FISCAL SUSTAINABILITY AND PUBLIC DEBT IN AN ENDOGENOUS GROWTH MODEL

by Jesús Fernández-Huertas Moraga
and Jean-Pierre Vidal

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In 2004 all publications will carry a motif taken from the €100 banknote.

CONTENTS

Abstract

This paper investigates fiscal sustainability in an overlapping generations economy with endogenous growth coming from human capital formation through educational spending. We assess how budgetary imbalances affect economic dynamics and the outlook for economic growth, thereby providing a rationale for fiscal rules ensuring sustainability. Our results show that the appropriate response of fiscal policy to temporary shocks is not trivial in the absence of fiscal rules. Fiscal rules allow for a timely reaction, thereby avoiding possibly disruptive fiscal adjustment in the future: the more adjustment is delayed, the larger is its necessary scale. We perform a rough calibration of the model to simulate the effects of a demographic shock (change in the population growth rate) under different fiscal policy scenarios.

JEL classification: E62, H63, H55, O41, E17

Keywords: Fiscal sustainability, public debt, overlapping generations

Non-technical summary

This paper investigates the implications of budgetary imbalances and undesirable debt dynamics for fiscal sustainability in a model where both interest rates and economic growth are affected by fiscal developments. A sustainable fiscal policy plan is compatible with the existence of an intertemporal equilibrium in every period; this in turn ensures that over time the government will be able to honor its payments without leading the economy to a catastrophe. Such a situation may occur in an economy where the financing of government debt fully crowds out physical capital, thereby leading to a disruption of productive activities.

A theoretical model aimed at a sustainability analysis needs to capture the impact of fiscal policy on economic developments. The overlapping generations model is suitable for that purpose, since government bonds crowd out physical capital in the absence of altruistically motivated bequests. The baseline model is here enriched with two elements. First, human capital formation, which is financed out of parental altruism, is the engine of growth and allows us to also account for the impact of fiscal policy on growth. Second, the public sector is fully specified. Both lump-sum and proportional taxes are levied to finance lump-sum pension benefits and government consumption. When the government revenue is not enough to cover public spending, the government issues public debt on which it has to pay interests in subsequent periods.

In the absence of fiscal rules stabilizing government debt, the economic equilibrium is in general unstable and unpleasant debt dynamics can lead the economy to an unsustainable situation. This is the case when the rate of interest exceeds the economic growth rate - a reasonable assumption. In such a situation an increase in the debt ratio results in an increase in the interest expenditure ratio, which in absence of tax increases or expenditure cuts has to be financed by borrowing, ultimately leading to a further increase in the debt ratio and endangering fiscal sustainability. The stabilization of the debt ratio in this model can be achieved by means of a simple fiscal rule, whereby for example the government's target is a constant debt ratio.

The savings of the economy are split between financing next period physical capital and the amount of public debt set in circulation by the government. As a result, issuing public debt reduces the capital level in the next period so that both the growth and the interest rate are affected (lower growth and higher interest rate). On the other hand, issuing public debt allows the current generation to enjoy higher levels of consumption that it would obtain were the government supposed to balance its budget every period. The current generation can also spend more on their children education, which has a positive effect on the growth rate through human capital accumulation. This trade-off generated by government debt is perfectly reflected in our model since both the growth rate and the interest rate are endogenously determined inside it. This is the main contribution with respect to other existing models in the literature since the difference between the interest and the growth rate of an economy are the key variables in determining whether a level of public debt can be made sustainable and both are jointly determined here in a general equilibrium setting.

By means of an example, calibrated to approximately match the reduction in population the European Union is projected to have in the next fifty years, we show how, starting from equilibrium, a change in the conditions of the economy (population growth rate in this case) can lead to an unsustainable situation unless fiscal policies are changed in response. In the example, we start from an equilibrium with a yearly growth rate of 1.85% and a yearly interest rate of 4.73%. The reduction in the population growth rate leads the economy to crash in five periods (here, approximately 150 years). However, when fiscal rules are in place, they keep the economy in a sustainable situation, leading ultimately to a new equilibrium in which the growth rate increases to 2.14% whereas the interest rate decreases to 4.59%. In addition, fiscal rules ensure a timely fiscal policy response, since we also show how delaying the answer to the shock in the absence of fiscal rules imposes a cost in terms of lower growth levels (and thus, lower consumption) for future generations (of course, the present generation can benefit from this delay).

The main conclusion that can be obtained from this paper is that fiscal sustainability can be extremely fragile in the absence of fiscal rules. On the contrary, fiscal rules make the economic equilibrium stable and ensure that public debt does not endanger fiscal sustainability. Furthermore, fiscal rules allow for a timely reaction, thereby avoiding disruptive fiscal adjustment in the future: the more adjustment is delayed, the larger is its necessary scale.

1 Introduction

This paper investigates fiscal sustainability in an overlapping generations economy with endogenous growth coming from human capital formation through educational spending. In the model, the government is allowed to run budgetary imbalances that can lead to the emission of public debt. The objective is to assess how budgetary imbalances affect economic dynamics and the outlook for economic growth, thereby providing a rationale for fiscal rules ensuring sustainability.

The tax and transfer system examined in this paper is fairly rich. Pension benefits are assumed to be paid in a lump-sum manner. Individuals pay a proportional tax on labor income and at the same time they either pay a lump-sum tax or receive a lump-sum transfer. Labor income taxation is characterized by high top marginal tax rates and relatively lower average effective tax rates, reflecting the progressiveness of income taxation. This is well captured in our model by combining a proportional income tax with a lump-sum transfer. In addition to the tax-benefit system, the government finances general public spending, which benefits individuals but does not affect their economic decisions, and issues bonds. As individuals maximize utility and therefore react to fiscal policies, the model provides a suitable framework to inquire about fiscal sustainability.

There is a large and growing literature on the convenience of imposing fiscal rules, as well as on their desirable properties in terms of sustainability and stabilization. Fiscal rules are in general justified as a way to prevent governments from choosing levels of taxation or government spending that do not maximize social welfare because of political biases (see, for example, Alesina and Perotti (1995) or Milesi-Ferretti (2004)). Political biases in fiscal policies can ultimately lead to increasing debt ratios, possibly endangering fiscal sustainability. In dynamic models, where absence of control of the debt dynamics may endanger sustainability, fiscal rules can help stabilize the dynamics of an economy that would otherwise be inherently unstable and maintain the economy on a sustainable path.

Overlapping generations models are suitable theoretical tools to address fiscal sustainability issues. As the Ricardian equivalence does not apply in these models and debt dynamics are in general unstable, fiscal rules are needed to maintain fiscal sustainability. In the classical Diamond model (Diamond (1965)) where the only source of economic growth is the accumulation of physical capital, De la Croix and Michel (2002) and Rankin and Roffia (2003) define and study fiscal sustainability. We extend here their analysis to an endogenous growth model in which the interest rate and the growth rate of the economy are jointly determined. There are two sources of economic growth in our model: the accumulation of physical capital and the formation of human capital. The accumulation of physical capital stems from individual savings. Endogenous growth results from the formation of human capital, which is assumed to result from parental education and educational spending, financed out of altruism.

The main findings are the following. First, the existence of steady states is

not sufficient to ensure fiscal sustainability. In particular, when the steady state is unstable, given an initial capital-labor ratio, there is a unique debt to GDP ratio ensuring fiscal sustainability. Second, in the presence of multiple steady states, the initial conditions in the economy matter for the long run equilibrium that will result from economic dynamics. Third, the stability properties of the economy depend on the set of fiscal instruments, i.e., on the adopted fiscal rules. Fiscal policy rules are generally needed to ensure the stability of equilibria that are dynamically efficient. In the literature dealing with public debt, two sorts of fiscal policy rules have in general been assumed[1]: the constant debt policy and the constant deficit policy. We will however concentrate on the former policy (to be precise, a constant debt to GDP ratio policy), since its implementation is much easier in our model and our main objective is to illustrate how rules change the dynamics of the economy (see Buiter (2003) for a comparison of suggested and currently implemented operational rules).

Not surprisingly, the set of economic equilibria and their associated dynamics are simpler in the case of a small open economy, where the debt dynamics are unstable if the interest rate is higher than the growth rate and stable otherwise. Only the first case is consistent with the assumption of a small open economy, since the external indebtedness of the small open economy would tend to infinity in the second case (see Marín (2002)). In the absence of fiscal rules, we get to the same conclusion for the case of a closed economy, that is, the debt dynamics are unstable if the interest rate is higher than the growth rate (below the golden rule of capital accumulation) and stable otherwise (above the golden rule).

These considerations clearly show that the appropriate response of fiscal policy to exogenous temporary shocks is not trivial in the absence of fiscal rules. If temporary *small* shocks occur in the neighborhood of a stable steady state, there is no strong case for adjustments to fiscal policy, as the economy can come back to its initial position by itself. However, when temporary shocks occur in the neighborhood of an unstable steady state, which is the standard case in an economy with public debt, they can alter the nature of the economic equilibrium, possibly endangering fiscal sustainability. Without timely reaction to such shocks, ensuring fiscal sustainability would require adjustment, possibly of a disruptive nature, in the future: the more the adjustment is delayed, the larger is its necessary scale. Fiscal rules preserving fiscal sustainability seem more appropriate to deal with small shocks, as they timely maintain the economy on a sustainable path and do not lead to disruptive adjustments.

The paper is organized as follows. In section 2 we present the main assumptions of our model. In section 3, the dynamics of the state variables (human capital, physical capital and public debt) are discussed. In section 4, we introduce a simulation exercise. We parameterize a baseline version of the model and then see what its properties are and, finally, as an illustration, how this baseline would react to a demographic shock similar to the one the European Union is expected to suffer in the next 50 years. Section 5 concludes the paper.

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[1] In an overlapping generations model Marín (2002) and Annicchiaricco and Giammarioli (2004) examine a fiscal rule whereby the primary balance is adjusted in function of the distance between the actual and the targeted levels of the debt and primary surplus ratios.

2 Model

The basic framework is an overlapping generations model (Allais (1947), Samuelson (1958), Diamond (1965)), in which parents have an altruistic concern for their children. Parents choose educational spending so as to maximize the expected net labor income of their children. They are therefore aware that the return to education is affected by labor income taxation. The tax system is fairly rich and encompasses both proportional and lump-sum labor taxes as well as old-age benefits. We consider both the case of balanced budget policies, whereby taxes levied on labor finance old-age benefits so that the tax system functions as a pay-as-you-go public pension scheme, and the case of public debt, whereby the government can run budget deficits or surpluses.

