only in some states of the world so they are risky.

We calibrate the range of values of γ we consider as follows. We assume that a borrower who sells a completely safe bond (b_t in our model) can do so at no cost. But if the bond sold is a risky one, we assume that it will carry a 10 basis points safety premium. This number is actually smaller than the safety premia found in the data.

The model counterpart to the real-life risky bond is a security which includes both the safe and risky bond which trade in our model $\left(b_t + b_t\left(A^{H}\right)\right)$. We therefore calibrate γ (the proportional transactions cost involved in issuing the risky security) to ensure that

$$\gamma b_{t}\left(A^{H}\right)=0.001\left(b_{t}+b_{t}\left(A^{H}\right)\right) \tag{47}$$

or

$$\gamma = 0.001 \left(\frac{b_t + b_t \left(A^H\right)}{b_t \left(A^H\right)} \right) \tag{48}$$

In equilibrium, our complete markets model economy generates a quantity of the risky security $b_t \left(A^H\right)$ which is a 1.5% fraction of the total debt issued $(b_t + b_t \left(A^H\right))$ hence the 10 basis points safety premium cost incurred (which applies to the entire bond $b_t + b_t \left(A^H\right))$ equates to an approximate 6% proportional transaction cost incurred in selling the risky security $b_t \left(A^H\right)$. Consistent with this, in our analysis below, we examine values of the transaction cost γ that vary between 0 and 0.06 in order to see how the use of state contingent debt responds and to see how the strength of collateral amplification varies with γ.[15]

5.2.3 Collateral amplification

Figures 1-4 display how the size of the market for state-contingent securities and the model's second moments vary as we increase γ from 0 to 0.06. Figure 1 displays the way the Arrow security market shrinks as a percentage of annual output as we increase the transaction cost γ. Figures 2 and 3 display, respectively, the standard deviations (in percentage terms) of the cyclical component of output and the land price and Figure 4 displays the standard deviation of the share of wealth held by high productivity entrepreneurs.

As discussed in the previous subsection, when $\gamma = 0$, the financial market allows for costless hedging and the second moments of this economy are identical to those in the first best. The volatility of the productive wealth share is zero as hedging removes any changes in the wealth distribution and the reallocation of land between productive and unproductive agents over the business cycle is completely shut down. As γ increases, the ratio of high state Arrow security issuance to output declines (Figure 1). As γ reaches 0.055, contingent debt completely disappears and the economy converges to the standard Kiyotaki-Moore model

[15] Alternatively we could calibrate the safety premium directly on $b_t \left(A^H\right)$. This security, taken on its own, is very risky because it pays nothing in the low state. Hence the maximum value of γ we consider (which is equal to 0.06) would be appropriate for such a security because it would give it an equity-like premium over ultra-safe assets.

In any case, as subsequent analysis will show, much smaller values of γ (in the region of $0.01 - 0.02$) will be sufficient to reduce significantly the issuance of contingent securities in equilibrium.

with only uncontingent debt.

As state contingent debt disappears from usage with higher values of γ the volatility of macroeconomic aggregates gradually converges to that in the standard Kiyotaki-Moore model. The interesting aspect of the evolution of the economy's second moments is that most of the increase in volatility occurs at relatively low levels of γ. For example the standard deviation of the logged land price increases from 2.2% to 3.6% per annum as γ rises from 0 to 0.02. Subsequently the increase in volatility is much more muted: the annual standard deviation of the logged land price rises from 3.6% to just over 4% per annum as γ rises from 0.02 to 0.05. This shows that large transactions costs for state contingent contracts are not needed in order to move the economy close to the Kiyotaki-Moore benchmark with significant amplification.

In addition, our analysis showed that even small safety premia (10 basis points) imply large costs of issuing state contingent securities. This discourages borrowing firms from issuing such securities in equilibrium and exposes their net worth to business cycle fluctuations. As a result, volatility is substantially higher because of the amplification arising from the interaction of collateral values and the wealth of leveraged borrowers.

6 Conclusions

This paper assesses quantitatively how the Kiyotaki and Moore (1997) model behaves under complete markets for aggregate risk. We find that, in line with the findings of Krishnamurthy (2003), complete markets completely kill off the collateral amplification mechanism. The collateral constraints continue to distort downwards the level of output in the economy. But the second moments of such an economy are identical to the ones of an economy with perfect credit markets. Once we allow for empirically realistic costs of using state contingent securities we find that this restores most of the strength of the collateral amplification mechanism.

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