Education in the family is a peculiar non-market activity, which has the following features. First, parents' ability to educate children can differ. Some parents may be more skilled than others in educating their children, which would result in heterogeneity in the levels of human capital of individuals belonging to the same cohort, even though these individuals are *ex ante* perfectly identical and endowed with the same skills. Second, there is no market for parental education. Children do not choose their parents, whereas the quality of their education and ultimately their level of human capital depend on their parents' abilities to educate them. Third, parents do not know *ex ante* their educational abilities, implying that they have to take educational decisions under uncertainty about the relationship between their efforts and their children's educational attainment. We model these features as a shock to the education technology; parents are either endowed with a highly productive or a standard education technology but they know their type only *ex post*. This generates a distribution of historical growth rates of human capital that converges to a normal distribution.

2.1 Households

The economy consists of a sequence of individuals who live for three periods: childhood, adulthood and old-age. Each individual belongs to a family or dynasty; there are I families in the economy. In the second period of their life, each individual gives birth to $1 + n_t$ children, so that all families grow at the same rate, which is also the population growth rate when $n_t = n_{t-1}$:

$$N_t = \sum_{i=1}^{I} N_t^i = \sum_{i=1}^{I} (1 + n_t) N_{t-1}^i = (1 + n_t) N_{t-1}$$

where N_t denotes the number of individuals born in period $t - 1$.

Individuals are educated during childhood and their human capital during adulthood mainly depends on their parents' educational spending and on their parents' human capital. The education technology is stochastic, and parents do not know *ex ante* whether they are endowed with a highly productive or a baseline education technology. The education technology is a characteristic

of parents; all children born in the same household are therefore educated according to the same technology. With probability p, parents are endowed with a highly productive education technology and their children's human capital, h_{t+1}, evolves according to:

$$h_{t+1} = (1 + \varepsilon)F_t e_t^{\delta} h_t^{1-\delta}$$

where $\varepsilon > 0$. e_t and h_t are parental spending on education and parents' human capital, respectively, while F_t denotes an economy-wide technological progress in the field of education. $\delta \in (0,1)$ stands for the elasticity of human capital with respect to private educational spending. Though education policies together with individual decisions clearly affect human capital formation, we do not address this issue in this paper, where technological progress in human capital formation is assumed to be exogenous. With probability $1-p$, they are endowed with the standard education technology:

$$h_{t+1} = F_t e_t^{\delta} h_t^{1-\delta}$$

Given levels of educational spending and parental human capital, the expected level of children's human capital is:

$$E_t[h_{t+1}] = (1 + p\varepsilon)F_t e_t^{\delta} h_t^{1-\delta} \tag{1}$$

During adulthood each individual born at $t-1$ supplies inelastically h_t efficiency units of labor, receives a gross labor income $w_t h_t$, pays proportional (τ_t) and lump-sum (η_t) taxes, consumes c_t, saves s_t and spends $(1+n_t)e_t$ on education:

$$c_t + s_t + (1 + n_t)e_t = (1 - \tau_t)w_t h_t - \eta_t \equiv \omega_t \tag{2}$$

where ω_t stands for labor income net of taxes. When old individuals consume the proceeds of their saving ($R_{t+1}s_t$), along with their pension benefits (Θ_{t+1}):

$$d_{t+1} = R_{t+1}s_t + \Theta_{t+1} \tag{3}$$

where d_{t+1} denotes old-age consumption.

We assume that individuals finance their children's education out of altruism[2]. They choose educational spending so as to maximize the expected net labor income of their children. There is no bequest motive in the model. The inclusion of bequests would allow parents to exercise their altruism in two different ways: through education or bequests. Depending on how they are modeled, operative bequests may lead to debt neutrality and consequently to the absence of real effects of fiscal policy when taxes and public transfers are lump-sum (see Lambrecht, Michel and Vidal (2001)). As our aim is to examine government debt in a model where fiscal policy is effective and there are risks to fiscal sustainability, we assume that there are no bequests.

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[2]For a survey of altruism in neoclassical growth models, see Michel, Thibault and Vidal (2004).

Each individual born at time $t-1$ is endowed with the following utility function:

$$U_t = (1-\beta)\left(\ln(c_t) + u\left(g_t\right)\right) + \beta\left(\ln(d_{t+1}) + u\left(g_{t+1}\right)\right) + \gamma\ln(E_t[\omega_{t+1}]) \quad (4)$$

where $0 < \beta < 1$ is a discount factor and $\gamma > 0$ is the degree of intergenerational altruism. g_t denotes the consumption of public goods in period t; as the utility is separable, the provision of public goods does not affect the first-order conditions. The expected net labor income of children is easily obtained from (1):

$$E_t[\omega_{t+1}] = (1-\tau_{t+1})w_{t+1}E_t[h_{t+1}] - E_t\left[\eta_{t+1}\right] \quad (5)$$

Individuals maximize (4) under their budget constraints (2)-(3) and (5). The first-order conditions of an individual's maximization problem are:

$$\frac{1-\beta}{R_{t+1}c_t} = \frac{\beta}{d_{t+1}} \quad (6)$$

$$\frac{(1-\beta)(1+n_t)}{c_t} = \frac{\gamma(1-\tau_{t+1})w_{t+1}(1+p\varepsilon)F_t\delta e_t^{\delta-1}h_t^{1-\delta}}{E_t[\omega_{t+1}]} \quad (7)$$

Equation (6) determines consumptions over the life-cycle: the marginal rate of substitution between adult and old-age consumption is equal to the rate of interest. Equation (7) determines parental educational choices. The utility loss in terms of reduced consumption of spending one euro on children's education is equal to the utility gain stemming from the increase in children's expected income out of altruism. Merging equations (2) and (3) gives an individual's life-cycle budget constraint:

$$c_t + \frac{d_{t+1}}{R_{t+1}} + (1+n_t)e_t = (1-\tau_t)w_t h_t - \eta_t + \frac{\Theta_{t+1}}{R_{t+1}} \equiv W_t \quad (8)$$

where W_t denotes life-cycle disposable income.

Regarding lump-sum taxes and pension benefits, we further assume that the tax-benefit system does not operate any redistribution across income classes. The tax system is focused on intergenerational redistribution and lump-sum taxes and benefits are therefore related to an individual's labor income class so that there is no intra-generational redistribution across income classes:

$$\eta_t = \tilde{\eta}_t w_t h_t \quad (9)$$
$$\Theta_{t+1} = \theta_{t+1} w_t h_t$$

Life-cycle disposable income is therefore given by:

$$W_t = \left(1 - \tau_t - \widetilde{\eta}_t + \frac{\theta_{t+1}}{R_{t+1}}\right) w_t h_t = (1 - T_t)\, w_t h_t \quad (10)$$

T_t indicates the implicit tax rate paid on labor income over the life cycle. This tax rate is positive if the rate of interest is higher than the implicit rate of return of public pensions: $R_{t+1} > \frac{\theta_{t+1}}{\tau_t + \tilde{\eta}_t}$.

Combining the life-cycle budget constraint (8) with condition (6), we can write:

$$c_t = (1 - \beta)(W_t - (1 + n_t)e_t)$$

$$d_{t+1} = \beta R_{t+1}(W_t - (1 + n_t)e_t)$$

Plugging the expression for c_t into condition (7), we obtain an equation characterizing the optimal choice of educational spending:

$$(e_t)^\delta - \frac{\gamma\delta}{(1 + n_t)(1 + \gamma\delta)}W_t(e_t)^{\delta-1} = \frac{\eta_{t+1}}{(1 + \gamma\delta)(1 - \tau_{t+1})(1 + p\varepsilon)F_t(h_t)^{1-\delta}} \quad (11)$$

Together with (9), equation (11) gives the optimal education spending:

$$
\begin{aligned}
e_t &= \frac{\gamma\delta(1 - \tau_{t+1})}{(1 + n_t)(1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma\delta(1 - \tau_{t+1}))}W_t \\
&= \frac{\gamma\delta(1 - \tau_{t+1})\left(1 - \tau_t - \tilde{\eta}_t + \frac{\theta_{t+1}}{R_{t+1}}\right)w_t h_t}{(1 + n_t)(1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma\delta(1 - \tau_{t+1}))}
\end{aligned}
\quad (12)
$$

The optimal consumption and saving can therefore be expressed as follows:

$$c_t = \frac{(1 - \beta)\left(1 - \tau_{t+1} - \tilde{\eta}_{t+1}\right)}{1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma\delta(1 - \tau_{t+1})}W_t \quad (13)$$

$$d_{t+1} = \frac{\beta R_{t+1}\left(1 - \tau_{t+1} - \tilde{\eta}_{t+1}\right)}{1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma\delta(1 - \tau_{t+1})}W_t \quad (14)$$

$$\frac{s_t}{h_t} = \sigma_t(R_{t+1})w_t \quad (15)$$

where we define:

$$\sigma_t(R_{t+1}) =$$

$$= \left(1 - \tau_t - \tilde{\eta}_t - \frac{\left(1 - \tau_t - \tilde{\eta}_t + \frac{\theta_{t+1}}{R_{t+1}}\right)\left((1 - \beta)\left(1 - \tau_{t+1} - \tilde{\eta}_{t+1}\right) + \gamma\delta(1 - \tau_{t+1})\right)}{1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma\delta(1 - \tau_{t+1})}\right)$$

2.2   Firms

In each period t, production occurs according to a Cobb-Douglas technology using two inputs, physical and human capital. Output is given by:

$$Y_t = A_t K_t^\alpha H_t^{1-\alpha} \tag{16}$$

where K_t and H_t denote the levels of physical and human capital in period t, respectively. A_t is a technology parameter, indicating exogenous technological progress over time, and $\alpha \in (0,1)$ is the share of physical capital.

The stock of capital in period t (K_t) comes from the total savings of the preceding period, public or private. The demand for labor (effective labor or human capital) maximizes profits:

$$\Pi_t = \max_{H_t} \left(A_t K_t^\alpha H_t^{1-\alpha} - w_t H_t \right)$$

The resulting wage is the competitive one:

$$w_t = (1-\alpha) A_t K_t^\alpha H_t^{-\alpha} \tag{17}$$

The remaining profits belong to capital owners so that we can define the return on physical capital as:

$$R_t = \frac{\Pi_t}{K_t} = \alpha A_t K_t^{\alpha-1} H_t^{1-\alpha} \tag{18}$$

2.3   Public sector

In each period, the government levies labor income taxes and pays pension benefits to retirees, finances public consumption $G_t = \sum_{i=1}^I N_{t-1}^i g_t^i$, reimburses the outstanding public debt B_{t-1} along with the accrued interests $(R_t - 1) B_{t-1}$ and issues government bonds B_t, which will be redeemed one period later. In period t, the government budget constraint is:

$$B_t = R_t B_{t-1} + \theta_t w_{t-1} \sum_{i=1}^I N_{t-2}^i h_{t-1}^i - N_{t-1} \eta_t - \tau_t w_t \sum_{i=1}^I N_{t-1}^i h_t^i + G_t$$

The relevant statistical concept for public debt in the model at hand is net debt, i.e. the difference between general government's liabilities and assets. We have already assumed that the lump-sum taxes and benefits are related to labor income when setting up the households' budget constraint. We do not consider the case of income redistribution across families, and focus on intergenerational redistribution by means of pay-as-you-go public pension and public debt. Under this assumption, we have:

$$B_t = R_t B_{t-1} + \theta_t w_{t-1} H_{t-1} - (\tilde{\eta}_t + \tau_t) w_t H_t + G_t \tag{19}$$

3 Intertemporal equilibrium

3.1 The economic growth rate

Up to now we have often omitted superscript i. Each family is characterized by a history of shocks affecting the education technology of each successive generation. Some generations in a given family are more successful than others in educating their children. The law of motion of human capital in family i is:

$$h_{t+1}^i = \left(1 + \widetilde{\varepsilon}_t^i\right) F_t \left(e_t^i\right)^\delta \left(h_t^i\right)^{1-\delta} \tag{20}$$

where $\widetilde{\varepsilon}_t^i = \varepsilon > 0$ with probability p and $\widetilde{\varepsilon}_t^i = 0$ with probability $1 - p$.

After T periods, we can calculate the expected level of human capital of a member of family i, whose ancestor had a level of human capital h_t^i in period t. This is given by:

$$E_t \left[h_{t+T}^i\right] = \sum_{j=0}^{T} \left(\binom{T}{j} p^{T-j} (1-p)^j h_{t+T}^j\right)$$

where h_{t+T}^j with $j = 0, ..., T$ represents the $T + 1$ possible different histories of shocks in T periods. h_{t+T}^0 is the human capital resulting from a history, according to which the family never benefited from a positive shock in T periods. h_{t+T}^j denotes the human capital of the descendant of a family that received j good shocks.

The optimal choice of education is governed by:

$$e_t^i = \frac{\gamma \delta \left(1 - \tau_{t+1}\right) \left(1 - \tau_t - \tilde{\eta}_t + \frac{\theta_{t+1}}{R_{t+1}}\right) w_t h_t^i}{(1 + n_t)(1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma \delta \left(1 - \tau_{t+1}\right))}$$

Plugging this into the law of motion, we have:

$$h_{t+1}^i = \left(1 + \widetilde{\varepsilon}_t^i\right) \Phi_t h_t^i \tag{21}$$

where:

$$\Phi_t \left(w_t, R_{t+1}\right) \equiv F_t \left(\frac{\gamma \delta \left(1 - \tau_{t+1}\right) \left(1 - \tau_t - \tilde{\eta}_t + \frac{\theta_{t+1}}{R_{t+1}}\right) w_t}{(1 + n_t)(1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma \delta \left(1 - \tau_{t+1}\right))}\right)^\delta$$

Assuming either that all the exogenous parameters and prices are constant over time, which would be the case of a small open economy, or else that prices have converged to their steady-state levels in the closed economy case, we can write for each family i:

$$h_{t+1}^i = \left(1 + \widetilde{\varepsilon}_t^i\right) \Phi \left(w, R\right) h_t^i$$

In this case, there is a simple expression for the previous expectation:

$$E_t \left[h_{t+T}^i \right] = \sum_{j=0}^{T} \left(\binom{T}{j} p^{T-j} (1-p)^j h_{t+T}^j \right) = \left(\Phi\left(w, R\right) \left(1 + p\varepsilon\right) \right)^T h_t^i$$

$\Phi\left(w, R\right)\left(1 + p\varepsilon\right) - 1$ is the expected average annual growth rate of human capital in family i. It will also be the average growth rate of the economy.

Labor supply in period t is equal to the aggregate level of human capital in the economy. The aggregation of the levels of human capital of the $N_t (= \sum_{i=1}^{I} N_t^i)$ individuals belonging to the I families can be written as:

$$H_t = \sum_{i=1}^{I} N_{t-1}^i h_t^i \tag{22}$$

Let us now turn to the dynamics of aggregate human capital. Using (21) we write the total level of human capital:

$$H_{t+1} = \sum_{i=1}^{I} N_t^i h_{t+1}^i = \sum_{i=1}^{I} \left(1 + n_t\right) \left(1 + \varepsilon_t^i\right) \Phi_t\left(w_t, R_{t+1}\right) N_{t-1}^i h_t^i$$

where ε_t^i is the realization of the shock affecting family i. Let us now calculate the expected growth rate of human capital $E_t\left(H_{t+1}\right)/H_t - 1$, where we have:

$$E_t\left(H_{t+1}\right) = \sum_{i=1}^{I} \left(1 + n_t\right) \Phi_t\left(w_t, R_{t+1}\right) N_{t-1}^i h_t^i E_t\left(1 + \widetilde{\varepsilon}_t^i\right)$$

Hence:

$$E_t\left(H_{t+1}\right) = \left(1 + p\varepsilon\right)\left(1 + n_t\right)\Phi_t\left(w_t, R_{t+1}\right) H_t \tag{23}$$

If the number of families I is large enough, aggregate human capital will be equal to its expectation at time t: $H_{t+1} = E_t\left(H_{t+1}\right)$. The growth rate of per capita human capital, which will also be the growth rate of per capita variables in the economy, can therefore be written as:

$$\rho_t = \left(1 + p\varepsilon\right) F_t \left(\frac{\gamma\delta\left(1 - \tau_{t+1}\right)\left(1 - \tau_t - \tilde{\eta}_t + \frac{\theta_{t+1}}{R_{t+1}}\right) w_t}{\left(1 + n_t\right)\left(1 - \tau_{t+1} - \tilde{\eta}_{t+1} + \gamma\delta\left(1 - \tau_{t+1}\right)\right)} \right)^{\delta} - 1$$

3.2   Physical capital and public debt

Savings finance both physical capital and public debt, so that the following capital market clearing condition holds:

$$K_{t+1} + B_t = \sum_{i=1}^{I} N_{t-1}^i s_t^i = \sum_{i=1}^{I} N_{t-1}^i \sigma_t \left(R_{t+1}\right) w_t h_t^i = \sigma_t \left(R_{t+1}\right) w_t H_t \qquad (24)$$

The intertemporal equilibrium in this economy can thus be defined as a sequence $\{K_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ satisfying the following equations:

$$
\begin{aligned}
K_{t+1} + B_t &= \sigma_t \left(R_{t+1}\right) w_t H_t \\
B_{t+1} &= R_{t+1} B_t + \theta_{t+1} w_t H_t - \left(\tilde{\eta}_{t+1} + \tau_{t+1}\right) w_{t+1} H_{t+1} + G_{t+1}
\end{aligned}
$$

Knowing that the evolution of aggregate human capital is governed by (23), the dynamics of the economy can be summarized by a system of two equations expressed in intensive terms:

$$k_{t+1} = \frac{\sigma_t \left(R_{t+1}\right) w_t - b_t}{(1 + p\varepsilon)(1 + n_t) \Phi_t \left(w_t, R_{t+1}\right)} \qquad (25)$$

$$b_{t+1} = \frac{R_{t+1} b_t + \theta_{t+1} w_t}{(1 + p\varepsilon)(1 + n_t) \Phi_t \left(w_t, R_{t+1}\right)} - \left(\tilde{\eta}_{t+1} + \tau_{t+1}\right) w_{t+1} + g_{t+1} \qquad (26)$$

where $k_t = K_t/H_t$, $b_t = B_t/H_t$ and $g_t = G_t/H_t$ are the ratio of physical to human capital, the ratio of public debt to human capital and public consumption per unit of human capital in period t, respectively.

It is well known that there are normally two steady state solutions when we introduce public debt in a standard overlapping generations economy. In the case of an endogenous growth model as the one presented here, there is no simple analytical expression for the solutions but the simulation below can help understand the functioning of the system. However, the analytical solution can be computed if we assume balanced budgets in every period and no government spending.

4   Simulations

In this section, we produce a rough calibration of the model with the objective of providing some examples about how fiscal sustainability can be guaranteed in the presence of diverse shocks. The way to proceed is first the calibration of a baseline scenario whose response to the mentioned shocks will then be assessed.

4.1   Baseline scenario

The calibration of 2 or 3-period overlapping generations economies is not very usual in the literature. The main difficulty in dealing with this issue is the need to simulate very long time periods. In the current example, we follow De

la Croix and Doepke (2003) in assuming time periods of 30 years. That is, a typical individual lives for 90 years. During the first 30 years (childhood), she just receives education; she works, consumes and saves during the following 30 years (adulthood); and, finally, she lives out of her retirement pension and savings for the last 30 years (retirement). This description is quite crude but the objective at this point is just to explain how such an economy would work without of course claiming that this is what actually happens in reality.

4.1.1 Macroeconomic assumptions

The macroeconomic assumptions refer to the choice of the parameters that ensure reasonable interest and growth rates in the baseline scenario. Of course, the final interest and growth rates depend also on the public finance assumptions.

The main parameters of the model with their chosen values are shown in the following table:

$\beta = 0.4252$	Equivalent to a yearly discount rate of 0.99
$\gamma = 0.169$	From De la Croix and Doepke (2004)
$\delta = 0.6$	From De la Croix and Doepke (2003)
$\varepsilon = 0.1$	Arbitrary
$p = 0.5$	Arbitrary; rapid convergence to Normal distribution
$\alpha = 0.3$	Standard range in the literature

For the particular choice of utility function selected in this model, β represents the weight of old age consumption in the utility of each individual. The value chosen is equivalent to a yearly discount factor of 0.99. This is higher than the usual estimate of 0.96 more frequently used in the literature. The qualitative conclusions of the model are the same with both values but this higher β allows us to produce more reasonable interest rates. γ is an important parameter in this model since it represents the degree of altruism that ultimately leads parents to worry about their children education. The value of 0.169 is taken from De la Croix and Doepke (2004). They give an alternative value of 0.271 in De la Croix and Doepke (2003) but the first one is preferred for the same reasons expressed in the choice of β. The same source is used for the choice of 0.6 as the value for δ, the elasticity of human capital with respect to educational spending. The value of the capital share ($\alpha = 0.3$) is taken in the usual range employed in the literature. The values chosen for the technology parameters A and F, which are just scale parameters, are respectively 50 and 1. Finally, the values for ε and p are taken arbitrarily. They affect the shape of the distribution of human capital and their effect on the aggregate behavior is identical to that of F_t. $\varepsilon = 0.1$ means that productive human capital producers are 10% more effective than the rest whereas $p = 0.5$ is chosen so that the distribution of historical family growth rates approaches a normal distribution more quickly.

The population growth rate assumed is of 0.34% per year. This last number represents the European Union population growth during the 90's (EUROSTAT 2002 Yearbook). This population growth and technology parameters, together

with the fiscal policy explained below give rise to a per capita output growth of 1.85% per year and a yearly long-term interest rate of 4.73%.

4.1.2 Public finance assumptions

The baseline scenario is calibrated for a case in which the budget is balanced every period and there is no net public debt. By zero net public debt, we mean that the government is neither a net lender to, nor a net borrower from, the other sectors of the economy. This does not mean that the government has zero liabilities but that government liabilities are equal to government assets. Although net government debt is the relevant concept in the model at hand, it departs from the statistical definition of general government gross debt as the consolidated liabilities of the ESA 95 general government sector. The other fiscal policy parameters have been set as follows:

$\tau = 0.6$	Marginal tax rate
$\tilde{\eta} = -0.2$	Consistent with an effective tax rate on labor income of 40%
$\theta = 0.2196$	Consistent with a pension expenditure ratio over GDP of 8%
$g = 17.6228$	Consistent with public consumption of 20% of GDP

The marginal tax rate on labor income (τ) and the lump-sum tax/transfer ($\tilde{\eta}$) have been chosen with a view to replicating both the effective tax rate and the high marginal tax wedge on labor income in the European Union. The marginal tax rate set at 60% ($\tau = 0.6$) captures the high marginal tax wedge on labor income in the European Union, where progressiveness in the statutory tax rates on personal income is moderated by a wide range of tax allowances and tax credits and the proportionality of social contributions (Joumard, 2002). Martinez-Mongay (2000) indicates that the effective tax burden on labor in the euro area was close to 40% in 1999, slightly higher than the effective tax burden in the European Union. These estimates are corroborated by those calculated by Mendoza *et al.* (1994) for Germany, France, Italy and the United Kingdom. The difference between the European Union and the euro area is attributable to the low effective tax rate on labor in the United Kingdom (about 25%). The choice of the lump-sum transfer ($\tilde{\eta} = -0.2$) is driven by the estimates for the effective tax burden on labor ($\tau + \tilde{\eta} = 0.4$). Alternatively, this calibration can be justified by the fact that taxes (13%) plus social contributions (15%) represent 28% of the GDP of the European Union for the period 1995-2000 (EUROSTAT 2002 Yearbook). Knowing this and given the assumption of a Cobb-Douglas production function, we can calculate:

$$0.28 Y_t = (\tilde{\eta}_t + \tau_t) \, w_t H_t = (\tilde{\eta}_t + \tau_t) \, (1 - \alpha) \, Y_t$$

$$(\tilde{\eta}_t + \tau_t) = \frac{0.28}{1 - \alpha} = 0.4$$

As for the pension parameter ($\theta = 0.2196$), its value has been generated endogenously from the choice of all the rest of parameters, which have been

chosen to obtain a pension expenditure to GDP ratio of 8%. This ratio was the actual number for the European Union in 2000. Under budget balance, we have:

$$\theta_t w_{t-1} H_{t-1} = (\tilde{\eta}_t + \tau_t) w_t H_t - G_t$$

We can then compute the pension expenditure to GDP ratio

$$
\begin{aligned}
\frac{\theta_t w_{t-1} H_{t-1}}{Y_t} &= \frac{(\tilde{\eta}_t + \tau_t) w_t H_t - G_t}{Y_t} \\
&= (\tilde{\eta}_t + \tau_t)(1 - \alpha) - \frac{G_t}{Y_t} = \\
&= 0.28 - 0.2 = 0.08
\end{aligned}
$$

Finally, $g = 17.6228$ is chosen so as to fix $\frac{G_t}{Y_t} = 0.2$, which is the value of government consumption expenditure in the European Union in the period 1995-2000.

In an alternative scenario, choosing $\beta = 0.2308$ (0.96 yearly discount rate), $\gamma = 0.271$ and $\alpha = 1/3$, and with fiscal policy parameters $\tilde{\eta} = -0.18$ and $\theta = 0.2360$ (which ensure again that the pension expenditure ratio over GDP is 8%), we generate a per capita output growth of 1.93% per year and a yearly interest rate of 7.88%.

Calibrating a two or three period overlapping generations model is a delicate exercise: the limited number of parameters leaves relatively little room for maneuver, possibly making the baseline calibration highly sensitive to change in assumptions. The sensitivity of our baseline calibration to changes in parameters can be observed in the following table, in which only one parameter differs from the baseline scenario in each variant:

	Growth Rate	Interest Rate	Debt ratio	Pension Expenditure
Baseline	1.85%	4.73%	0%	8%
$\beta = 0.4$	1.80%	4.90%	-0.01%	8.12%
$\beta = 0.45$	1.90%	4.57%	0.11%	7.89%
$\gamma = 0.15$	1.70%	4.49%	-0.36%	8.38%
$\gamma = 0.2$	2.07%	5.06%	0.44%	7.50%
$\alpha = 0.25$	1.75%	4.18%	1.39%	8.84%
$\alpha = 1/3$	1.92%	5.12%	-0.63%	7.47%
$\varepsilon = 0.05$	1.80%	4.65%	-0.13%	8.14%
$\varepsilon = 0.2$	1.97%	4.88%	0.24%	7.74%
$p = 0.25$	1.80%	4.65%	-0.13%	8.14%
$p = 0.75$	1.91%	4.80%	0.12%	7.87%
$\tilde{\eta} = -0.18$	1.78%	4.99%	0.93%	8.19%
$\tilde{\eta} = -0.22$	1.95%	4.42%	-1.39%	7.79%
$\theta = 0.2$	1.82%	4.78%	0.57%	7.35%
$\theta = 0.2360$	1.88%	4.68%	-0.51%	8.53%

4.1.3 Steady states: existence and stability

The existence and characterization of steady states depend critically on the assumptions about the fiscal policy that the government is implementing.

First of all, let us assume that the government fixes taxes and pension benefits, as well as government consumption as a fraction of GDP and lets the economy work by itself. The dynamics of such an economy would be governed by the following dynamic system:

$$k_{t+1} = \frac{\sigma_t\left(w_t, R_{t+1}\right) - b_t}{(1+p\varepsilon)(1+n_t)\Phi_t\left(w_t, R_{t+1}\right)}$$

$$b_{t+1} = \frac{R_{t+1}b_t + \theta_{t+1}w_t}{(1+p\varepsilon)(1+n_t)\Phi_t\left(w_t, R_{t+1}\right)} - \left(\tilde{\eta}_{t+1} + \tau_{t+1}\right)w_{t+1} + g_{t+1}$$

The first question is whether or not such an economy will converge to some steady state. To study this, we can fix all the parameters and assume $k_{t+1} = k_t = k$ and $b_{t+1} = b_t = b$. After solving for b in the second equation, we obtain a single non linear equation in k. The representation of such an equation is shown in figure 1.

Steady state of the baseline calibration
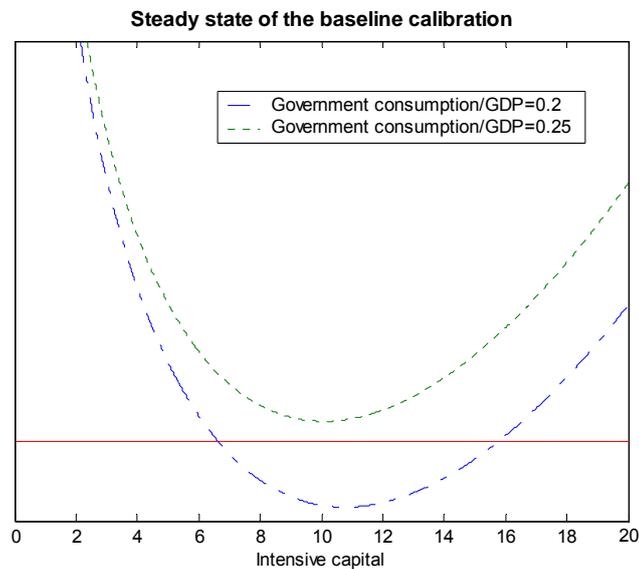
Figure 1:

The intersection of the dash line and the x-axis represents the set of steady states for the baseline calibration. There are two steady states resulting from

this choice of parameter values. The existence of steady states is fragile and crucially depends on fiscal policy. For example, increasing government consumption from 20% to 25% of GDP is sufficient to disrupt the long-term economic equilibrium, as it can be seen on figure 1. Similar results can stem from excessive public expenditure ratios and either excessive or insufficient taxation. With our specification of the economy, steady states are multiple (two), unless they do not exist, except for the special case of a tangency solution. From these two steady states, the first one is associated with dynamic efficiency (under-accumulation of capital), while the second is in general in the area of dynamic inefficiency.

The dynamics of the system can better be observed in the following phase diagram, which corresponds to the baseline calibration (see figure 2). First, the two steady states identified by figure 1 are located at the intersection of the curves $k_{t+1} = k_t$ and $b_{t+1} = b_t$. Second, we can observe what is the associated net debt to GDP ratio for each of the two steady states: zero for the first steady state (it was calibrated for this to be the case) and a net lending position of 19.89% of GDP for the second one.

Phase diagram of the baseline calibration
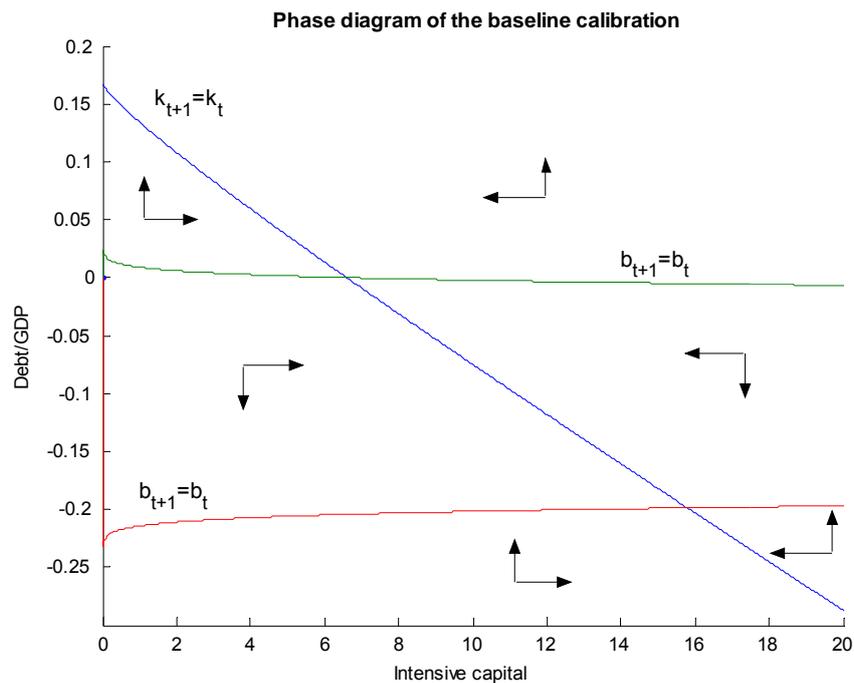
Figure 2:

It can be seen from the diagram that the first steady state is unstable. This means that, for any initial intensive capital value, there is only one debt

level that would lead the economy to this first steady state, characterized by under-accumulation of capital. This steady state would be stable in the saddle path sense, with a saddle path slightly decreasing in the net debt ratio in the proximity of the steady state. Debt, however, is a stock variable and there is no mechanism in this economy that can set the debt ratio in the saddle path so that a small perturbation will lead the economy to diverge from the steady state. We can notice that if the perturbation leads to a debt level below the saddle path, the economy, evolving according to its own laws of motion, will converge to the second steady state. However, if the perturbation leads to a higher debt level, the economy will inexorably diverge to reach a 0 capital level, unless some kind of discretionary fiscal policy is adopted or new countervailing shocks happen.

As for the second steady state, it is situated to the right of the golden rule intensive capital level. The golden rule intensive capital level is defined as the solution to the equation:

$$R = (1+n)(1+\rho) = (1+p\varepsilon)(1+n)\Phi(w,R)$$

For the baseline calibration, this value is 14.5925, which is lower than the second steady state, situated at a level of intensive capital of 15.8052. At this steady state, thus, the economy is in an area of dynamic inefficiency, as defined in Cass (1965). The growth rate of the economy (in per capita terms) at this steady state is 2.53% per year with a yearly real interest rate of 2.62%. The pension expenditure ratio is now reduced to 6.57% of GDP.

Why do we not consider so carefully this stable steady state? The reason, in addition to the fact that it is associated to an unusually high net lending position of general government, is the number of studies that have established how extraordinary is that an economy is in a dynamically inefficient steady state (Mankiw, Romer and Weil (1992)). It must be noticed, though, that this steady state is robust to small perturbations. Even more, it can be said that once the economy is in this steady state, an unreasonably big shock is required to endanger fiscal sustainability. Another justification for not concentrating on the study of the over-accumulation steady state could be based on political economy arguments. Once governments start running budget surpluses, it is very unlikely that they will keep the same budget policy instead of spending their extra resources. A good example of this can be found in the United States debate about what to do with the fiscal surplus at the end of the 90's.

Coming back to the first steady state, theory tells us that if the initial conditions of the economy are such that we start from the first steady state, the economy should remain there forever. However, we claim that this steady state is inherently unstable, making fiscal sustainability very fragile. Suppose that the economy situates itself with a slightly higher debt to GDP ratio than that implied by the steady state. An example can be seen in figure 3. The initial conditions are set at the under-accumulation steady state plus some arbitrary small amount of additional government liabilities (0.1% of GDP). Such a small

Trajectory of the baseline economy starting from steady state
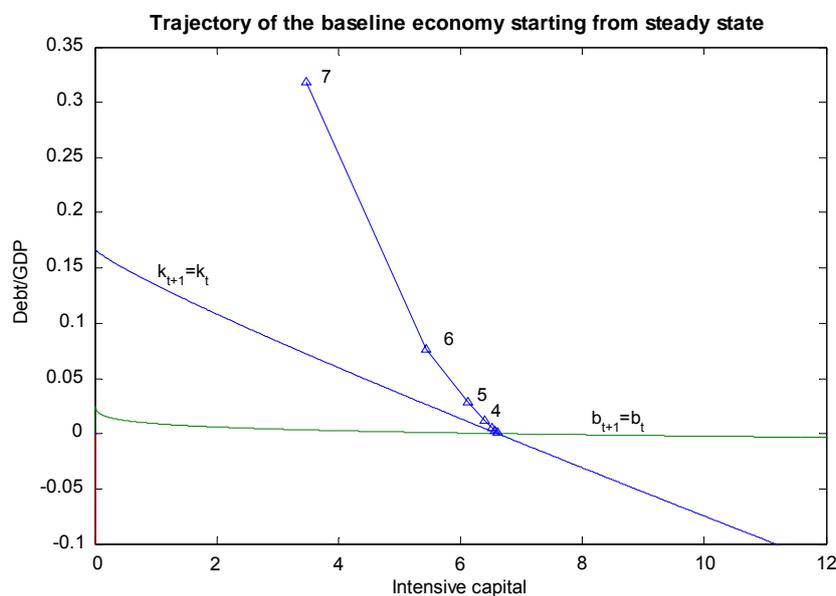
Figure 3:

perturbation (we can make it arbitrarily small) is sufficient to disequilibrate the system. If the same fiscal parameters are nevertheless kept, public debt starts to accumulate, very slightly in the first periods but at an increasing rhythm. In this example, the economy would crash at the eighth period. Remember that this is a discrete time model so that we have jumps in the phase diagram; the jump after the seventh period would lead us to negative capital levels, that is, a collapse of the economy.

It could be argued that this should not be a major concern since eight periods in this model represent actually 240 years. It is true that it is very extreme to assume that nothing else would change in 240 years, although some civilization may have collapsed out of fiscal problems in such a time span. However, the best way to reconcile this model with the behavior of real economies is to think that the economy is never in steady state and that the phase diagram is continuously changing, as the long-run steady state also depends on fiscal policy decisions. In this case, fiscal sustainability can be a more delicate issue than the previous numerical analysis would suggest, since the instability of the system greatly multiplies when the economy is far from the steady state.

Is there any way to limit this intrinsic instability of the first steady state? The government can proceed in two different ways in this setup. One solution is to adopt discretionary policies. For example, the pension system can be extremely generous during one generation putting the economy in an explosive

path but then policies can change so as to approximate the economy again to a new steady state.

A more systematic solution would come from adopting a fiscal rule. The adoption of a fiscal rule changes the dynamics of the system. The fiscal rule that will be considered in this paper is that of a fixed net debt policy. Suppose then that the government fixes a desired net debt ratio and adjusts its fiscal policy instruments. In the ensuing simulations, we will keep fixed the tax parameters and will use either the pension benefits (θ_t) or government consumption (G_t) as equilibrating variables. In either case, the dynamics of the system will be described by figure 4, which has been drawn for the case in which the targeted net debt ratio is 0 (budget balance). The steady state is unique here and it is globally stable, as we know is the case in general whenever we have budget balance. We obtain the same result for all feasible debt ratios. The set of feasible objective debt ratios under a fixed debt policy and their associated intensive capital values are depicted in figure 5. Any policy aiming at sustaining a debt level over the depicted line would lead the economy to a negative capital stock or, in other words, would not be sustainable.

Capital dynamics for a fixed debt policy (b=0)
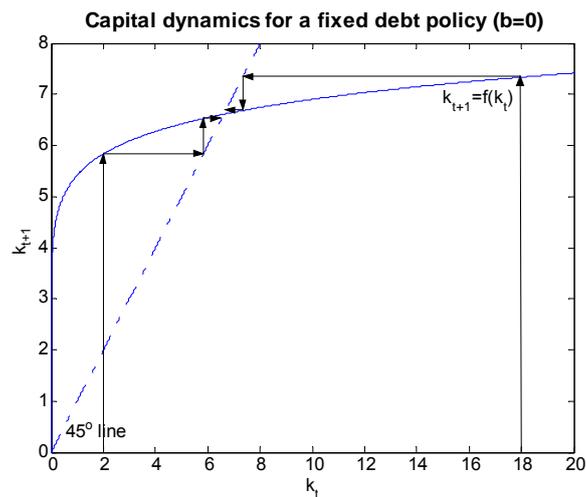
Figure 4:

There are of course intermediate solutions between the two extremes of keeping the same fiscal structure (figure 3) or adopting a fiscal rule (figure 4 or 5). On the theory side (see for example Schmitt-Grohe and Uribe (2004)), fiscal rules usually set the tax revenue as an increasing function of government liabilities. The adjustment speed of this function to some target levels is then chosen by the government so as to minimize some kind of loss function or, equivalently, to maximize national welfare. Studying optimal fiscal policy or even the optimal fiscal rule goes beyond the scope of this paper, but the effect of alternative

Dynamics of different fixed debt policies
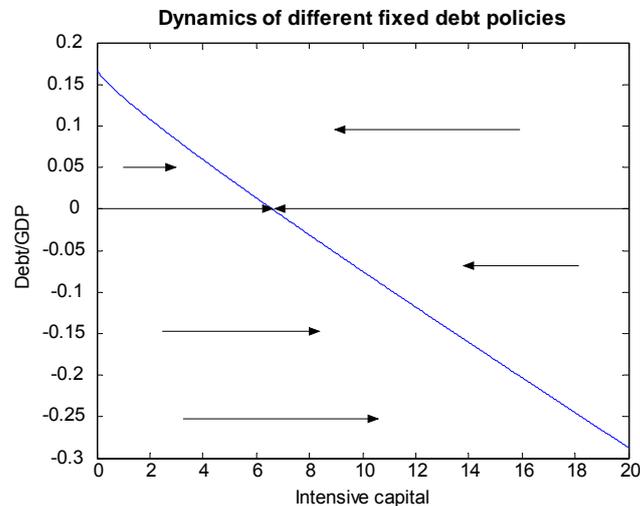
Figure 5:

policy scenarios can be appreciated through the following experiment. Take the example of figure 3 in which the economy starts from the steady state capital level but with a small amount of public debt equivalent to 0.1% of GDP. Now suppose that instead of not reacting to the fiscal crisis, the government decides to revert to a fiscal rule (one implying zero debt in this case) at some point in time. Figure 6 shows what the effects of this policy on the average utility[3] level in the economy depending on when the government decides to recourse to the fiscal rule. The costs of delaying the response to fiscal problems are reflected in a high cost of fiscal stabilization during the period in which the economy reverts to the rule and in the fact that the economy settles in a lower path of growth after stabilization than the one that would have been attained if the stabilization policy had been undertaken in time.

In this experiment, the way in which fiscal stabilization is obtained is by reducing the level of public expenditure so as to go back to a level of zero public debt. As a result, an indirect measure of the cost of fiscal stabilization at different points in time is given by the reduction of the percentage of public expenditure over GDP that has to be given up in order to achieve it at different points in time. For example, suppose that the government decides to "stabilize" after period 1 in which the level of public debt has been of 0.1% of GDP. In that case, public expenditure needs to go from 20% of GDP to 19.77% in period 2 (it immediately goes back to 20% again in period 3 once the stabilization has been obtained). If "stabilization" took place after period 2, public expenditure

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[3]Average utility is obtained by assuming that $u(g_t) = \ln(g_t)$ so that the total utility of any given individual would be:
$$U_t = (1 - \beta)(\ln(c_t) + \ln(g_t)) + \beta(\ln(d_{t+1}) + \ln(g_{t+1})) + \gamma \ln(E_t[\omega_{t+1}])$$

would need to be reduced to 19.48% of GDP for one period. After period 3, the number would be 18.80% of GDP. After period 4, we would need to reduce public expenditure to 17.14% of GDP. After 5, the reduction would have to go till 12.36% of GDP. Finally, for "stabilization" to take place after period 6, a negative ratio of public expenditure to GDP would be required of -11.84% of GDP, which amounts to say that going back to a zero public debt level in one period would be impossible just by reducing the level of public expenditure and either other measures (reducing pension benefits) or more gradual policies would be required.

Average utility

Legend:
- No stabilization
- Stabilization after period 1
- Stabilization after period 2
- Stabilization after period 3
- Stabilization after period 4
- Stabilization after period 5
- Stabilization after period 6

Periods
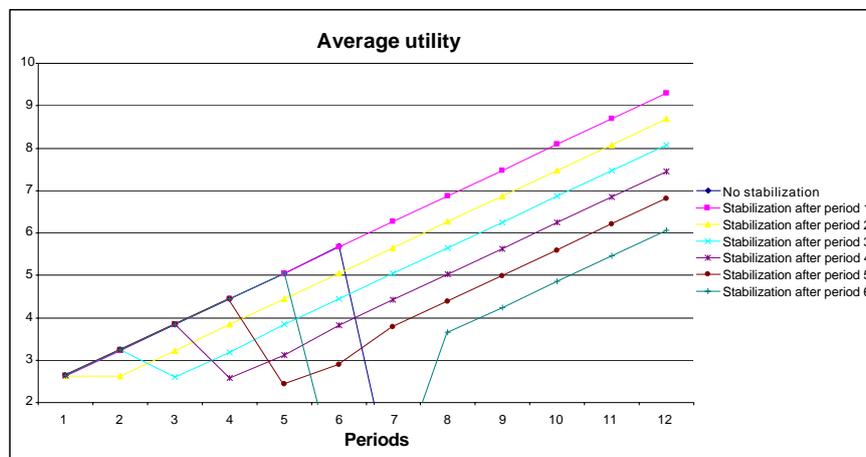
Figure 6:

Another way of stabilizing would be to choose a different target level after the shock takes place. Continuing with the previous example, one possibility would be to target a new debt level once the shock takes place. Suppose that the government decides to stabilize at a level of 0.1% debt over GDP once the shock takes place. In the end, this is a matter of intergenerational redistribution. The brunt of the cost will come in the period in which stabilization is decided. Setting a higher debt target would imply a lower capital steady state that would be reflected in lower growth rates and lower utility for the rest of the periods.

4.2   Fiscal sustainability and fiscal consolidation

In this part, we present one example of how fiscal sustainability can be affected by changes in some of the parameters that we have considered exogenous. The issue of fiscal sustainability has become increasingly popular in Europe in relation with the possibility or impossibility of keeping unchanged the public pension arrangements in view of the estimates of population decrease over the next 50

years. We will analyze what is the prediction of our model for this particular case.

4.2.1 Demographic shock

The demographic shock consists of a change in the population growth rate. The EPC-Budgetary challenges posed by population ageing projects (page 12) that the total population of the European Union (before enlargement) will go from 376.4 million in 2000 to 364.2 million in 2050. This gives us a yearly population decrease of 0.07% per year that we must compare to the yearly population growth of 0.34% assumed in the baseline scenario. The way we introduce the shock is by assuming that -0.07% will be the generation growth rate after two periods (in the new steady state), that is: $n_{t+2} = (1 - 0.0007)^{30}$. The transition is modeled by choosing n_{t+1} so that the total population after two periods is $\left(\frac{364.2}{376.4}\right)^{\frac{60}{50}}$ times that of the baseline[4].

In principle, we can just compare the new dynamic system that is generated when we apply the shock without changing any of the policy parameters. This is done in figure 7. In figure 8, we focus on the dynamically efficient steady state. We can see that the new steady state is characterized by higher intensive capital level and a net government debt of (-0.27%). The per capita growth rate associated with the new steady state would be 2.15% (compare to 1.85% in the baseline) and the interest rate 4.57% (compare to 4.73%). Keeping the same policy parameters, the pension expenditure ratio would increase to 8.28% of GDP (0.28% of GDP higher than the baseline).

The reason why the new steady state features higher growth comes from the structure of the model. Growth is generated here by the investment in human capital. A lower population growth rate allows each parent to invest more on each of the remaining children so that they will have a higher per capita human capital that will show up as a general higher growth rate in all the relevant variables in the economy.

Importantly, it is misleading and dangerous to compare directly two steady states. By looking at figure 8, we can realize that the economy would not go to the new steady state. In fact we can run a simulation to see what would happen if policy parameters remain unchanged. The result can be seen in figure 9, illustrating how the economy would crash five periods after the population shock happens under no policy changes.

It could be of interest seeing the evolution of some significant variables in the economy along the dynamic path. This can be observed in figures 10, 11, 12 and 13. It is remarkable that the effect on the interest and growth rate is not really alarming at any point and it is very difficult to deduce from it that the economy is heading to a disaster. In this respect, neither growth nor interest rates seems to be useful indicators of fiscal sustainability by themselves.

Phase diagram after a demographic shock (from 0.34% to -0.07% yearly population increase)

Figure 7:

Phase diagram after a demographic shock (from 0.34% to -0.07% yearly population increase)
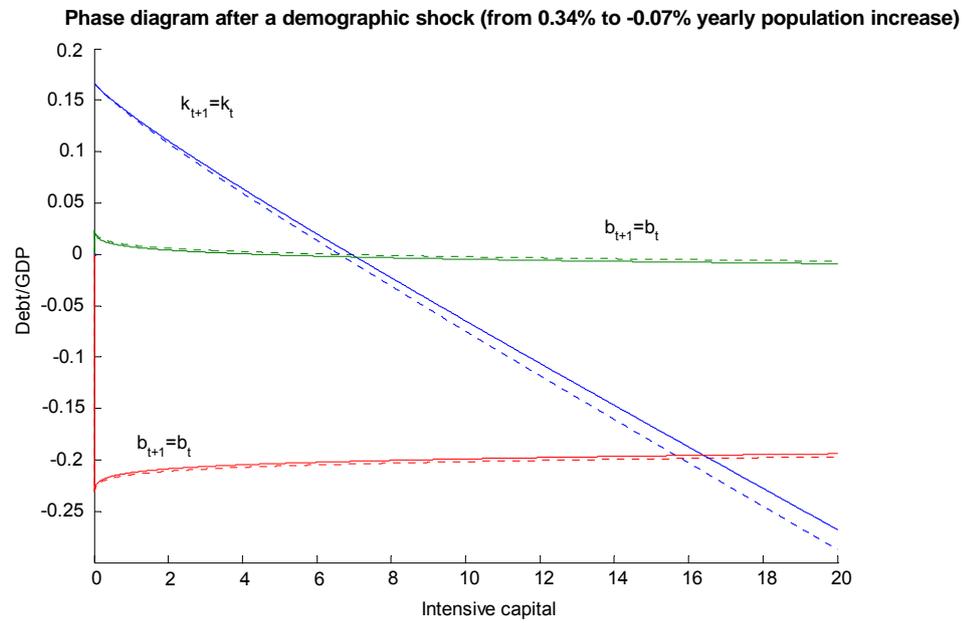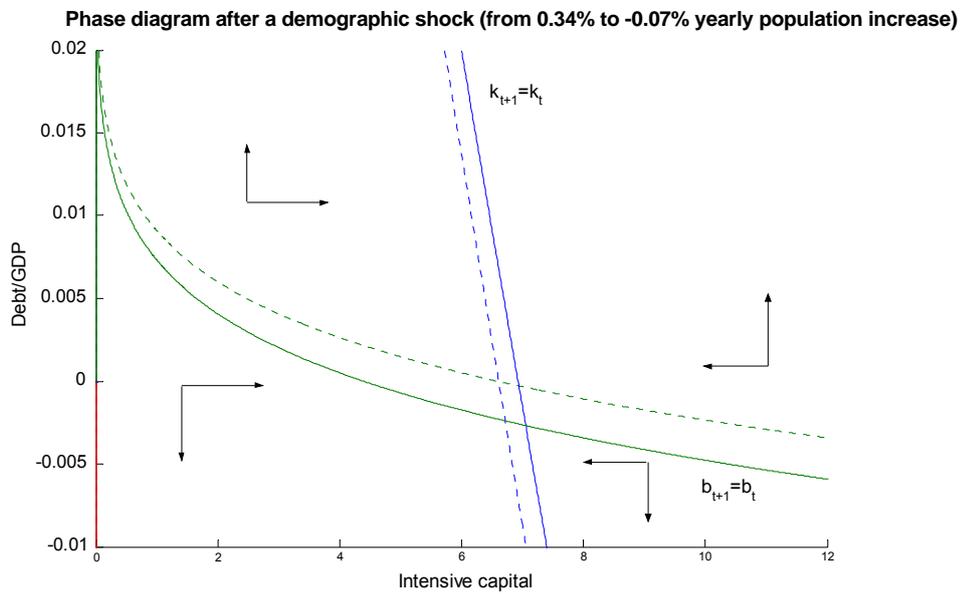
Figure 8:

Figure 9:

Figure 10:

Effects of the demographic shock

Figure 11:

Effects of the demographic shock

Figure 12:

Effects of the demographic shock on pension expenditure over GDP
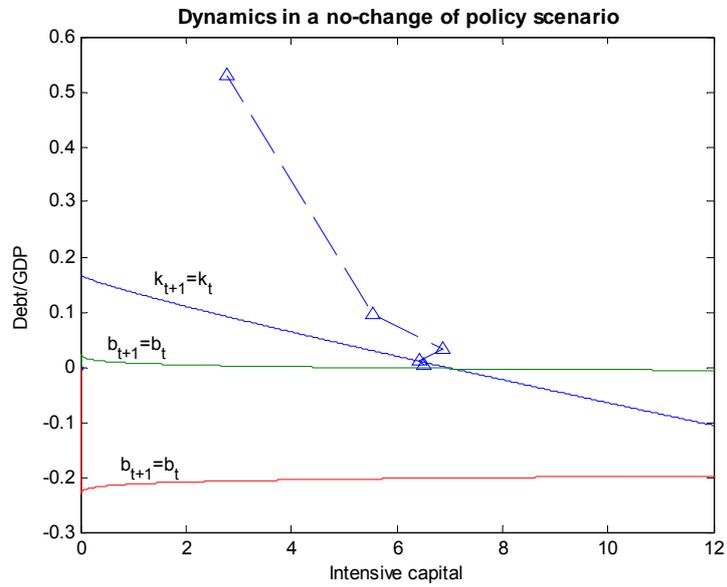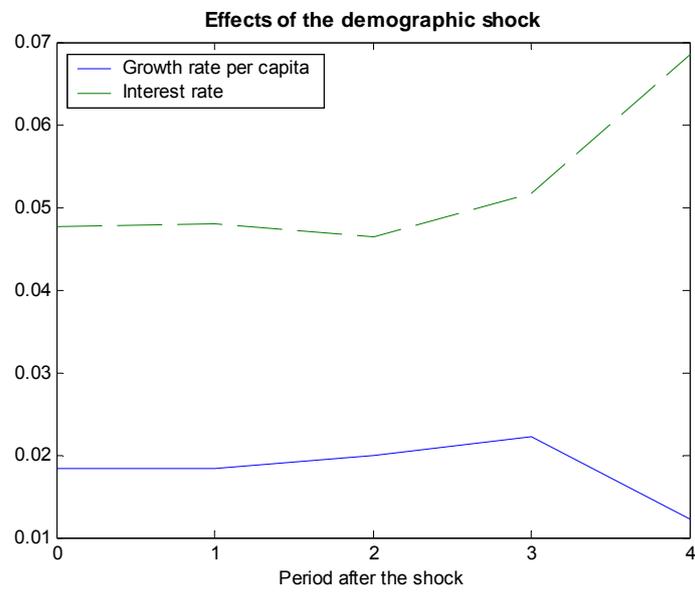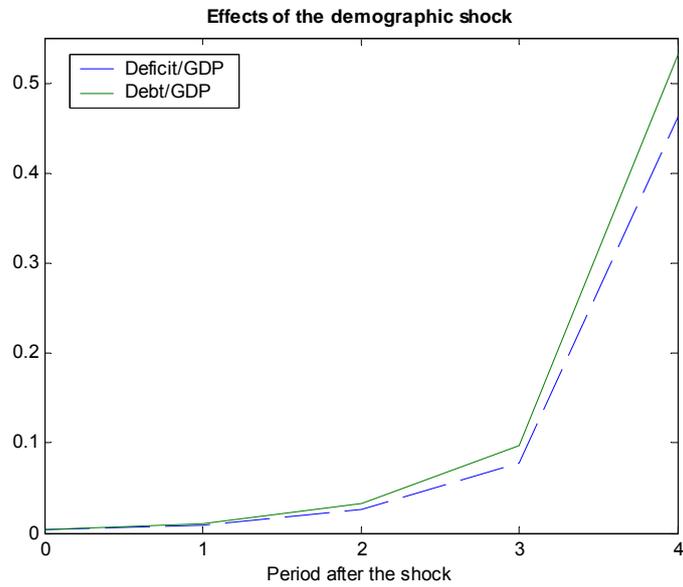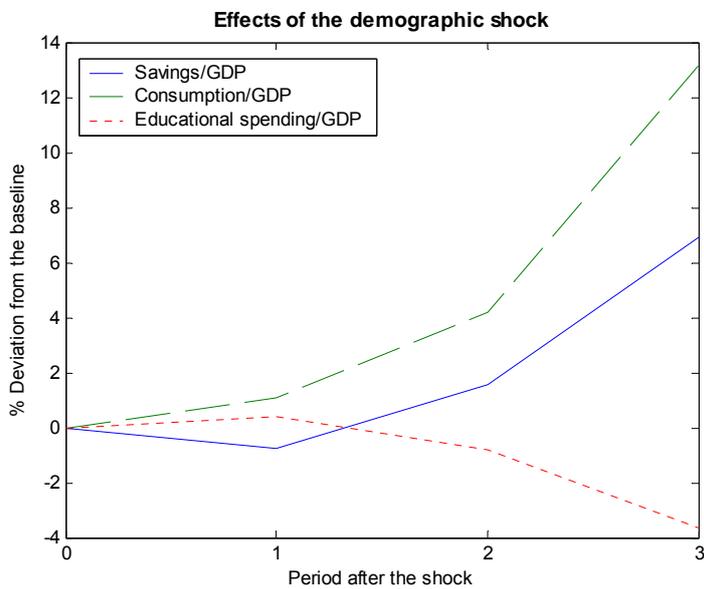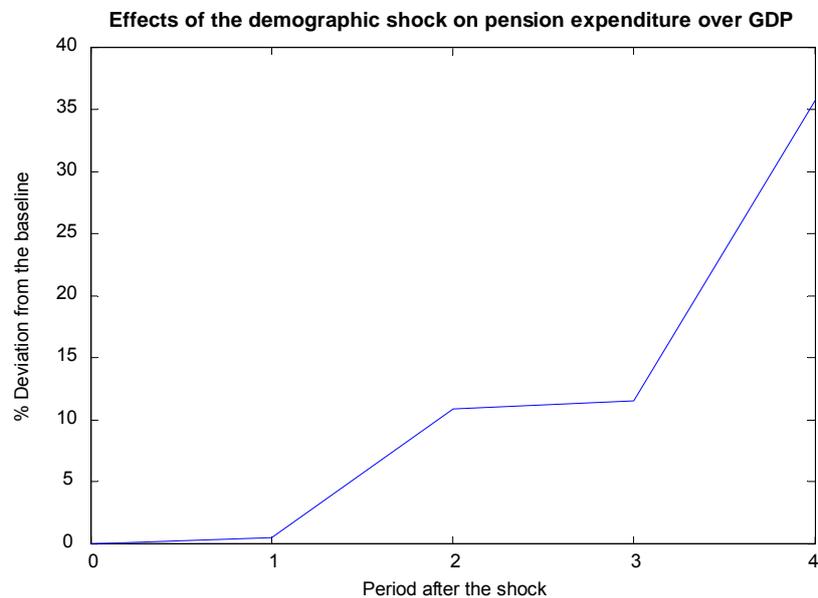
Figure 13:

The picture is different if we concentrate on other variables. For example, the behavior of debt and deficit is clearly explosive (figure 11). Figure 12 shows that the main ratios in the economy: consumption to GDP (around 62.7% in the baseline), savings to GDP (14.4% in the baseline) and private education to GDP (2.9%), also react importantly to the demographic shock. Finally, the ratio of pension expenditure to GDP also increases along the dynamics caused by the shock (figure 13).

We discussed in the previous section that fiscal rules could prevent the economy from entering in unsustainable dynamics. In this case, we will first analyze what is the effect of adopting a fiscal rule of budget balance (zero net debt). When we want to impose budget balance or any other fixed debt policy, we can use either government consumption or pension benefits as equilibrating variables. If we use the latter (pension reform), most of the relevant variables in the economy stay unchanged (the ratios presented in the previous charts). There are of course repercussions on the growth rate per capita and interest rate that can be observed in figure 14.

The change in the pension benefits does not affect the pension expenditure to GDP ratio, but it is nevertheless interesting to know by how much pension benefits have to decrease to attain budget balance. This calculation is presented

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[4] We elevate to the power of 60 because we take two complete 30-year periods.

Effect of the demographic shock under budget balance
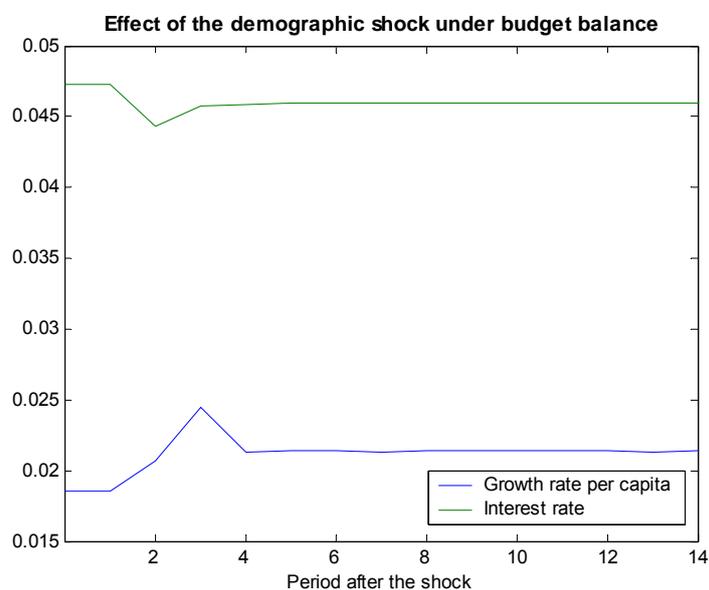
Figure 14:

in figure 15. Benefits can remain unchanged for the first period but must be reduced by more than 8% in the following one. Once the economy stabilizes in the new steady state, benefits can be upgraded slightly but would remain 4% below their original level in spite of the higher growth rate associated to the new steady state.

Another possibility for the government to ensure budget balance is to reduce government consumption, while maintaining pension benefits. The effect is exactly the same that was observed in figure 14 for the interest and growth rate per capita. However, the reaction of other macroeconomic variables is different, as it can be seen in figures 16 and 17. Figure 16 shows that the consumption to GDP ratio increases 0.6% in the new steady state with respect to the baseline, whereas the savings ratio goes down by some 0.5% of GDP with respect to the baseline and educational spending with respect to GDP also increases by 0.28% with respect to the baseline. In figure 17, we can see how the pension expenditure to GDP ratio evolves when we keep benefits unchanged. It increases by 3.9% of GDP with respect to the baseline in the new steady state. Finally, the necessary reaction of the government consumption to equilibrate the economy implies a decrease by 1.6% of GDP with respect to the baseline.

Evolution of social security benefits after the shock to equilibrate the budget
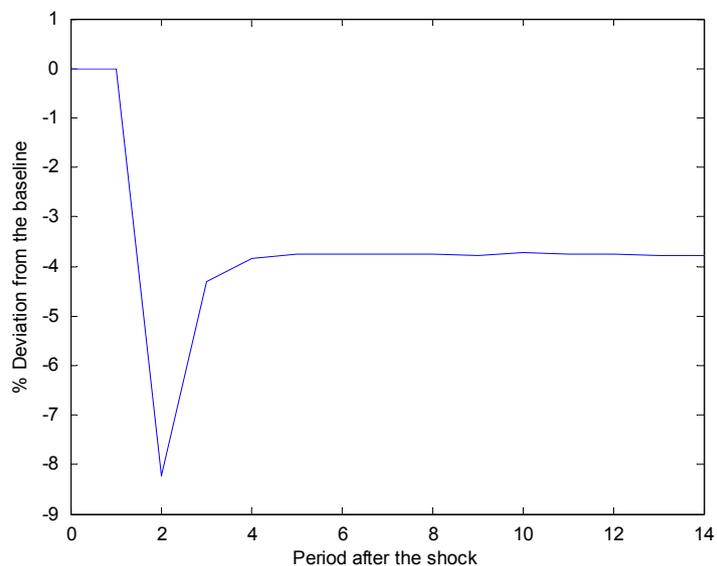
Figure 15:

Effect of the demographic shock under budget balance

Figure 16:

Effect of the demographic shock under budget balance
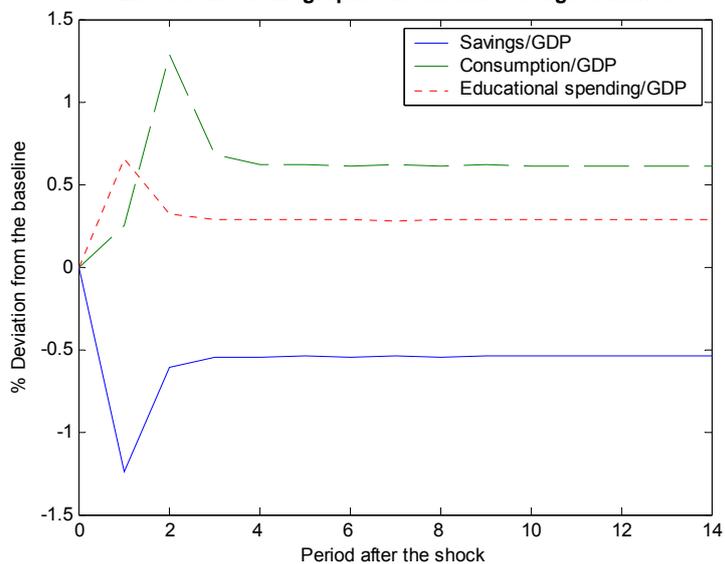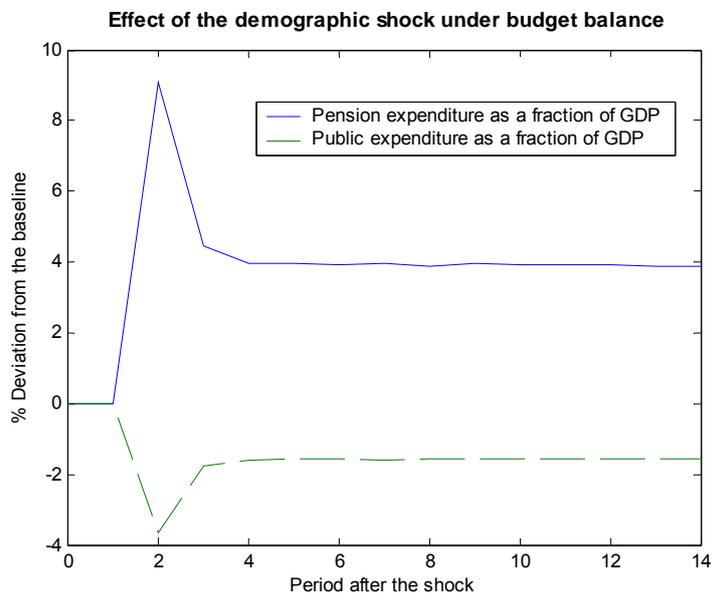
Figure 17:

4.3 Inequality

Before concluding the paper, some comments must be made about the distributional results of the model. These results are completely analogous to those obtained by Glomm and Ravikumar (1992) and by De la Croix and Doepke (2003 and 2004). This is why we will not comment very extensively on them.

In figures 18, 19 and 20, we can see the evolution of the distribution of normalized (divided by the mean) levels of human capital. Even if everybody starts from the same level of human capital, the assumed law of motion of human capital increases the heterogeneity of the population in every period. Figure 18 shows the distribution after the first period. Figure 19 presents the distribution after 100 periods and figure 20 shows how the distribution converges to a lognormal distribution after 200 periods. Finally, we can see how inequality increases over time in the evolution of the coefficient of variation (figure 21).

We can do the same analysis with respect to the evolution of family growth rates of human capital. As above, figure 22 shows the distribution of growth rates of human capital after the first period. Figure 23 shows the same distribution after 100 periods and, finally, figure 24 presents how the distribution of family growth rates of human capital converges to a normal distribution. However, contrary to the inequality in levels of human capital, the inequality
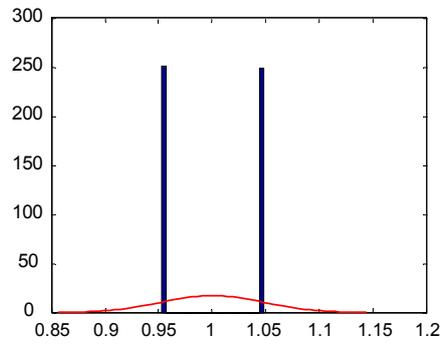
Figure 18:
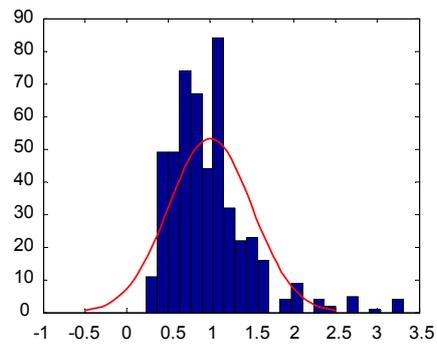
Figure 19:
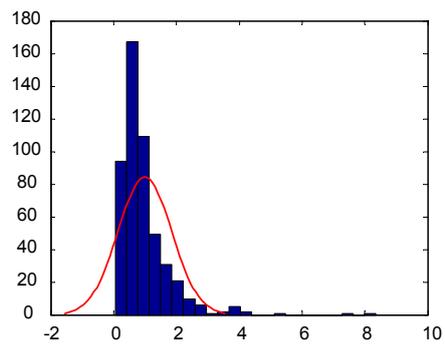
Figure 20:
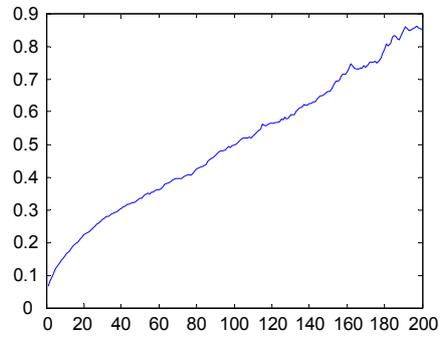
Figure 21:

in family growth rates of human capital (measured again as the coefficient of variation) decreases with time, as it can be seen in figure 25.
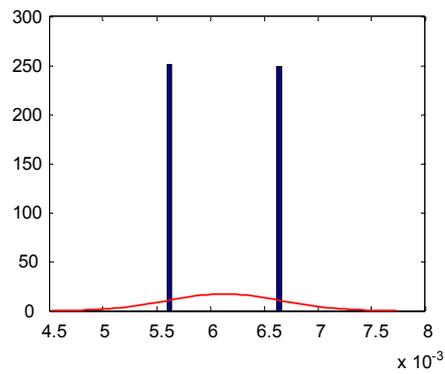
Figure 22:
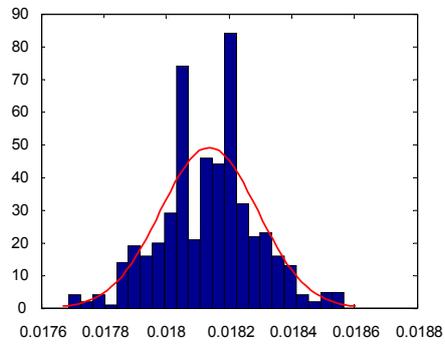
Figure 23:
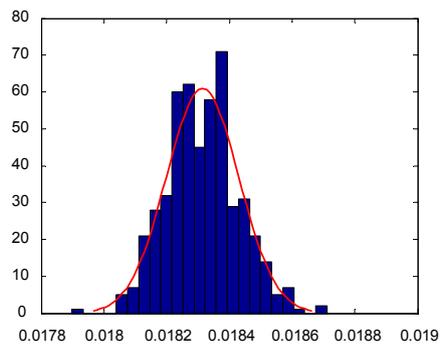
Figure 24:
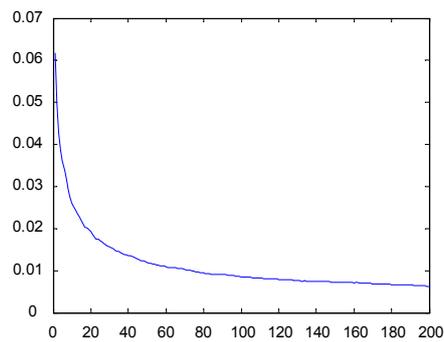
Figure 25:

5 Conclusion

The objective of this paper was to analyze fiscal sustainability in an overlapping generations economy in which government imbalances are allowed. The conclusions that can be drawn from this study are the following.

First, it is not sufficient to concentrate on steady state analysis. Fiscal policy is sustainable if the economic equilibrium exists in all periods. This is a general obvious point that we want to remark here because of its particular relevance in this model. For example, in the demographic shock studied in this paper, it is clear that the new steady state is characterized by higher growth rates and lower interest rates than the one before the shock. However, the economy, let to its own dynamics without any change in fiscal policy, would never attain the new more favorable steady state but would instead be led to an unsustainable situation.

Second, it must be emphasized that fiscal sustainability can be extremely fragile. All the indicators in an economy may remain at "normal" levels, while the economy is actually in an unsustainable situation.

Third, fiscal policy rules seem to be necessary to ensure fiscal sustainability. The government cannot remain passive in front of the shocks that affect the economy but it must react to those shocks in a way that situates the economy on sustainable paths. Under fiscal rules, such as a constant debt policy in the framework developed in this paper, the dynamic properties of the economy change and the equilibrium becomes stable under "normal" shocks.

These are the main conclusions that can be drawn from such a study of fiscal sustainability. It would be interesting, as a direction for future research, to check the robustness of these conclusions to different time structures, which would also ease the calibration of the model. Finally, the modeled economy is rich enough as to assess the optimality of different fiscal policies.

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